DEVELOPMENT OF NUMERICAL CODE FOR SELF-CONSISTENT WAKE FIELD ANALYSIS WITH CURVED TRAJECTORY ELECTRON BUNCHES*

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Abstract

Strongly interacting phenomena of electromagnetic radiation fields and ultra-relativistic electron is one of great interests in accelerator science such as in electron beam dynamics at the bunch compressor. The phenomena are described by time domain boundary value problem for the Lienard-Wiechert solutions. Authors develop a time domain boundary element method for self-consistent wake field analysis of electromagnetic fields and charged particles. To use boundary integral equation for describing the electromagnetic fields, the time domain boundary value problems for the Lienard-Wiechert solution can be naturally formulated and we can simulate the wake field phenomena with electron beam dynamics. In this paper, beam dynamics of curved trajectory electron bunches inside uniform beam tube are numerically simulated by using 2.5 dimension time domain boundary element technique. Various effects of closed beam tube for ultra-relativistic electron dynamics are considered comparing with the Lienard-Wiechert solutions in free space.

INTRODUCTION

Authors have been working in development of a Time Domain Boundary Element Method (TDBEM) for wake field analysis of particle accelerators [1-3]. Advantages of use of the TDBEM compared with conventional finite difference schemes in wake fields analysis are numerical modelling with smooth boundary, grid dispersion free property in wake potential calculations and treatment of arbitrary charged particle trajectories. Up to now, the TDBEM wake fields analysis was developed for a given particle trajectory and good agreements in wake potentials with analytical and conventional numerical solutions are confirmed. Then, to fully induce the advantage of the TDBEM wake field analysis, self-consistent analysis of charged particles and wake fields is one of most meaningful applications. Indeed, strongly interacting phenomena of electromagnetic radiation fields and ultra-relativistic electron is one of great interests in accelerator science such as in electron beam dynamics at the bunch compressor. And those phenomena can be analyzed by only use of the self-consistent analysis of charged particles and wake fields. Especially the TDBEM can naturally include the Lienard-Wiechert solution in its formulation and describes the electromagnetic radiation fields strongly interacting with high-energy charged particles. This paper presents a numerical scheme of the self-consistent wake fields analysis based on the TDBEM.

SELF-CONSISTENT WAKE FIELDS ANALYSIS BASED ON TDBEM

Overview of Self-consistent Wake Fields Analysis

Calculation flow of the self-consistent wake fields analysis based on the TDBEM is shown in Fig.1. The self-consistent simulation consists of mainly the TDBEM wake field calculation and charged particle tracking calculation, and these calculations are iteratively performed until the solutions reach to convergence. That is, the TDBEM wake fields calculation is performed for given charged particle trajectory which is calculated by the charged particle tracking. Then the electromagnetic fields produced by the charged particles are calculated by using the Lienard-Wiechert solutions, accordingly this self-consistent simulation can treat arbitrary shape trajectory and arbitrary energy of high energy charged particles. And then the charged particle tracking calculation is performed under the assumption of given

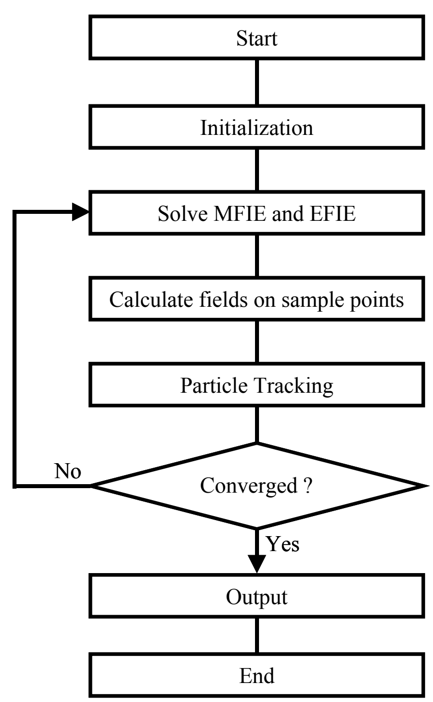

Figure 1: Flow chart of self-consistent simulation.

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electromagnetic field distribution which is calculated by the TDBEM. The convergence check is done by comparison of the charged particle trajectory shape at the previous and new iteration steps.

TDBEM Wake Field Calculation

The TDBEM wake fields calculation is based on the following Kirchhoff' integral representations of the electric and magnetic fields [1]:

$$\mathbf{E}(t,\mathbf{r}) = \mathbf{E}_{LW}(t,\mathbf{r}) - \frac{1}{4\pi}\int_S \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial}{\partial t}\left(\mathbf{B}(t',\mathbf{r}')\times\mathbf{n}'\right) + \left[\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\left(\mathbf{E}(t',\mathbf{r}')\cdot\mathbf{n}'\right) + \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\frac{\partial}{c\partial t}\left(\mathbf{E}(t',\mathbf{r}')\cdot\mathbf{n}'\right) \right]\right) dS' \tag{1}$$

$$\mathbf{B}(t,\mathbf{r}) = \mathbf{B}_{LW}(t,\mathbf{r}) - \frac{1}{4\pi}\int_S \left[\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\times\left(\mathbf{B}(t',\mathbf{r}')\times\mathbf{n}'\right) + \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\times\frac{\partial}{c\partial t}\left(\mathbf{B}(t',\mathbf{r}')\times\mathbf{n}'\right)\right] dS' \tag{2}$$

where $\mathbf{r}$ is the observation point in the bounded analytical region, $\mathbf{r}'$ is the point on the boundary surface S, t' is the retarded time denoted by $t' = t - |\mathbf{r} - \mathbf{r}'|/c$, $\mathbf{n}'$ is the unit normal vector on the boundary surface, and $\mathbf{E}_{LW}$ and $\mathbf{B}_{LW}$ are the Lienard-Wiechert (LW) fields, which are calculated from trajectories and velocities of charged particles. The boundary values $\mathbf{B}\times\mathbf{n}$ and $\mathbf{E}\cdot\mathbf{n}$ are interpreted as the surface current and charge densities excited by the LW fields. This means that the influence of the charged particle motion is reflected on the surface current and charge densities through the LW fields, and the induced surface current and charge densities produce the wake fields in the accelerator structures which is described by the boundary integral terms of (1) and (2). The boundary integral representations (1) and (2) are interpreted as time domain boundary value problems for the Lienard-Wiechert free space solutions.

In the practical numerical simulation based on the TDBEM, (2) are discretized for time and 3D space, and we can finally obtain the following multi-matrix equation:

$$[M_0][B_n] = [B_{LW}] - \sum_{l=1}^{L}[M_l][B_{n-l}] \tag{3}$$

where $[B_l]$ denotes an unknown vector which consists of tangential components of magnetic fields on the boundary elements at time $t = l\Delta t$ (l = 0, ···, L), $[M_l]$ denotes a coefficient matrix determined by the boundary integral of (2) on S, L is the total number of the matrices, $[B_{LW}]$ is a given vector calculated from the inner products of the tangential unit vectors and the LW fields. By iteratively solving (3) at each time step, the surface current density induced on the boundary surface of an accelerator structure can be obtained. The surface charge density is calculated by (1) from the surface current $[B_l]$ and $[E_{LW}]$ in the similar way. Once the boundary values over all time steps are calculated, the wake fields at any position inside the domain surrounded by S can be obtained from (1) and (2).

In general, the TDBEM requires huge computer memory and calculation cost, especially full 3D TDBEM simulation is not impossible even in supercomputers. This paper therefore assumes axis-symmetry in the numerical model of the accelerator tube, but allows to be 3D arbitrary motion for the charged particles (so-called 2.5 D simulations).

Particle Tracking with Macroparticle Model

The particle tracking is performed based on the following standard Lorentz force equation of motion for relativistic charged particles.

$$d\dot{\mathbf{p}}_i / dt = q_i\left(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}\right) \tag{4}$$

$$dE_i / dt = q_i \mathbf{E}\cdot\mathbf{v}_i\,. \tag{5}$$

where $\mathbf{p}_i$ and E_i are individual momentum and energy of the charged particle q_i in the bunch. Then, to suppress the number of particles in the bunch by reasonable one for practical numerical simulation, a macro-particle model is adopted here. The macro-particle modelling of the bunch is 1 D distribution along the bunch trajectory under the assumption of sliced bunch macro-particle perpendicular to the direction of the motion.

The trajectory of the individual particle itself has very important information for the calculation of the Lienard-Wiechert fields because the LW fields is evaluated at the following retarded time t_r , (see Fig.2)

$$t_r = t - \frac{|\mathbf{r} - \mathbf{r}_i(t_r)|}{c} \tag{6}$$

It is know that this causality relation is strongly connected to high frequency property of the LW solution [4-6], therefore (6) should be carefully evaluated and the retarded time t_r is exactly obtained for the calculation of the LW fields.

NUMERICAL EXAMPLES

As an example of the self-consistent wake fields analysis based on 2.5D TDBEM, charged particle bunch

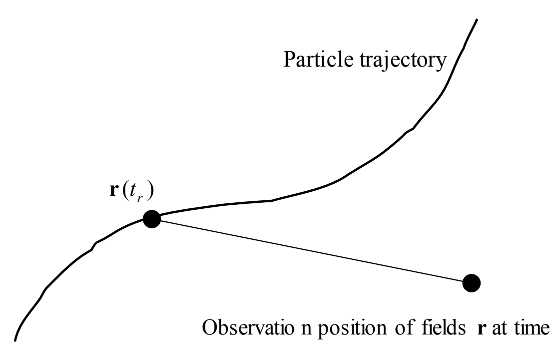

Figure 2: Causality relation on retarded time.

motions inside uniform cylindrical beam pipe are simulated here (see Fig.3). It is assumed that the initial charged particle energy is 1 MeV, total charge of the bunch is 4.807×10^{-16} C, RMS length σ = 3mm, and the number of macro-particles is taken to be 300. The figures 4 shows the tangential components of magnetic fields on the surface which correspond to the surface current induced by the beam of straight and curved (bending radius R = 1.5 m) bunch trajectories. Clear disturbance of surface currents is observed in simulation of the curved trajectory.

CONCLUSION

This paper has presented a numerical scheme of the self-consistent wake fields analysis based on the TDBEM. The TDBEM is formulated as a time domain boundary value problem of the Lienard-Wiechert free space solution coupling with particle tracking calculation. The numerical example of bunch motion in uniform beam pipe tells us that self-consistent simulations show obviously different result on surface currents from conventional given straight trajectory simulations. The numerical examples for a bunch with low energy have been demonstrated in this paper because computing environments are not enough. Calculations of realistic beam parameters will be done with the developed code in future.

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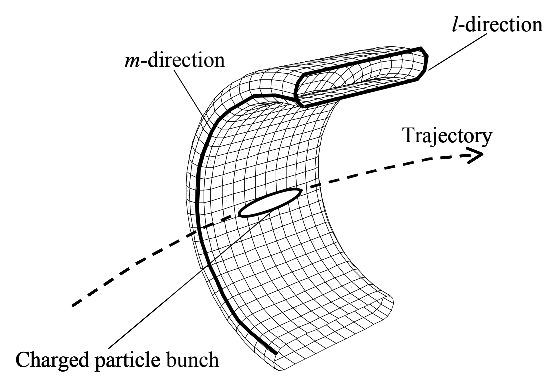
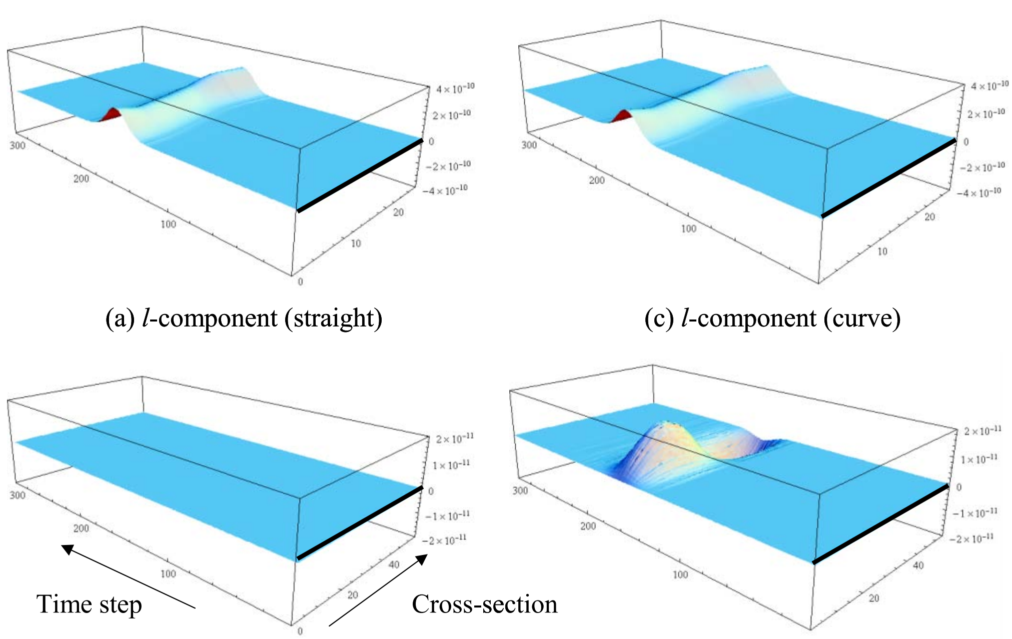


Figure 3: Numerical models of self-consistent wake field analysis of charged particle in uniform beam pipe.



(a) *l*-component (straight) (c) *l*-component (curve)

(b) *m*-component (straight) (d) *m*-component (curve)

Figure 4: Time evolution of surface current induced by a bunch of two different trajectories: straight (left) and curve (right).