# ACCELERATION VOLTAGE PATTERN FOR J-PARC RCS 

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## Abstract

The calculation code for the acceleration voltage pattern is usually based on the differential equation of the longitudinal synchrotron motion. We have developed the method based on the forward-difference equation which satisfies the synchronization with the bending magnetic field ramping accurately.

## INTRODUCTION

In the case of the Rapid Cycling Synchrotron (RCS), since the bending magnetic field is usually ramped by sinusoidal wave form using resonant network system and the cycle is fast. It is very important for the longitudinal particle tracking code to calculate the synchronous particle motion precisely, because the change of the energy, the rf frequency and the amplitude of the rf voltage is large. Furthermore, when we perform the particle tracking including the field error of the bending magnet [1], it is very complicated to calculate the energy gain of the synchronous particle turn by turn. In the case of the J-PARC RCS, the bending magnetic field is not pure sinusoidal wave, but have some higher harmonic components.

The phase stability of the accelerated particles around the synchronous particle has been investigated well, but the motion for the synchronous particle is not so much investigated so far. The basic principle of the synchrotron [2,3] is that the energy gain $\Delta E$ per a turn has a relation with the bending magnetic field $B$ as

$$
\begin{equation*}
\Delta E=V_{\mathrm{rf}} \sin \phi_{s}=2 \pi \rho R \frac{d B}{d t} \tag{1}
\end{equation*}
$$

where $V_{\mathrm{rf}}$ is the amplitude of the acceleration voltage, $\phi_{s}$ is the synchronous phase, $\rho$ is the bending radius and $R=C / 2 \pi$ is the average radius which relates to the circumference $C$ of the accelerator ring. This is a well-known equation to know how much acceleration voltage the rf cavity feeds into the particle. In order to get the acceleration voltage from the energy gain, other conditions should be considered to define $\phi_{s}$. The J-PARC RCS employs the condition that the longitudinal emittance $\varepsilon_{L}$ and the momentum filling factor $P_{f}$ should be kept constant during almost all of the acceleration period [4]. The calculation code "RAMA $[5,6]$ " manages such things.

However, when we use the acceleration voltage calculated turn by turn from eq.(1) for the longitudinal particle

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tracking code, we found that the energy gain is slightly different from the expected value. This comes from the fact that eq.(1) consists of the differential of the bending magnetic field, whereas what we have to know is the difference of the bending field turn by turn.

Furthermore, in the conventional longitudinal particle tracking code, the frequency of the accelerating voltage $f_{\mathrm{rf}}$ is proportional to the revolution frequency of the synchronous particle $f_{\text {rev }}$ as

$$
\begin{equation*}
f_{\mathrm{rf}}=h f_{\mathrm{rev}} \tag{2}
\end{equation*}
$$

where $h$ is a harmonic number. It seems to be indifferent to the synchronous phase $\phi_{s}$, that is, such frequency pattern does not satisfies the energy gain $V_{\mathrm{rf}} \sin \phi_{s}$ turn by turn accurately.

In this paper, we investigate how to get the acceleration voltage pattern, which is based on the difference equation of the longitudinal motion and the acceleration voltage tracking scheme.

## DIFFERENCE EQUATION FOR THE SYNCHRONOUS PARTICLE

Let us consider the energy gain per turn as shown in Fig. 1. The bending magnetic field has a value of $B_{n}$ on n-th turn at the time $t_{n}$. The particle passes through the rf cavity at that time, then gets the energy $\Delta E=V_{\mathrm{rf}} \sin \phi_{s}$. The total energy of the particle changes from $E^{n-1: n}$ to $E^{n: n+1}$, where the suffix $n-1: n$ shows "from (n-1)-th turn to n -th turn" and $n: n+1$ shows "from n -th turn to ( $\mathrm{n}+1$ )-th turn". The momentum of the particle becomes $p^{n-1: n}$ and $p^{n: n+1}$. The revolution period of the particle from ( $\mathrm{n}-1$ )-th turn to n-th turn is $T_{\mathrm{rev}}^{n-1: n}$ and $T_{\mathrm{rev}}^{n: n+1}$ from n -th turn to $(\mathrm{n}+1)$-th turn, respectively.

It is considered that there is the relation between the momentum and the bending magnetic field as

$$
\begin{align*}
p^{n-1: n} & =e B_{n} \rho  \tag{3}\\
p^{n: n+1} & =e B_{n+1} \rho \tag{4}
\end{align*}
$$

where $e$ is the elementary charge. These equations mean that, for example, the particle gets the energy on $n$-th turn as that momentum matches with the bending magnetic field on $(\mathrm{n}+1)$-th turn. This is the principle of the synchrotron that the energy gain per turn should match with the change of the bending magnetic field turn by turn.

From the relativistic relations,

$$
\begin{align*}
p^{n: n+1} & =m_{0} c \beta^{n: n+1} \gamma^{n: n+1}  \tag{5}\\
E^{n: n+1} & =m_{0} c^{2} \sqrt{1+\left(\beta^{n: n+1} \gamma^{n: n+1}\right)^{2}} \tag{6}
\end{align*}
$$

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Figure 1: The schematic view of the energy gain turn by turn.
where $m_{0}$ is the rest mass of the particle, $\beta$ is the ratio of the velocity to the speed of light $c$ and $\gamma=1 / \sqrt{1-\beta^{2}}$, respectively. Combining the eq.(4), (5) and (6), we obtain that

$$
\begin{equation*}
E^{n: n+1}=m_{0} c^{2} \sqrt{1+\left(\frac{B_{n+1} \rho}{m_{0} c}\right)} \tag{7}
\end{equation*}
$$

On the other hand, the revolution period from $n$-th turn to the $(n+1)$-th turn is

$$
\begin{equation*}
T_{\mathrm{rev}}^{n: n+1}=\frac{C}{\beta^{n: n+1} c} \tag{8}
\end{equation*}
$$

where we assume the orbit of the synchronous particle traces the center of the ring. From the another expression of eq.(6),

$$
\begin{equation*}
E^{n: n+1}=\gamma^{n: n+1} m_{0} c^{2} \tag{9}
\end{equation*}
$$

Combining the eq.(5), (8) and (9) we obtain

$$
\begin{equation*}
E^{n: n+1}=\frac{B_{n+1} c^{2} \rho}{C} T_{\mathrm{rev}}^{n: n+1} \tag{10}
\end{equation*}
$$

Since the equations (7) and (10) should have the same value,

$$
\begin{align*}
& m_{0} c^{2} \sqrt{1+\left(\frac{B_{n+1} \rho}{m_{0} c}\right)^{2}} \\
& =\frac{B_{n+1} c^{2} \rho}{C} T_{\mathrm{rev}}^{n: n+1} \tag{11}
\end{align*}
$$

Since the time $t_{n+1}=t_{n}+T_{\text {rev }}^{n: n+1}$, then we rewrite eq.(11) using the revolution period as

$$
\begin{align*}
& m_{0} c^{2} \sqrt{1+\left(\frac{B\left(t_{n}+T_{\mathrm{rev}}^{n: n+1}\right) \rho}{m_{0} c}\right)^{2}} \\
= & \frac{B\left(t_{n}+T_{\mathrm{rev}}^{n: n+1}\right) c^{2} \rho}{C} T_{\mathrm{rev}}^{n: n+1} \tag{12}
\end{align*}
$$

This is the equation of the synchronous particle motion by the forward-difference method. We can get the revolution period from eq.(12), and when the bending magnetic field pattern is defined, the revolution period is also defined uniquely.

Once the revolution period is obtained, we can calculate the energy gain per turn as

$$
\begin{align*}
\Delta E= & V_{\mathrm{rf}} \sin \phi_{s}=E^{n: n+1}-E^{n-1: n} \\
= & m_{0} c^{2} \sqrt{1+\left(\frac{B\left(t_{n}+T_{\mathrm{rev}}^{n: n}\right) \rho}{m_{0} c}\right)^{2}} \\
& -m_{0} c^{2} \sqrt{1+\left(\frac{B\left(t_{n}\right) \rho}{m_{0} c}\right)^{2}} \tag{13}
\end{align*}
$$



Figure 2: The difference between the conventional method and the forward-difference one.

In the longitudinal particle tracking code, the conventional method to track the synchronous particle is that the energy gain per turn is obtained from eq.(1) at first, then the revolution period $T_{\text {rev }}^{n}$ is calculated according to the energy at the time $t_{n}$. It is not confirmed precisely that the period is really synchronized with the bending magnetic field. In the case of the forward-difference method, the revolution period $T_{\text {rev }}^{n: n+1}$ is obtained associated with the bending magnetic field and the energy gain per turn.

The Figure 2 shows the difference between the conventional method and the forward-difference one. The condition of the J-PARC RCS is used, that is, the proton is accelerated from 181 MeV to 3 GeV within 20 ms . The thick line in the upper figure is the total energy difference, dot line is the frequency difference, and the lower figure shows the time difference on each turn. In these case, the final energy becomes 3.0 GeV exactly in the case of the forward-difference, though 2.9998 GeV in that of conventional one.

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## ACCELERATION VOLTAGE TRACKING

Let us consider "acceleration voltage tracking" as shown in Fig. 3, where $h=2$ is chosen for J-PARC RCS as an example. The synchronous particle is accelerated by the acceleration voltage with the frequency of $f_{\mathrm{rf1}}^{n: n+1}$ on the phase $\phi_{s 1}^{n: n+1}$ at $n$-th turn. The suffix " 1 " shows the first bucket. In this case, there is no assumption for $f_{\mathrm{rf} 1}^{n: n+1}$, this means $f_{\text {rf1 }}^{n: n+1} \neq h f_{\text {rev }}$ is acceptable. The particle circulates the ring with the revolution period $T_{\text {rev } 1}$ calculated from eq.(12), then the particle sits on the phase $\phi_{s 1}^{n+1: n+2}$ at $(\mathrm{n}+1)$-th turn.


Figure 3: The schematic view of the relation between the frequency change and the synchronous phase.

In order to satisfy this condition, $f_{\mathrm{rf} 1}^{n+1: n+2}$ on ( $\mathrm{n}+1$ )th turn should have a relation, which consists of $f_{\text {rf1 }}^{n: n+1}$, $f_{\mathrm{rf} 2}^{n: n+1}$ and $T_{\mathrm{rev}}^{n: n+1}$ as

$$
\begin{align*}
T_{\mathrm{rev} 1}^{n: n+1}= & \frac{1}{2} \frac{1}{f_{\mathrm{rf} 1}^{n: n+1}}-\frac{\phi_{s 1}^{n: n+1}}{2 \pi f_{\mathrm{rf1}}^{n: n+1}}+\frac{1}{f_{\mathrm{rf} 2}^{n: n+1}} \\
& +\frac{1}{2} \frac{1}{f_{\mathrm{rf1}}^{n+1: n+2}}+\frac{\phi_{s 1}^{n+1: n+2}}{2 \pi f_{\mathrm{rf1}}^{n+1: n+2}} \tag{14}
\end{align*}
$$

This equation shows the frequency of the acceleration voltage is defined so that its wave length should trace the change of the synchronous phase turn by turn. In order to calculate the $(\mathrm{n}+1)$-th turn's frequency $f_{\mathrm{rf1}}^{n+1: n+2}$ from the n-th turn's one, we have to consider about $f_{\text {rf2 }}^{n: n+1}$ for the second bucket. We can calculate the revolution period for the second bunch $T_{\text {rev2 }}$ from eq.(12), then we also obtain the same kind of relation for $f_{\text {rf2 }}^{n+1: n+2}$ as

$$
\begin{align*}
T_{\mathrm{rev} 2}^{n: n+1}= & \frac{1}{2} \frac{1}{f_{\mathrm{rf} 2}^{n: n+1}}-\frac{\phi_{s 2}^{n: n+1}}{2 \pi f_{\mathrm{rf} 2}^{n: n+1}}+\frac{1}{f_{\mathrm{rf} 1}^{n: n+1}} \\
& +\frac{1}{2} \frac{1}{f_{\mathrm{rf} 2}^{n+1: n+2}}+\frac{\phi_{s 2}^{n+1: n+2}}{2 \pi f_{\mathrm{rf} 2}^{n+1: n+2}} \tag{15}
\end{align*}
$$

This equation includes $f_{\text {rf1 }}^{n+1: n+2}$, so eq.(14) and (15) should be solved simultaneously. The Figure 4 shows the difference between $f_{\text {rf1 }}^{n+1: n+2}$ and $h f_{\text {rev }}$ turn by turn in the case of the J-PARC RCS. Since the synchronous phase $\phi_{s}$ 04 Hadron Accelerators
increases until 15 ms , the frequency of the acceleration voltage pattern becomes a little bit higher than $h f_{\text {rev }}$.


Figure 4: The frequency difference between $f_{\mathrm{rf} 1}$ and $h f_{\text {rev }}$.

## SUMMARY

We investigate how to get the acceleration voltage pattern which is based on the difference equation of the longitudinal motion and the acceleration voltage tracking scheme. By using this scheme, the more accurate longitudinal particle tracking is performed, and it is very useful when we simulate under the conditions of unexpected difference between the acceleration voltage pattern and the bending magnetic field.

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