

Instabilities and Space Charge Resonances in High Intensity Ring Accelerators

Review of the theory with some 'new' applications

Contents:

- o Space charge effects and impedance sources
- o Landau damping and space charge
 - ✓ Transverse 'loss of Landau damping'
 - ✓ Landau damping due to octupoles
 - ✓ Resonance crossing with space charge
 - ✓ Longitudinal 'loss of Landau damping'
- o Microwave instabilities with space charge
- o Summary

Space charge and image charge effects

transverse and longitudinal space charge parameters



Transverse incoherent tune shift:

$$\Delta Q \propto -\frac{q^2 NR}{mB_f \beta_0^2 \gamma_0^3 \epsilon}$$

Transverse coherent tune shift ('space charge impedance')

$$\Delta \Omega_c \propto -i \frac{q^2 N}{m \gamma_0 Q_0} Z_{\perp}$$

$$Z_{\perp}^{sc} = -i \frac{Z_0 R}{\beta_0^2 \gamma_0^2 b^2}$$

(Ω : frequency of coherent beam center oscillations)

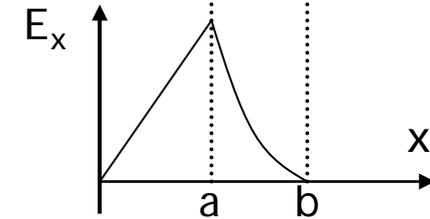
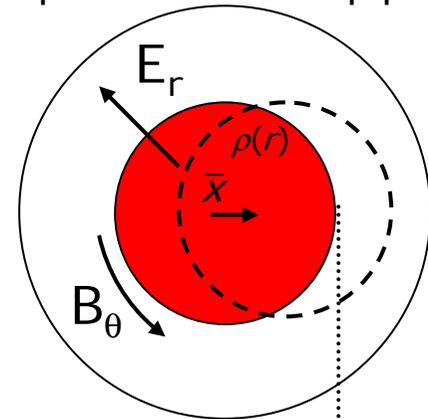
RF voltage reduction and longitudinal space charge parameter

$$V(\phi) = V_{rf}(\phi) + V_s(\phi) \quad V_s \propto -N \left| \frac{Z_P^{sc}}{n} \right| \frac{\partial I(\phi)}{\partial \phi} \quad (\text{space charge induced voltage})$$

$$\Sigma = \frac{1}{V_{rf} / V_s - 1} \approx \frac{V_s}{V_{rf}}$$

Relative incoherent synchrotron frequency shift $\frac{\Delta \omega_s}{\omega_s} \approx -\frac{1}{2} \Sigma$

transverse beam profile inside a pipe



Space charge and impedances in ring machines at a reference energy

	E [GeV]	ΔQ_v	Σ	impedances	remarks
SNS	1	-0.15	-	e-cloud, kicker, wall	1 ms accumulation uncontrolled loss < 10 ⁻⁴
CERN PS	1.4	-0.25	0.06	kickers	
SPS	26	-0.07	0	e-cloud, kickers	
SIS 18	0.01	-0.5*	0.4*	wall, kicker, rf	$\Delta p/p$ preservation
SIS 100 (FAIR project)	0.2	-0.3*	0.3*	wall, kicker, rf	1 s accumulation, $\Delta p/p$ preservation uncontrolled loss < 10 ⁻⁴

final bunch
compression
requirements.

*
performance goals !

Some general comments:

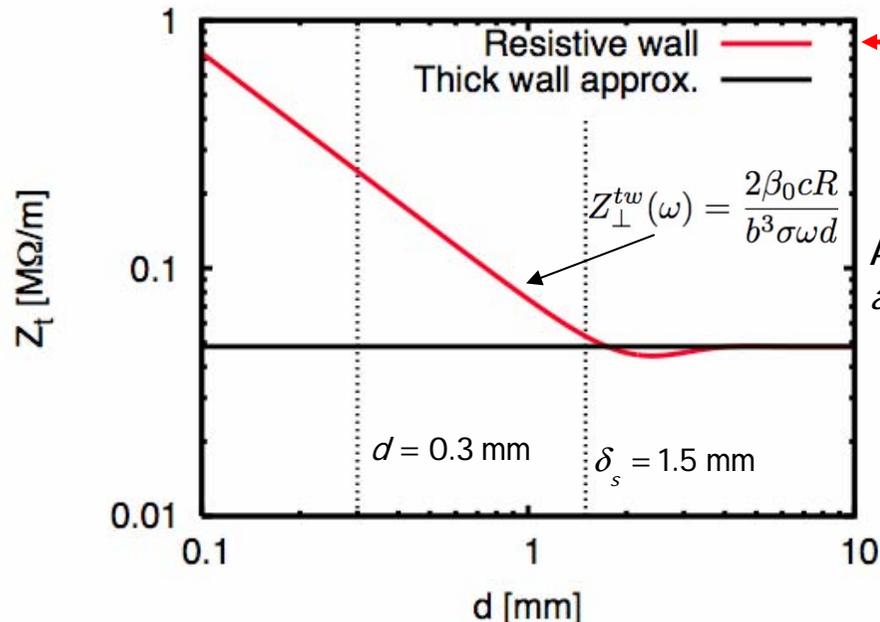
- o Space charge induced (ΔQ) resonance crossing can cause fast or gradual beam loss
- o Space charge alone (below transition energy) 'usually' does not cause coherent instabilities
- o However, instability thresholds and 'impedance budgets' may be modified by space charge.

This 'modification' of instability thresholds is one of the main topics in this review

Impedance sources

relevant for SIS 18/100 and other high intensity synchrotrons

- ✓ Transverse resistive (thin) wall impedance in fast ramping synchrotrons.



Resistive wall impedance in SIS 18 at 160 kHz.

Transmission coefficient as a function of d:

Al-khateeb, Hasse, et al., *Transverse coupling impedance in a smooth resistive pipe for arbitrary energies*, [THPCH034](#)

- ✓ Ferrite loaded kickers
 - o e.g. B. Doliwa, et al., *Numerical Impedance Calculations for the GSI SIS-100/300 Kickers*, [WEPOCH113](#)
- ✓ Magnetic alloy or ferrite loaded rf cavities
 - o e.g. P. Hülsmann, *Broadband Acceleration RF System for the SIS18 Upgrade*, GSI note (2004)
- ✓ 'Electron-clouds'
 - o G. Rumolo, et al., *Simulation study of TMCI threshold in the SPS*, [MOPLS096](#)

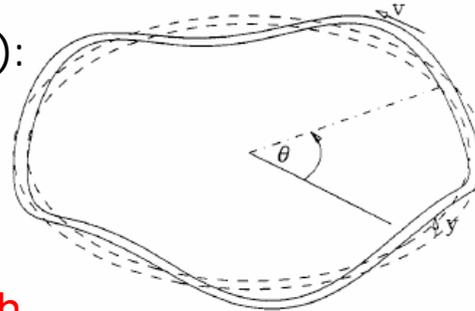
Transverse resistive wall instability with space charge

Example: coasting Ar¹⁸⁺ beam in SIS 18 at injection (11.4 MeV/u)

Transverse beam 'offset' oscillations $\bar{x}(\theta, t) = \hat{x} \exp(in\theta - i\Omega t)$

Coherent frequency shift ('cold beam'):

$$\Delta\Omega_c \propto -i \frac{q^2 N}{m\gamma_0 Q_0} Z_{\perp}$$



$$\Omega_0 = (4 - Q_v)\omega_0 = 2\pi 160\text{kHz}$$

Resulting 'cold beam' instability growth time due to resistive wall:

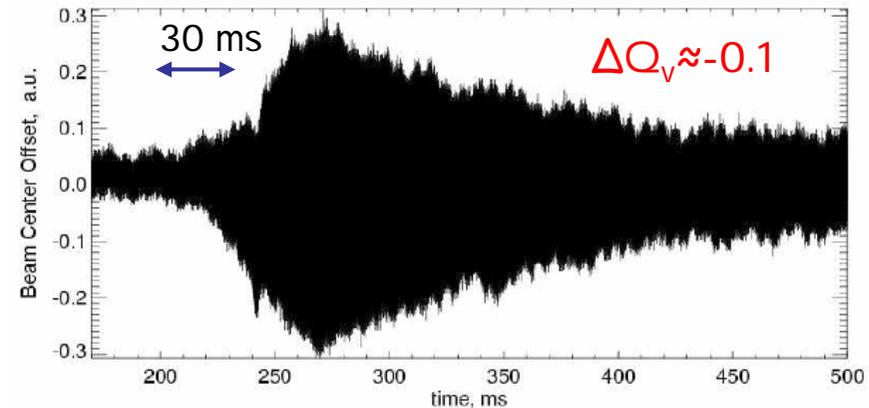
$$\tau_l = \left(\omega_0 \Im\Delta\Omega_c\right)^{-1} \approx 30\text{ ms}$$

Tune spread and damping rate:

$$\delta Q = \frac{S}{\omega_0} \frac{\Delta\rho}{\rho}; 0.001 \quad \tau_L = \left(S \frac{\Delta\rho}{\rho}\right)^{-1} \approx 1\text{ ms}$$

$$S = \omega_0 (\xi - (n - Q)\eta_0)$$

Beam offset vs. time observed in SIS 18



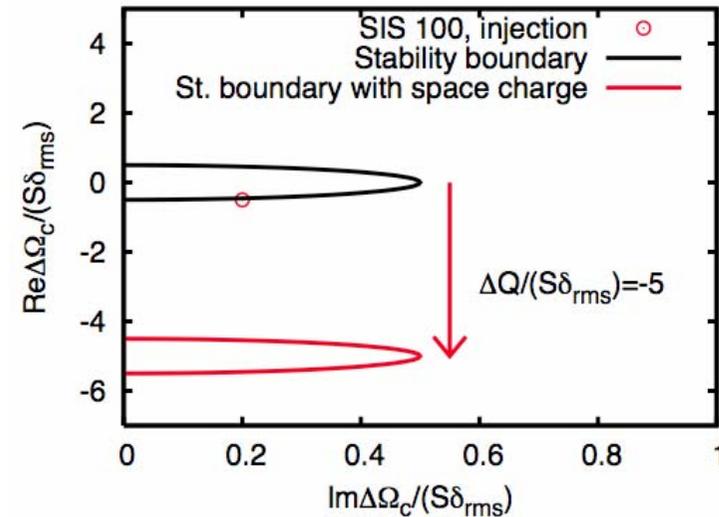
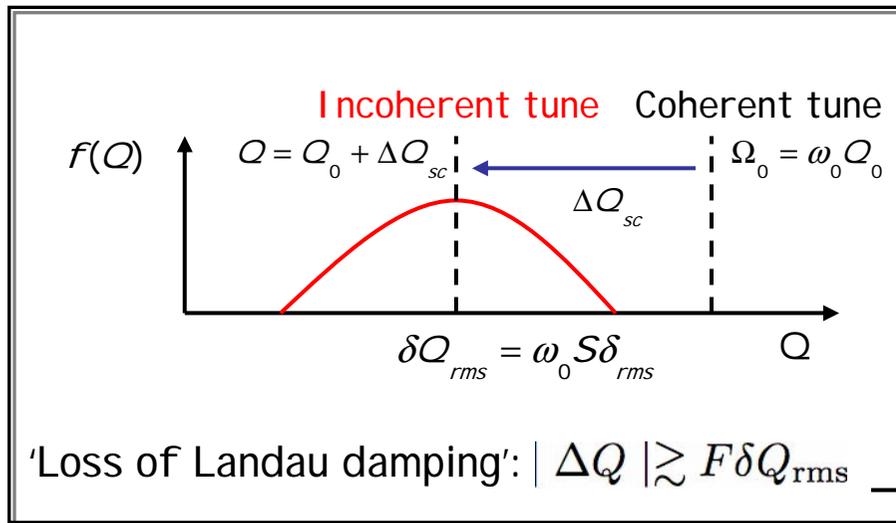
The space charge tune shift $\Delta Q_v \approx -0.1$ removes Landau damping and the beam reacts like a 'cold' beam.

Space charge induced loss of Landau damping and Application to SIS 100

Similar observations e.g. in the FNAL recycler ring:

K.Y.Ng, *Resistive-Wall Instability in the Fermilab Recycler Ring*, AIP Conf. Proc. 773, 365 (2005)

beam with rms momentum spread δ_{rms} and $\Delta Q_{sc} > 0$



Stability boundary ('circle'):

$$|\Delta \Omega_c - \omega_0 \Delta Q| \lesssim F S \delta_{rms}$$

Possible cures:

- ✓ increase momentum spread (not in SIS 18/100)
- ✓ (reactive) broadband feedback system (shifts coherent tune Ω)

o bandwidth: $f_{\min} = (1 - [Q]) f_0$ $f_{\max} = \frac{f_0 \Delta Q}{|\eta_0| F \delta_{rms}} \approx 100 \text{ MHz (SIS 100)}$

- ✓ increase tune spread using octupoles



Nonlinear space charge and octupoles

Stabilization of transverse instabilities

D. Möhl, H. Schönauer, Proc. IX Int. Conf. High Energy Acc. (1974)

M. Blaskiewicz, Phys. Rev. ST Accel. Beams 4, 044202 (2001)

E. Metral, F. Ruggiero, Proc. of EPAC 2004

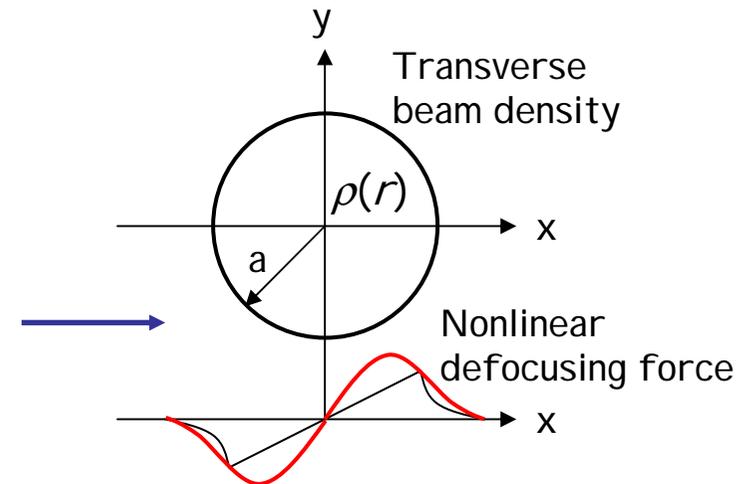
o The **tune spread induced by external nonlinear forces** (e.g. octupoles) can stabilize transverse instabilities.

o The **nonlinear component of the internal space charge defocusing force** also generates a tune spread.

o The amplitude dependent tune due to octupoles and space charge is

$$Q_x(\hat{x}, \hat{y}) = Q_0 - \underbrace{\Delta Q_x}_{\text{space charge}} G(\hat{x}, \hat{y}) + \underbrace{S_x^{ext}}_{\text{octupole}} \hat{x}^2$$

$$\text{with } G(\hat{x}, \hat{y}) \approx 1 - S_x^{sc} \hat{x}^2 - S_y^{sc} \hat{y}^2$$



Solution of dispersion relations predicts:

✓ the tune spread due to nonlinear space charge alone causes no Landau damping.

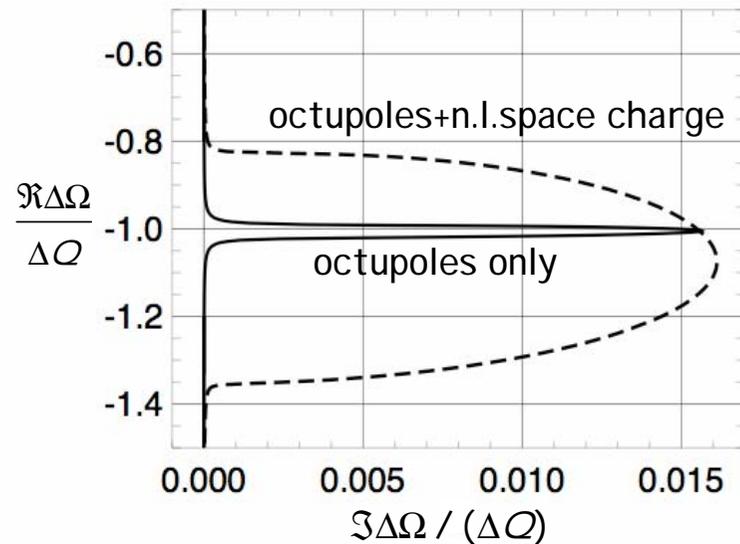
✓ octupole-induced damping is enhanced.

Landau damping due to nonlinear space charge and octupoles

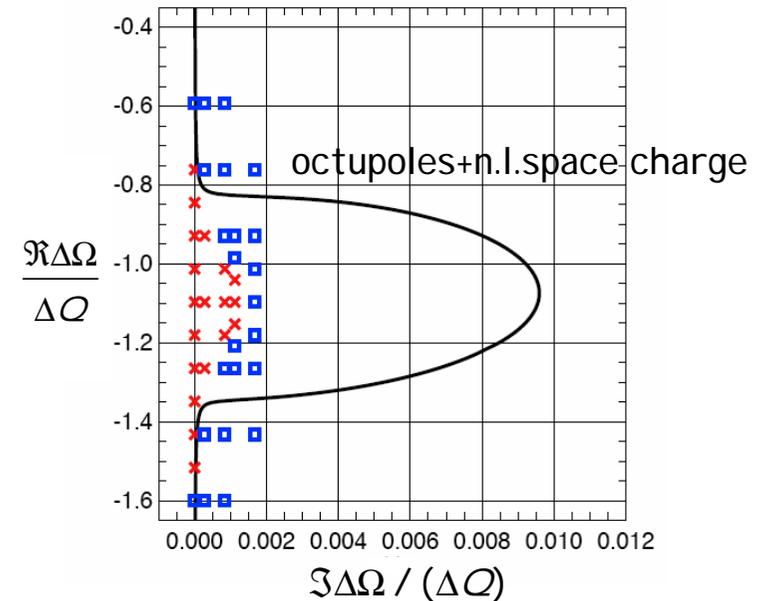
Dispersion relation and simulation scans

V. Kornilov, et al., Proc. of ICFA-HB2006 workshop (2006)

Stability boundaries from dispersion relation (Möhl, Schönauer):



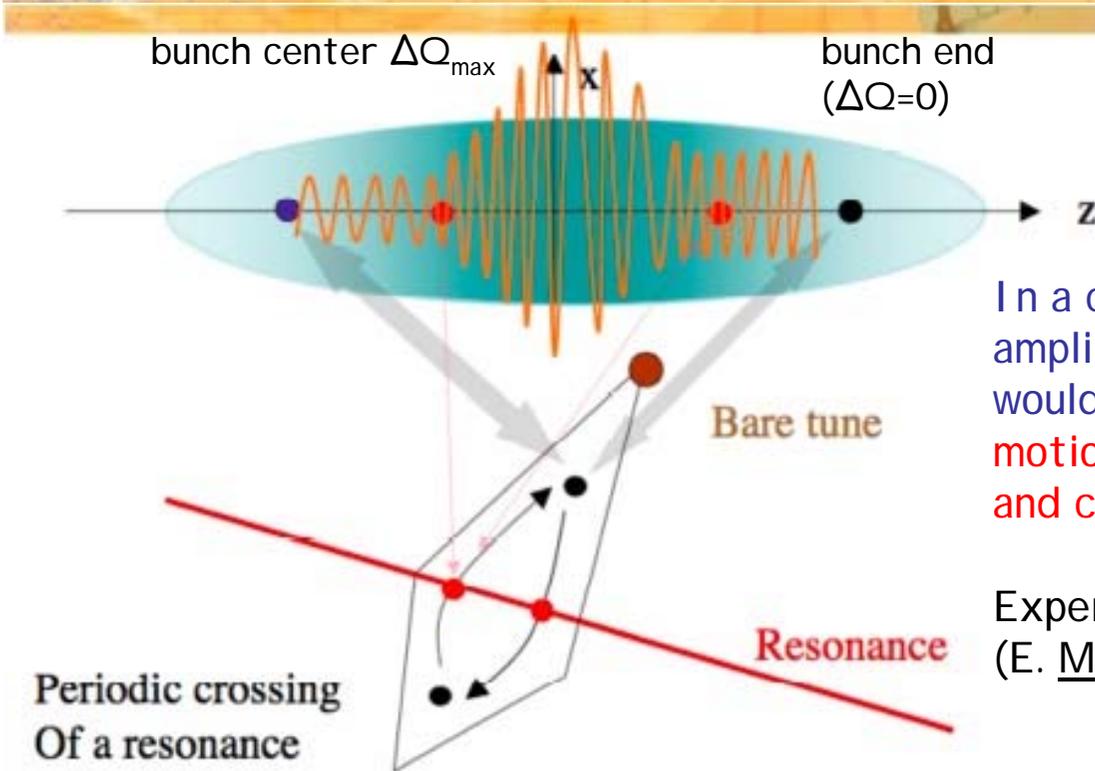
Simulation scan using the PATRIC tracking code with self-consistent space charge and impedances:



- For SIS 100 studies must be continued and extended to long bunches.
- Octupole-induced nonlinear resonances and corresponding beam loss ?

Octupoles and space charge

Gradual beam loss in bunches due to resonance trapping



G. Franchetti, et al., *Space charge induced resonance trapping in high-intensity synchrotrons*, [THPCH004](#)

In a coasting beam the detuning with amplitude caused by nonlinear space would limit the increase: **Synchrotron motion is essential for 'resonance trapping' and corresponding gradual beam loss.**

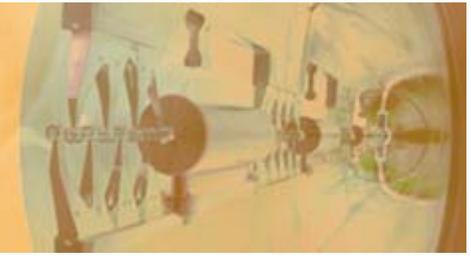
Experiments performed in the CERN PS (E. [Metral et al.](#)) support this loss mechanism.

- o Octupoles and nonlinear space charge cause **enhanced Landau damping** (previous slide).
- o However, their combined interaction can also cause **enhanced beam loss in bunches**.
- o In case of **SIS 100** studies are being performed in order to find **a optimized working point**:

G. Franchetti, et al., *Considerations for the SIS 100 high intensity working point*, [THPCH005](#)

Longitudinal dipole modes in rf buckets

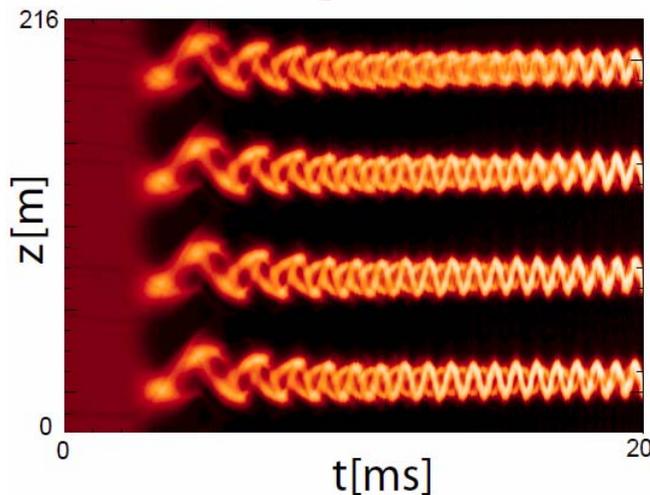
Loss of Landau damping due to space charge



Observation: Persistent dipolar or quadrupolar bunch oscillations above a threshold intensity.

e.g. A.Hofmann and F. Pedersen, IEEE Trans. Nucl. Sci. 26, 3526 (1979)

Persistent dipole oscillations in SIS 18: $\Sigma \approx 0.2$



Synchrotron frequency in a rf bucket ($S=1/16$):

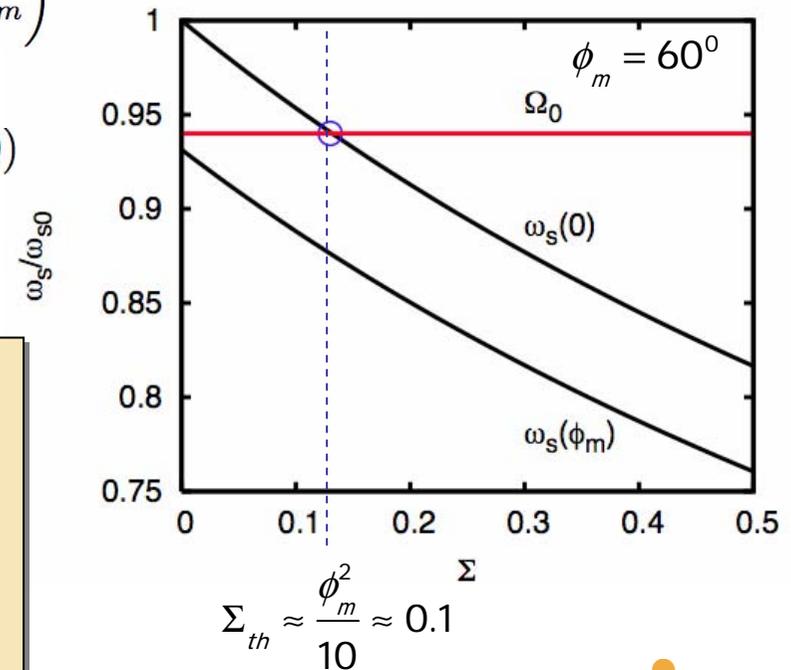
$$\omega_s(\hat{\phi}) = \frac{\omega_{s0}}{\sqrt{1+\Sigma}} (1 - S\hat{\phi}^2) \approx \omega_{s0} + \Delta\omega_s - S\hat{\phi}^2$$

Frequency of 'rigid' dipole oscillations (bunch length ϕ_m):

$$\Omega_0 \approx \omega_{s0} \left(1 - \frac{4}{5} S\phi_m^2\right)$$

Landau damping:

$$\omega_s(\phi_m) < \Omega < \omega_s(0)$$



Possible cures:

- + (reactive) feedback to decrease dipole frequency
- + inductive insert to increase synchrotron frequency
- double rf wave to increase frequency spread:

not effective with space charge ! See:

O. Boine-Frankenheim, T. Shukla, Phys. Rev. ST-AB (2005)

Longitudinal bunch stability with Space Charge

Stability boundary and simulation scans

B. Zotter, *Longitudinal Stability of Bunched Beams*, CERN SPS/81-19 (1981)

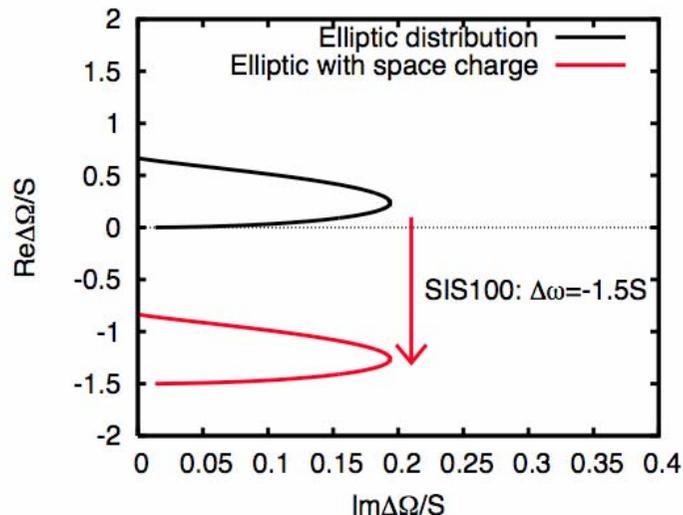
K.Y. Ng, *Comments on Landau Damping due to Synchrotron Frequency Spread*, FERMI LAB (2005)

Coherent frequency shift with effective dipole impedances:

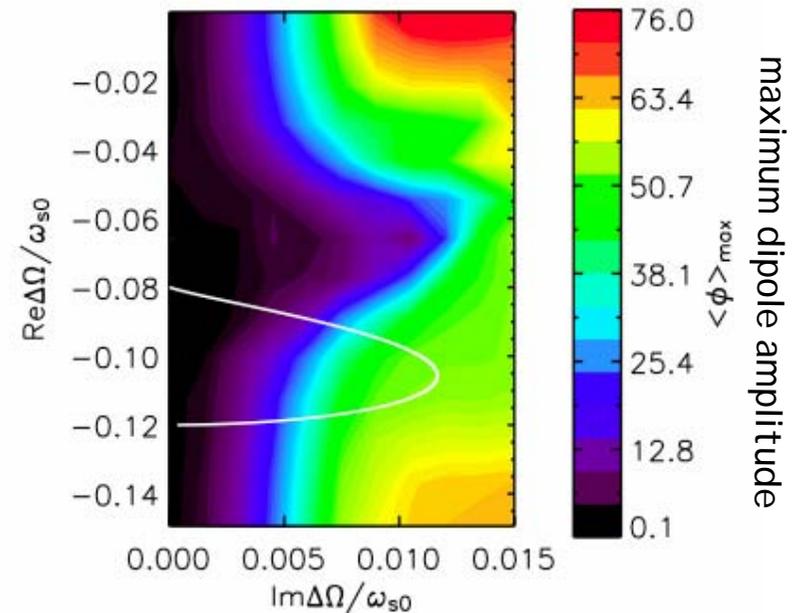
$$\Delta\Omega_c \approx \frac{i\omega_{s0}}{2} (Z_R^{\text{eff}} + iZ_I^{\text{eff}})$$

Dispersion relation for a nonlinear rf bucket

$$1 = -\pi(\Delta\Omega_c - \Delta\omega) \int \frac{df(\hat{\phi})}{d\hat{\phi}} \frac{\hat{\phi}^2 d\hat{\phi}}{\Omega - \omega_s(\hat{\phi})}$$



Simulation scan (LOBO code) performed with self-consistent space charge



The **complex interplay** of different effects requires a **validation** of the range of applicability of analytic stability boundaries.

Dipole instability in flat-topped bunches

'Pure' space charge driven instability

A. Hofmann, F. Pedersen, 1979: [Flat-topped bunches](#)

R. Baartman et al., PAC 1989: [Dipole instability in hollow bunches](#)

O. Boine-Frankenheim, T. Shukla, Phys. Rev. ST Accel. Beams 8 (2005)

Cures:

+ reactive feedback !

+ double rf !

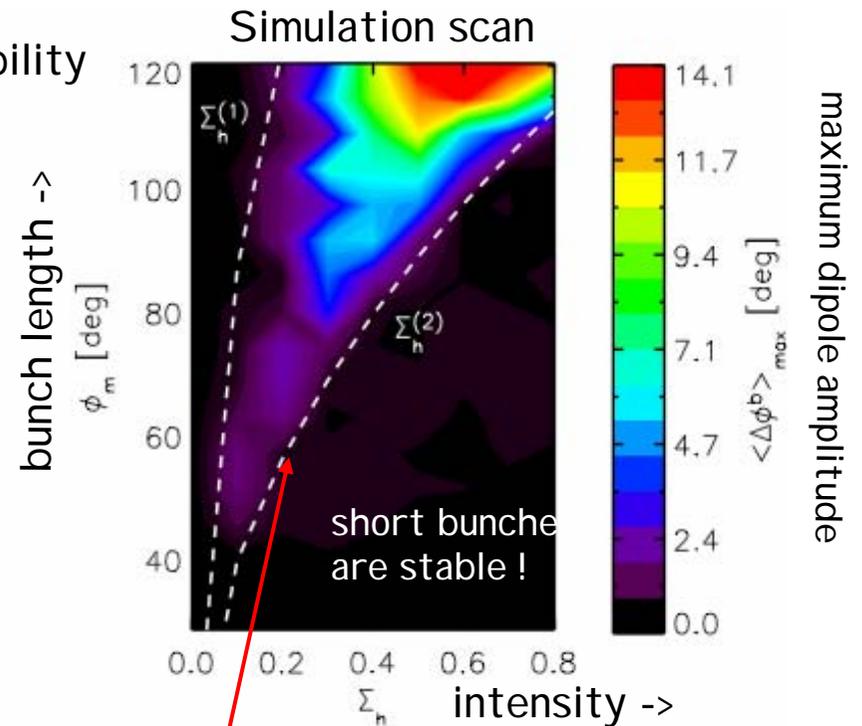
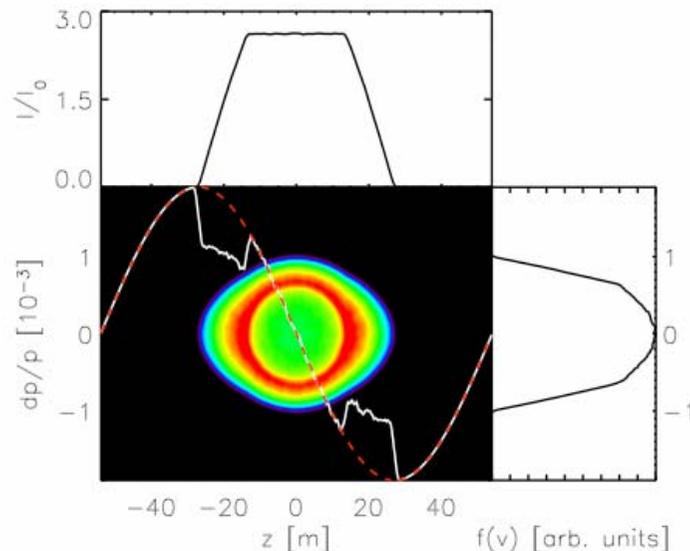
+ Lower bunching factor and ΔQ_{SC}
without new rf system (option in SIS 100)

- 'Decay' through a space charge driven dipole instability

Flat-topped (not hollow !) bunch:

$$f(H) = C_1 \sqrt{H_m - H} - C_2 \sqrt{H_h - H}$$

'main bunch' 'inner bunch'



$$\Omega \approx \omega_s(\phi_m, \Sigma_{th}) \text{ 'Stability threshold'}$$

Longitudinal microwave instability (MI)

Instability threshold in barrier rf buckets with space charge

O. Boine-Frankenheim, I. Hofmann, Phys. Rev. ST Accel. Beams 6, 034207 (2003)

Interaction of a long bunch with a broadband resonator (resonant wave length \ll bunch length).

Longitudinal space charge impedance

$$\frac{Z_n^{sc}}{n} = -i \frac{gZ_0}{\beta_0 \gamma_0^2} \frac{1}{1 + (n/n_c)^2}$$

Velocity of space charge waves

(U: norm. space charge impedance):

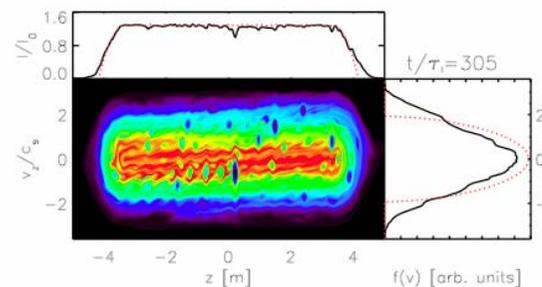
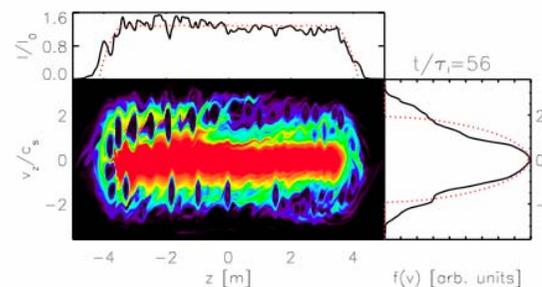
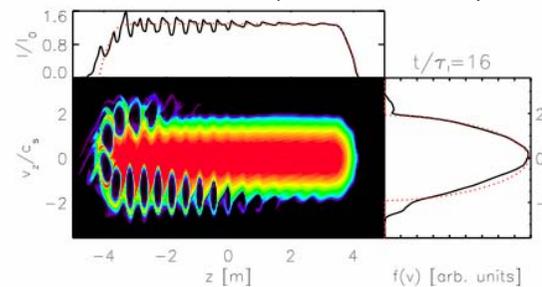
$$c_s = \frac{R}{n} \Delta \Omega_R \approx \frac{1}{2} R S \sqrt{U_n}$$

'Reflection time' (bunch length l):

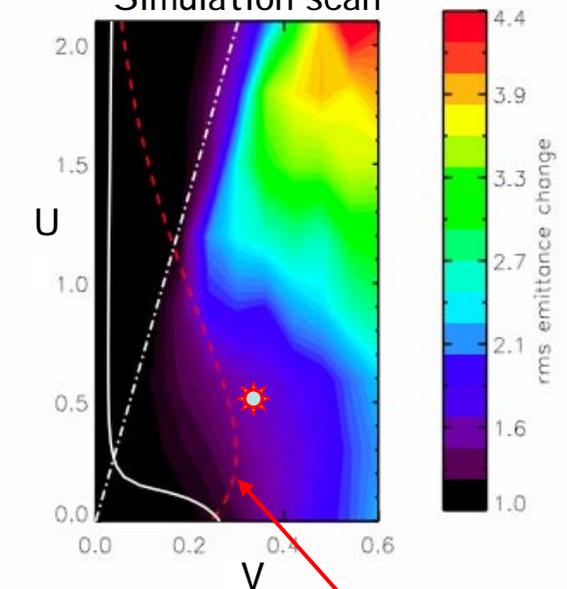
$$\tau \approx l/c_s$$

Space charge induced 'convection' (coherent shift) can damp the longitudinal MI in long bunches !

Simulation (LOBO code):



Simulation scan

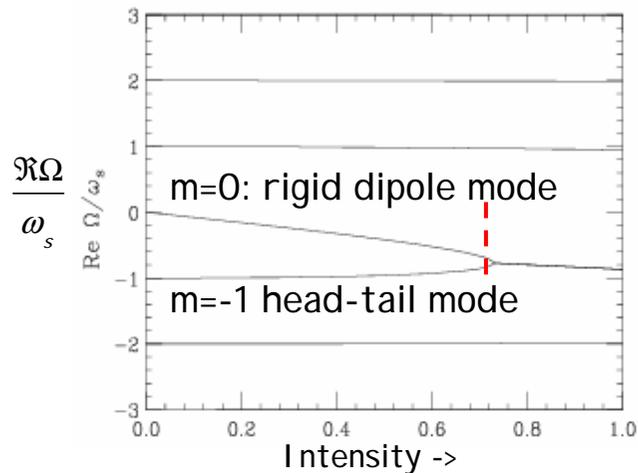


coasting beam stability boundary ('Boussard criteria')

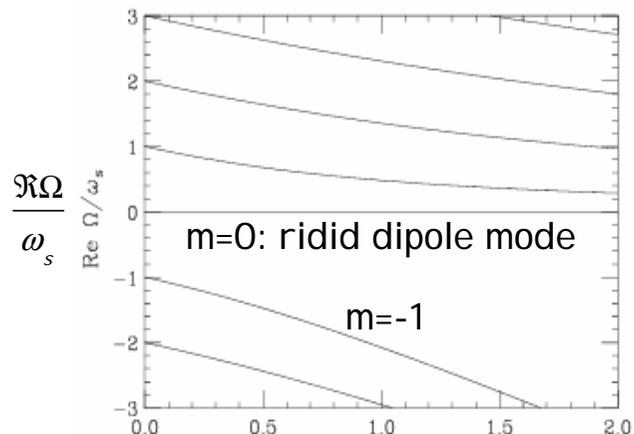
Fast head-tail instability with space charge

Transverse Mode Coupling Instability (TMCI) in the SPS

Coherent frequency shift due to a transverse resistive wake force



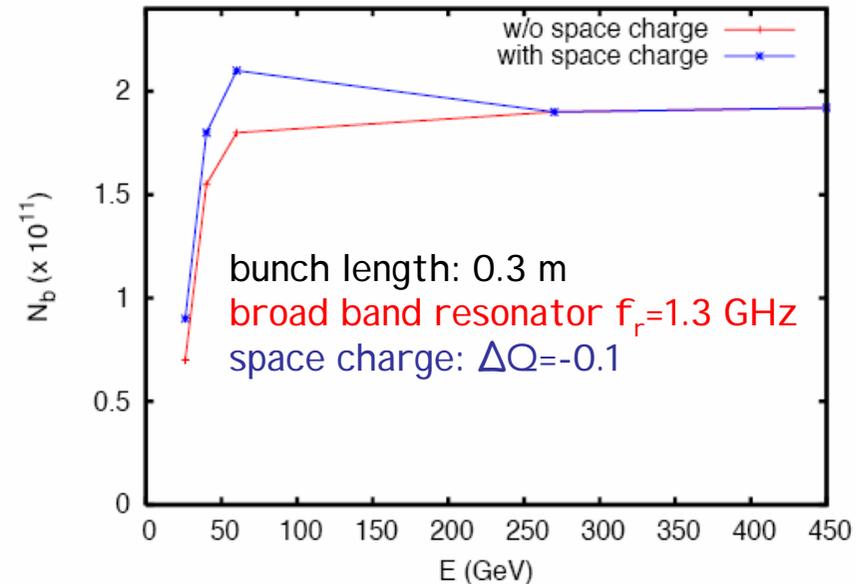
Same, but with space charge only:



M. Blaskiewicz, *Phys. Rev. ST Accel. Beams* 4, 044202 (2001)
 K.Y. Ng, *Physics of intensity dependent beam instabilities*, USPAS

G. Rumolo, et al., *Simulation study on the energy dependence of the TMCI threshold in the SPS*, [MOPLS096](#):

HEADTAIL simulations: TMCI threshold in the SPS



Space charge induced **coherent mode shifts** can **inhibit the mode coupling instability** for **sufficiently strong space charge**.

Summary

Space charge modifies coherent instability thresholds and 'impedance budgets'.

- 1) Lower instability thresholds ('Loss of Landau damping' due to incoherent tune shift)
 - a) transverse resistive wall instability
 - b) longitudinal dipole oscillations in nonlinear rf buckets
- 2) Increased instability thresholds (coherent frequency shifts)
 - a) microwave instabilities in long bunches
 - b) fast head-tail instability
- 3) Space charge driven instabilities:
 - a) longitudinal dipole mode in flat-topped bunches

(Some) possible cures:

- a) Controlled phase space blow-up.
- b) (Reactive) feedback systems.
- c) Increase tune spread due to external nonlinear focusing:
 - + octupoles and nonlinear space charge (space charge resonances and working point !).
 - double rf (tune spread not effective with space charge)

Large scale simulation scans (on parallel computers) and dedicated experiments:

+ important to validate simplified analytic approaches.