

# INSTABILITIES AND SPACE CHARGE EFFECTS IN HIGH INTENSITY RING ACCELERATORS \*

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## Abstract

In this contribution beam dynamics in circular proton and ion accelerators with high beam intensity and space charge effects is reviewed. The main focus is on recent theoretical and experimental results related to collective instabilities with space charge and possible cures. We outline the effect of space charge on collective instability thresholds and impedance budgets. The stability of longitudinal bunched beam modes and of transverse dipole modes in the presence of space charge and external nonlinearities is discussed. Finally recent work related to longitudinal and transverse microwave instabilities with space charge is summarized.

## INTRODUCTION

Space charge effects play an important role in many existing and future high intensity ring accelerators, especially at injection energies or close to transition. In combination with linear or nonlinear machine resonances space charge can cause fast [1] or gradual transverse beam losses [2, 3] resulting in the so called 'incoherent space charge limit' [4]. Below transition energy and for 'well behaved' beam distribution functions space charge alone does not cause coherent instabilities. However, space charge can modify the instability thresholds, the growth rates and the saturation levels of coherent instabilities driven by impedance sources, like the resistive wall, kickers or other ring components (see e.g. [5, 6, 7, 8, 9]). In order to accurately predict impedance budgets for future high current machines, like the SIS 100 at GSI [10], a detailed understanding of the interplay of space charge, nonlinear focusing fields and impedances is required. In many cases of practical interest stability boundaries can be obtained from simplified analytical models. Because of the complexity space charge adds to the analysis the resulting instability thresholds require a verification by computer simulation or experiments, especially if nonlinear and collective effects are important. The simulation studies greatly benefit from the possibility to perform large-scale parameter studies on modern computers. In order to give an example of the relevance of space charge effects in different machines we list their space charge parameters in Tab. 1.

Table 1: Longitudinal ( $\Delta\nu_s$ ) and transverse ( $\Delta Q_v$ ) space charge tune shifts (\*performance goals) and important impedance sources for a reference energy in different machines [3, 10, 11, 27].

machine	GeV/u	$\Delta Q_v$	$\Delta\nu_s/\nu_s$	impedance
SNS	1	-0.15	—	kicker, e-cloud
CERN PS	1.4	-0.25	-0.03	wall, kicker
SPS	26	-0.07	< 0.01	kicker, e-cloud
SIS 18	0.01	-0.5*	-0.2*	wall, kicker, rf
SIS 100	0.2	-0.3*	-0.15*	wall, kicker, rf

## TRANSVERSE RESISTIVE WALL INSTABILITY

One of the most severe instabilities in high current ring machines is the transverse resistive wall instability. In the presence of space charge the threshold intensity for the onset of the instability can be much lower [7, 12]. This phenomenon is usually explained in terms of the 'loss of Landau damping' due to the space charge induced shift of the incoherent betatron tune

$$\Delta Q \propto -\frac{Z^2 N R}{A B_f \beta_0^2 \gamma_0^3 \epsilon} \quad (1)$$

relative to the frequency of coherent dipole oscillations  $\Omega = (n - Q)\omega_0 + \Delta\Omega$ , that is not affected by the direct space charge force. Here  $Z$  and  $A$  are the particle charge and mass numbers,  $N$  is the particle number,  $R$  the ring radius,  $B_f$  the bunching factor,  $\epsilon$  the transverse emittance and  $\omega_0$  the revolution frequency.  $\Delta\Omega$  is the coherent frequency shift and  $n$  the mode number. In case of a 'cold' beam without incoherent frequency spread  $\Delta\Omega$  is directly determined by the transverse wall impedances

$$\Delta\Omega_c = -\frac{i}{8\pi^2} \frac{q^2 N}{m\gamma_0 c Q_0} Z_{\perp} \quad (2)$$

with the bare tune  $Q_0$ . At the lowest coherent frequency  $\Omega \approx (1 - [Q])\omega_0$ , with the fractional part of the tune  $[Q]$ , the wall in fast ramping synchrotrons, like SIS 100, is thin relative to the skin depth and the following expression for the driving resistive wall impedance applies [13]

$$Z_{\perp}^{tw} = Z_{\perp}^{tw}(\Omega) = \frac{2\beta_0 c R}{b^3 \sigma \Omega d} \quad (3)$$

with the wall thickness  $d$ , the pipe radius  $b$  and conductivity  $\sigma$ . The effect of the image currents in the wall is represented through the space charge impedance

$$Z_{\perp}^{sc} = -i \frac{Z_0 R}{\beta_0^2 \gamma_0^2 b^2} \quad (4)$$

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For relatively thick beams, like in SIS 100 at injection energy or in the SNS accumulator ring [14], the coherent frequency shift due to image currents can be important. Introducing the effective chromaticity with the slip factor  $\eta_0$

$$S = \omega_0 (\xi - (n \pm Q)\eta_0) \quad (5)$$

the stability boundary resulting from the dispersion relation for a beam with finite rms momentum spread  $\delta_{\text{rms}}$  can be approximated through a modified 'Zotter-Keil' criteria

$$|\Delta\Omega_c - \omega_0\Delta Q| \lesssim FS\delta_{\text{rms}} \quad (6)$$

with a form factor  $F$  that depends on the momentum spread distribution. If wall effects can be neglected ( $\Delta\Omega_c = 0$ ) loss of Landau damping due to direct space charge occurs for  $|\Delta Q| \gtrsim FS\delta_{\text{rms}}$ . As an example, during accumulation in SIS 100 one expects  $\Delta Q/(FS\delta_{\text{rms}}) \approx -10$  (see [12]), which is well outside the stability boundary for a Gaussian momentum distribution. Without Landau damping even weak resistive impedances can cause unstable growth of dipole oscillations. The resulting condition for the bandwidth of a feedback system is

$$\omega_{\min} = (1 - [Q])\omega_0, \quad \omega_{\max} = \frac{\omega_0\Delta Q}{|\eta_0|F\delta_{\text{rms}}} \quad (7)$$

In order to restore Landau damping in a coasting beam it would be sufficient to shift the real part of the coherent frequency by  $\Delta\Omega = \Delta Q$  using a purely reactive broadband feedback. A feedback is termed reactive if it causes only a coherent frequency shift whereas a system causing pure damping is called resistive [15]. In the case of bunched beams the analysis of growth rates and damping mechanisms with space charge is more complex. Firstly in a finite beam the lowest mode numbers cannot be driven as efficiently. Secondly, there can be additional damping mechanisms due to the spreads in the incoherent and in the coherent tunes along the bunch. Especially the coherent tune spread induced by the space charge impedance Eq. (4) can lead to stabilization (see e.g. [14]).

## LONGITUDINAL DIPOLE MODES

Persistent longitudinal dipole or quadrupole oscillations attributed to space charge in intense bunches have been observed e.g. in the CERN PS booster [16, 17] and also in SIS 18 [18]. The space charge induced loss of longitudinal Landau damping can be a severe problem. The bunches are unstable with regard to every coupling impedance that drives dipole or quadrupole modes. In the following we will review work related to dipole oscillations in nonlinear rf buckets and the effect of space charge. The relative longitudinal space charge tune shift and also the relative tune spread due to the nonlinear rf focusing field can be much larger than in the transverse plane. Therefore analytic approaches have to be confirmed by simulation studies. In the following an elliptic bunch distribution will be assumed [16]. The resulting bunch profile is realistic for

strong space charge [4] and leads to a space charge force proportional to the external rf force [18]. The longitudinal space charge effect (below transition) is measured through the parameter

$$\Sigma = \frac{1}{V_{rf}/V_s - 1} > 0, \quad \text{with } V_s \propto -qN_b \left| \frac{Z_{\parallel}^{sc}}{n} \right| \frac{\partial \lambda}{\partial \phi} \quad (8)$$

with ratio of the rf voltage  $V_{rf}$  and the space charge induced voltage  $V_s$ . Here  $N_b$  is the number of particles in the bunch,  $Z_{\parallel}^{sc}/n$  is the constant, long wavelength, longitudinal space charge impedance,  $\lambda$  is the line density profile and  $\phi$  the rf phase of the bunch particles. For small  $\Sigma$  and bunch length  $\phi_m$  (parabolic bunch profile) the incoherent synchrotron frequency in a rf bucket as a function of the particle amplitude is

$$\omega_s(\hat{\phi}) = \frac{\omega_{s0}}{\sqrt{1 + \Sigma}} (1 - S\hat{\phi}^2) \approx \omega_{s0} + \Delta\omega_s - S\hat{\phi}^2 \quad (9)$$

with the synchrotron frequency  $\omega_{s0}$  for small amplitudes and the space charge induced synchrotron frequency shift  $\Delta\omega_s/\omega_{s0} \approx -\Sigma/2$ . The synchrotron frequency spread due to the nonlinear rf is  $\delta\omega_s \approx S\phi_m^2$  with  $S = 1/16$ . The frequency of 'rigid' dipole oscillations of a short parabolic bunch in a rf wave is

$$\Omega_0 = \omega_{s0} + \Delta\Omega_0 \approx \omega_{s0} \left( 1 - \frac{4}{5}S\phi_m^2 \right) \quad (10)$$

with the zero intensity coherent frequency shift  $\Delta\Omega_0$  (see [16, 18, 19]). Landau damping is lost when the coherent frequency (here the 'rigid' dipole mode) is outside the band of incoherent synchrotron frequencies [16]. For Landau damping the following condition should apply

$$\omega_s(\phi_m) < \Omega < \omega_s(0) \quad (11)$$

Below transition space charge reduces the synchrotron frequency and only the upper limit applies

$$\frac{\Delta\omega}{S} < \frac{4}{5}\phi_m^2, \quad \text{or } \Sigma < \frac{\phi_m^2}{10} \quad (12)$$

For  $\phi_m = 60^\circ$  we obtain a threshold space charge parameter of  $\Sigma_{th} \approx 0.1$ . It is important to note that the  $\Sigma_{th}$  describing the loss of Landau damping in a double rf wave, used also for bunch flattening, is much smaller [18]. Therefore a double rf system cannot be used to restore Landau damping, even though the incoherent frequency spread in the double rf wave is much larger compared to a single rf wave. In the following we analyze the effect of coherent synchrotron tune shifts and the corresponding bunch stability thresholds. We introduce an effective normalized dipole impedance  $Z^{\text{eff}}$  and the induced coherent synchrotron frequency shifts for a very short bunch in the form

$$\Delta\Omega_c \approx \frac{i\omega_{s0}}{2} (Z_R^{\text{eff}} + iZ_I^{\text{eff}}) \quad (13)$$

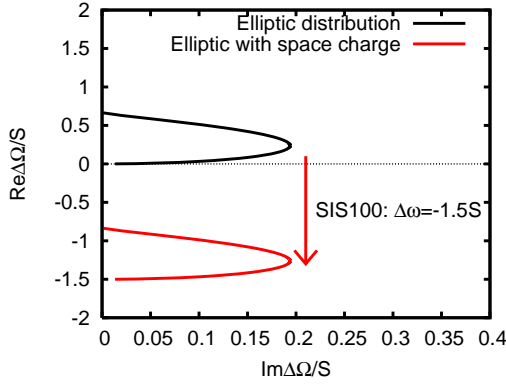


Figure 1: Stability boundary from Eq. (14). The shifted boundary corresponds to  $\Sigma = 0.3$  and  $\phi_m = 90^\circ$ , which are the expected bunch parameters in SIS100 during accumulation.

The coherent frequency shift in a nonlinear rf bucket can be obtained from the dispersion relation [19, 5]

$$1 = -\pi(\Delta\Omega_c - \Delta\omega) \int \frac{df(\hat{\phi})}{d\hat{\phi}} \frac{\hat{\phi}^2 d\hat{\phi}}{\Omega - \omega_s(\hat{\phi})} \quad (14)$$

with the synchrotron frequency distribution  $f(\hat{\phi})$ . In deriving Eq. (14) mode coupling has been neglected and a 'rigid' dipole oscillation has been assumed. In Fig. 1 the resulting stability boundary for a elliptic momentum spread distribution is shown. The shifted boundary corresponds to  $\Sigma = 0.3$  and  $\phi_m = 90^\circ$  (SIS 100). In this case the space charge induced frequency shift is  $\approx -0.12$ , which is about twice the frequency spread in the nonlinear rf bucket. An important question is the validity of the stability boundary obtained from Eq. (14). Fig. 2 shows the result of an impedance scan using a self-consistent particle tracking code [18] solving the synchrotron equation of motion including the effective impedances and space charge. For  $\Sigma = 0$  we obtain perfect agreement with the stability boundary predicted by Eq. (14). However, for  $\Sigma = 0.3$  one can see in Fig. 2 that the stability area and the shift of the stability boundary differ. The actual stable area (black) and also the shift obtained from the simulation are both smaller. The reason for this discrepancy possibly lies in the complex interplay of nonlinear and collective dynamics that cannot be fully captured with the simplified analysis. The limits of applicability of Eq. (14) become apparent if one considers flat-topped bunch profiles. Mathematically a flat-topped bunch can be created by subtracting two elliptic distributions with different bunch lengths [16]. Different methods have been employed to generate flat-topped bunches and to increase the bunching factor and so the transverse space charge limit. Flat-topped bunches are unstable in the presence of longitudinal space charge. They 'decay' through a dipole instability [20, 18] above a certain threshold space charge parameter. The stability boundary for flat-topped bunches cannot be predicted ignoring mode coupling [21].

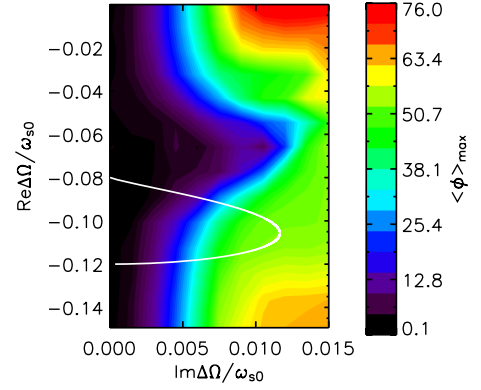


Figure 2: Simulation scan of induced dipole oscillations for different effective impedances  $Z^{\text{eff}}$ . Plotted is the maximum (saturated) dipole amplitude obtained from the simulation. The initial condition is a matched elliptic distribution with a bunch length  $\phi_m = 60^\circ$  and  $\Sigma = 0.3$ . The curve represents the stability boundary obtained from Eq. (14).

It is interesting to point out that simulation results show that flat-topped bunches are stabilized if  $-Re\Delta\Omega_c \gtrsim -\Delta\omega$ . In this case the coherent dipole mode is shifted below the minimum incoherent synchrotron frequency in the bunch. Such a shift of the coherent dipole frequency could be achieved using a purely reactive longitudinal feedback system.

## NONLINEAR SPACE CHARGE

In the foregoing sections we treated Landau damping due to linear and nonlinear tune spreads induced by external forces. The direct space charge field is another internal source of nonlinear tune spread. In the transverse plane the betatron amplitude dependent tune due to (nonlinear) space charge and an external nonlinear force is [8]

$$Q_x(\hat{x}, \hat{y}) = Q_0 - \Delta Q_x G(\hat{x}, \hat{y}) + S_x^{\text{ext}} \hat{x}^2 \quad (15)$$

with  $G(\hat{x}, \hat{y}) \approx 1 - S_x^{\text{sc}} \hat{x}^2 - S_y^{\text{sc}} \hat{y}^2$ . Here  $\Delta Q_x$  is the linear space charge tune shift,  $S_x^{\text{ext}}$  is the tune spread coefficient representing e.g. an octupole,  $S_x^{\text{sc}}$  is the tune spread coefficient for small amplitudes (relative to the beam radius  $a$ ) due to nonlinear space charge. For rough estimates the tune spread for a parabolic density profile due to nonlinear space charge can be approximated as  $\delta Q^{\text{sc}} \approx S_x^{\text{sc}} a^2 \Delta Q = 3/8 \Delta Q$ . In Ref. [8] (with refinements in Ref. [22]) it was shown that nonlinear space charge can enhance Landau damping due to octupoles. Nonlinear space charge alone does not cause any Landau damping, because it is an internal force that cannot affect the beam centroid motion. The external nonlinearity is needed in order for nonlinear space charge to become effective (see also [23]). The results obtained in Refs. [8, 22] exhibited a strong dependence on the polarity of the octupole. A dependence of Landau damping

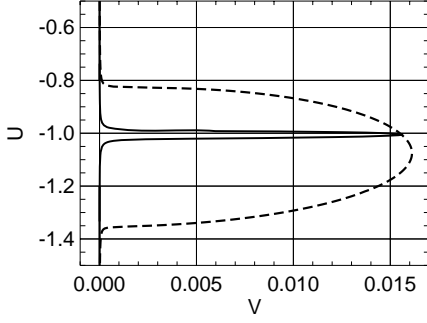


Figure 3: Stability boundary provided by octupoles with (dashed curve) and without (solid curve) nonlinear space charge.  $U + iV = \Delta\Omega_c / \Delta Q_x$ .

on the octupole polarity has also been obtained in simulations studies for the SNS using a bunched beam [14]. The dispersion relation used in Refs. [8, 22] is

$$1 = \int (\Delta\Omega_c - \omega_0 \Delta Q(\hat{x}, \hat{y})) \frac{d\rho(\hat{x})}{d\hat{x}} \frac{\hat{x}^2 g(\hat{y}) d\hat{y} d\hat{x}}{\Omega - \omega_0 Q_x(\hat{x}, \hat{y})} \quad (16)$$

with  $\Delta\Omega_c$  determined through Eq. (2) and with the horizontal and vertical betatron amplitude distributions  $f(\hat{x})$  and  $g(\hat{y})$ . Fig. 3 shows the stability boundary obtained from Eq. 16 with and without nonlinear space charge for octupole strengths and beam parameters relevant for SIS 18 and 100. Eq. (16) relies on a number of simplifying assumptions, e.g. radial mode coupling induced by nonlinear space charge has been neglected. In SIS 18 and 100 nonlinear space charge together with octupoles and the negative coherent frequency shift induced by image currents could potentially stabilize the resistive wall instability [12]. In order to obtain more accurate results a comparison of the stability boundary obtained from Eq. 16 with a simulation scan using the self-consistent particle tracking code PATRIC has been performed [12]. The simulations confirmed that there is no Landau damping due to nonlinear space charge alone. Furthermore the enhancement of the octupole-induced Landau damping by nonlinear space charge has been confirmed. Also the shift and the width of the stability boundary along the  $Re\Delta\Omega_c$  axis agrees very well with the simulation results. However, the results of the simulation scans indicate that Eq. (16) overestimates the stable area in the  $\Delta\Omega_c$  plane. Another important finding is that the effect of the octupole polarity on the stability boundary is much weaker than predicted by Eq. (16). Further theoretical as well as experimental studies are required in order to consolidate these results and to extend them also to bunched beams. In this context also the question of possible long-term beam loss due to resonances induced by octupoles [2] needs to be addressed.

## LONGITUDINAL MICROWAVE INSTABILITY

Here we consider the interaction of long bunches with a broadband resonator. The bunch length is assumed to be long relative to the resonant wavelength. The local frequency spread in the bunch is

$$S = \frac{1}{2} \omega_0 \eta_0 \left( \frac{\Delta p}{p} \right)_{\text{fwhm}} \quad (17)$$

The normalized impedance is defined as

$$V_n - iU_n = \frac{|\eta_0| I_0 q}{2\pi m R^2 \gamma_0 S^2} \left[ \text{Re} \left( \frac{Z_n}{n} \right) + i \text{Im} \left( \frac{Z_n}{n} \right) \right] \quad (18)$$

with the longitudinal impedance  $Z_n$  at  $\omega_n = n\omega_0$ . We will assume that the imaginary impedance is dominated by the longitudinal space charge impedance [24]

$$\frac{Z_n^{sc}}{n} = -i \frac{g Z_0}{\beta_0 \gamma_0^2} \frac{1}{1 + (n/n_c)^2} \quad (19)$$

We define the space charge parameter as  $U = U_{n=1}^{sc}$ . The coherent frequency shift of longitudinal waves is defined as  $\Delta\Omega = \omega - n\omega_0$ . If we assume that the impedance spectrum is dominated by space charge we obtain the phase velocity of space charge waves on the bunch (see e.g. [25])

$$c_s = \frac{R}{n} \Delta\Omega_R \approx \frac{1}{2} R S \sqrt{U_n} \quad (20)$$

together with the corresponding instability growth rate

$$\tau_I^{-1} = \Delta\Omega_I \approx \frac{1}{2} n S \frac{V_n}{\sqrt{U_n}} \quad (21)$$

The time needed for a perturbation starting at the bunch head to reach the bunch end is  $\tau \approx l/c_s$ , with the bunch length  $l$ . If we assume that all waves will be damped due to incomplete reflection or 'wave breaking' at the bunch end we can formulate the following stability criteria [6] and substitute Eqs. 20 and 21

$$\frac{c_s \tau_I}{\alpha_e l} \gtrsim 1 \quad \Rightarrow \quad \frac{V_n}{U_n} \lesssim \frac{2 R \alpha_e}{n l} \quad (22)$$

with the effective number of  $e$ -foldings  $\alpha_e$  from the bunch head to the end. Fig. 4 shows the comparison of this stability criteria with a simulations scan (see Ref. [6]) for a beam confined between two barrier rf waves and interacting with a broadband resonator with the normalized shunt impedance  $V$  centered at a wave length  $\lambda \approx l/10$ . One can see that for  $U \gtrsim 1$  the stability boundary follows Eq. 22 if we choose  $\alpha_e = 6$ . Also shown are the coasting beam stability boundaries for a parabolic and for a Gaussian momentum spread distribution.

## TRANSVERSE MICROWAVE INSTABILITIES WITH SPACE CHARGE

At frequencies above  $\omega_{\text{max}}$  from Eq. (7) Landau damping due to the finite momentum spread becomes effective.

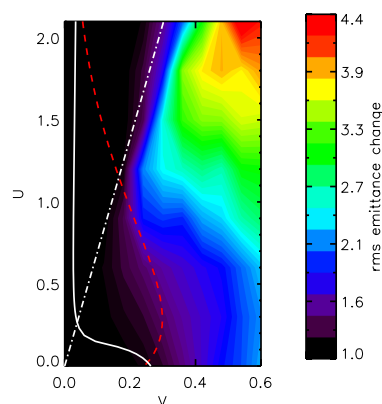


Figure 4: Emittance increase in a barrier bucket beam as a function of initial  $(U, V)$  values. The black area corresponds to a stable beam.

Therefore the loss of Landau damping is usually not an issue for high frequency. An exception is the operation close to transition. In long bunches the coherent frequency shift induced by the space charge impedance can lead to a phase velocity of coasting beam-like slow modes on the bunch. If unstable modes are damped at the bunch end the transverse microwave instability can be stabilized, similar to the stabilization obtained in the longitudinal plane (see previous Section). As an example we studied the beam breakup instability observed in the CERN PS [26] near transition using the PATRIC code. In this case the synchrotron motion can be regarded as 'frozen'. The simulation showed the importance of space charge ( $\Delta Q_v \approx -0.2$ ) for the instability threshold and especially for the reproduction of the observed transverse beam along the beam tail.

In short bunches space charge can lower the threshold for the fast head-tail instability. This has been verified within analytical and numerical models using linear space charge forces [9, 27]. In Ref. [27] a strong transverse emittance increase below the instability threshold was observed in the simulations that was attributed to the interaction of space charge and the broadband impedance.

## CONCLUSIONS

Space charge can strongly affect the damping mechanisms and so the impedance budget in high current ring machines. In our review we showed that the loss of Landau damping due to the space charge induced incoherent tune shift can lower the threshold currents for the coasting beam transverse resistive wall instability and for longitudinal dipole instabilities in nonlinear rf buckets. Possible cures are external damping systems. In order to restore Landau damping a reactive feedback might be sufficient, which could be used also to stabilize the space charge driven dipole instability in flat-topped bunches. Another cure is to increase the momentum spread. In case of a

tight budget for the tolerable momentum spread blow-up, like in SIS 100, another option is to increase the incoherent frequency spread by external nonlinear focusing fields. However, in case of longitudinal dipole modes in double rf waves, one finds that the increase in synchrotron frequency spread cannot become effective in the presence of space charge. A potential cure for the transverse resistive wall instability are octupoles in combination with nonlinear space charge. Simulation scans show that the effect of nonlinear space charge is not as strong as predicted by a simplified dispersion relation. With regard to microwave instabilities different effects have been identified that can suppress the instability in long ('coasting beam modes') and in short bunches ('head-tail modes').

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