PARAMETER SCANS AND ACCURACY ESTIMATES OF THE DYNAMIC APERTURE OF THE CERN LHC

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Abstract

Techniques to make use of large distributed computing facilities allow for denser parameter scans of the dynamic aperture, i.e., the domain in phase space where bounded single-particle motion prevails. Moreover, one can also increase the number of 'seeds' each of which represents a possible realization of multipolar components around the machine. In this paper the dependence of the dynamic aperture on the step size of the grid of initial conditions and on the number of seeds is studied. Estimates on the accuracy of the dynamic aperture are derived and the definition of an improved protocol for numerical simulations is presented.

INTRODUCTION

The dynamic aperture (DA), i.e. the amplitude of the region in phase space where stable motion occurs, is a key quantity in the evaluation of the performance of the future LHC. Therefore, an accurate numerical estimate is mandatory as well as a good knowledge of the error associated with the protocol used to compute the DA (see Ref. [1] for a detailed account on the subject). The computation of such a quantity relies on numerical simulations, performed with the MAD-X [2] and/or the Sixtrack [3] codes. For the case of the LHC studies, the number of turns N is equal to 10^5. A polar grid is defined in the physical space (x, y), at a location where the optical parameters satisfy $\alpha_x = \alpha_y = 0$, where 5 angles, corresponding to different transverse emittances ratio y/x, are considered. Along each of these radial directions a number of initial conditions are considered, namely 30 initial conditions are uniformly distributed over an amplitude range of 2σ (each initial condition is in fact split into two nearby conditions to allow chaos detection by means of the computation of the maximal Lyapunov exponent [4, 5]). The momentum off-set of the initial conditions is set to 0.75×10^{-3}, corresponding to $3/4$ of the bucket half-height.

The use of such an approach should guarantee an accuracy in the computation of the DA of about 0.5σ [6]. It is worth mentioning that it is customary to express the DA in units of transverse beam size, i.e. sigmas.

Indeed, the need of taking into account the influence of random magnetic errors, requires that the DA computation is repeated for a number of different sequences of generated error so as to evaluate minimum, maximum, and average values of the DA over the ensemble of realizations of random errors. An analysis of the statistical error is carried out in Ref. [7] showing that one needs an unbiased sample of 60 realizations of the LHC with magnetic field errors to find, with a 95 % confidence level, that only 5 % of the total number of all possible LHC realizations have a DA lower than the lowest one found by particle tracking. As far as the LHC model is concerned, the optics version V6.4 is used for the discussion presented in the next sections, where the magnetic errors, both systematic and random, are assigned to the main dipoles, cold separation dipoles and main quadrupoles, all at injection energy. The error values are collected in the error table number 0210. Even though this configuration is certainly not the most up-to-date, it represents a realistic test-bench for the study on DA focusing on the accuracy of the DA estimate on the number of space phase angles and on the number of realizations.

The major computational tasks for 1000 angles, 6 amplitudes and 60 seeds, some 360,000 two hour jobs, were run on the CERN Modular Physics Screensaver (CPSS [8]) system using WINDOWS desktop PCs (see also Beam–Beam simulations [9]).

DA DEPENDENCE ON PHASE SPACE ANGLE

A special series of simulations was performed by increasing the number of phase space directions ϑ up to 1000. For each value of the total number of angles N, the actual values are given by:

$$\vartheta_{N,i} = \frac{90^\circ\, i}{N+1} \qquad 1 \leq i \leq N. \tag{1}$$

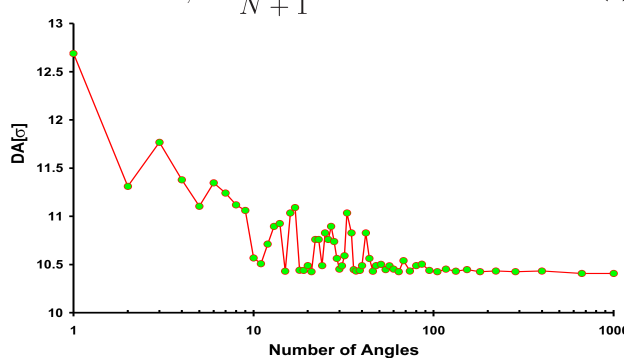

Figure 1: Minimum DA as a function of number of phase space angles.

The graph shows three different regimes:

- Up to 10 Angles: Clearly the sampling of the angles is not good enough and the DA decreases logarithmically. As most of our tracking has been done with 5 angles one has to reduce the stated DA by the ratio of 10.5 to 11.1, i.e, roughly 5%.
- 10 to 50 Angles: In this regime the long-term minimum is reached but there is a very large variation

of about 8% to larger DA values. Originally, it was hoped that 30 angles might be sufficient. However, this analysis shows that regime 2 is not good enough.

- More than 50 Angle: As of 50 angles very little variation can be reported $\sim 1\%$. The minimum itself is not falling any further. It can therefore be concluded that a value between 50 and 100 angles is sufficient to determine the minimum over angles.

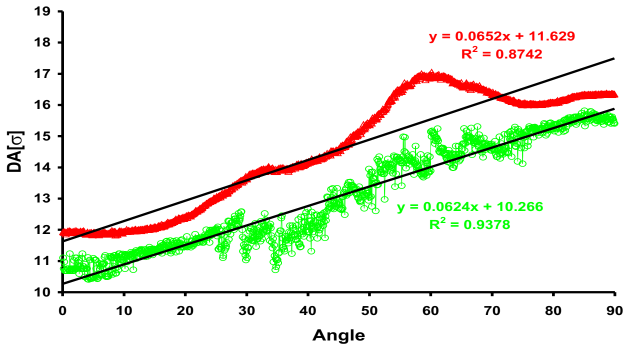

Figure 2: Average (red markers) and minimum DA (green markers) as a function of phase space angles.

This graph 2 shows that the minimum of 60 seeds as a function of phase space angle is following on average a straight line with high R^2 value. Not surprising, there is quite some variation and there are slight indications of the effect of resonances where the DA changes up to 1σ. It is interesting to note that each individual seed does not show this nice linear behavior: in Fig. 3 the result of two seeds are shown together with the minimum over all seeds. The individual seeds depict very large variations with respect to the phase space angle.

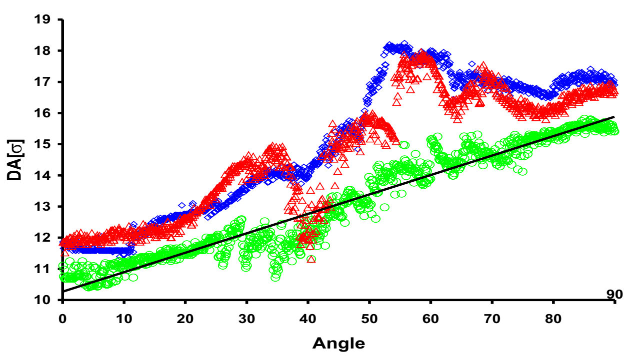

Figure 3: Minimum DA over sixty realizations (green markers) compared to DA of two individual realizations (red and blue markers), all as a function of phase space angles.

The average values of the DA (Fig. 2) has much less microscopic variations but the average DA does not follow a straight line at all. It seems to us that this smooth behavior of the minimum DA as function of phase space angle is revealing an important underlying feature of the motion. Further studies would be required to fully understand if it can be exploited for simplifying the tracking effort.

DA DEPENDENCE ON THE NUMBER OF REALIZATIONS

A special series of numerical simulations was performed for a maximum number of 600 realization of the LHC lattice, while scanning the transverse phase space by means of five angles, only. The resulting data were analyzed in view of determining the minimum number of realizations required to derive a reliable estimate of the DA. In Fig. 4 the DA distribution for two values of the phase space angle is shown. For each plot the distribution computed using the 600 realization is plotted together with the one obtained for only 60 realizations, i.e. the nominal number of realizations for the standard protocol. The actual shape of the

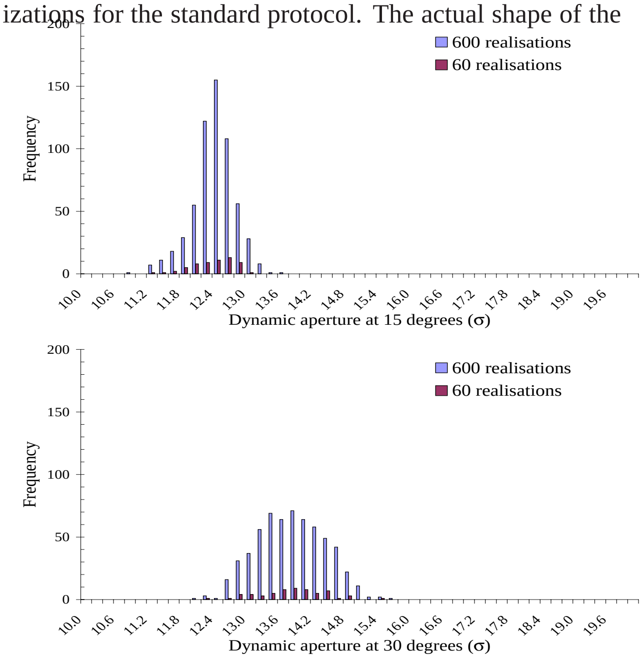

Figure 4: Distribution of the DA for two values of the phase space angle ϑ.

distribution strongly depends on the phase space angle and, of course, the tails for the two cases, namely 600 or 60 realizations, differ considerably.

To determine how many realizations are required, it is possible to compute the key descriptors of the distribution, i.e. minimum, average and maximum value, as a function of the number of realizations (see Fig. 5). For the case of $\vartheta = 15°$ a strong dependence on the number of realizations is found, and, the minimum DA settles down after 300 realizations. On the other hand for $\vartheta = 30°$ the dependence is less pronounced. From these considerations it emerges that the dynamics at $15°$ gives rise to a richer phenomenology, hence, in the following, the analysis will be limited to this case. The next step consist in deriving the Cumulative Distribution Function (CDF) for the DA distribution. Given an ordered set of N values of the DA resulting from the numerical computation on N realizations of the LHC, the CDF can be computed as

$$CDF(DA_i) = \frac{i}{N}, \qquad i < N. \tag{2}$$

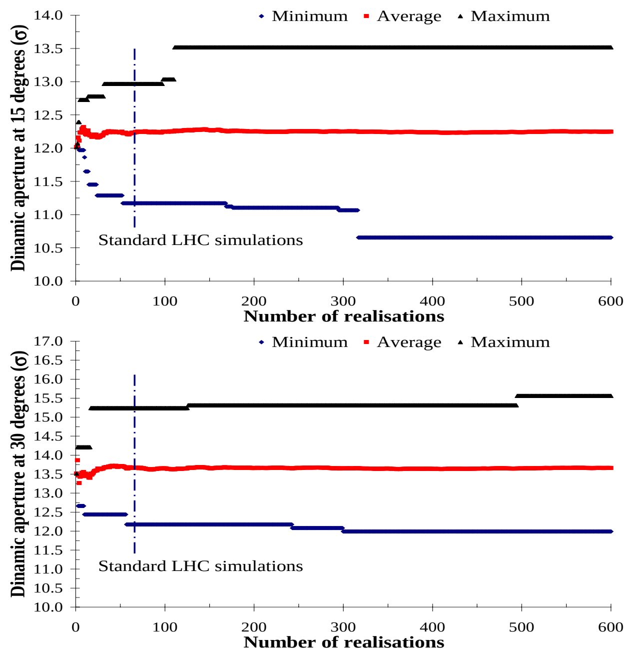

Figure 5: Distribution of the DA for two values of the phase space angle ϑ.

An example is shown in Fig. 6, where the CDF is shown as computed from a set of 60 or 600 realizations It is

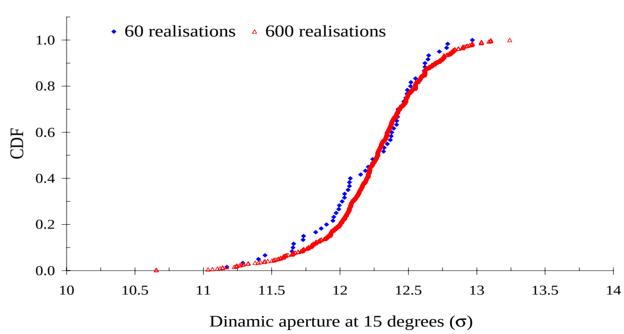

Figure 6: CDF as a function of the DA for $\vartheta = 15°$ computed using 60 or 600 realizations of the LHC.

clearly seen that the curve obtained using 600 realizations is smoother and, hence, more accurate. From the knowledge of the CDF it is possible to extract the value of the probability to obtain a DA smaller than, e.g., 12σ at $15°$. In particular, to assess the minimum number of realizations required, it is possible to determine such a probability as a function of the number of realizations used. Such a quantity is plotted in Fig. 7. The probability is a decreasing function of the number of realizations. Furthermore, this quantity seems to indicate that the asymptotic value is reached for about 300 realizations, thus confirming the observations made on other quantities.

It is worth mentioning that Extreme Value Theory (EVT) could also be applied to study the statistical behaviour of the DA, similarly to what is presented in Ref. [10].

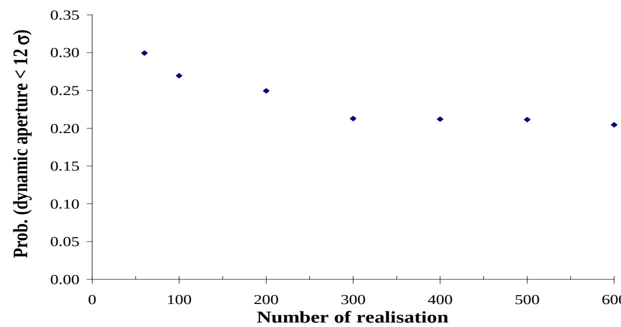

Figure 7: Probability for the DA at $15°$ to be smaller than 12σ as a function of the number of realizations used to computed the CDF.

CONCLUSIONS

In this paper a detailed analysis of the accuracy of the numerical computation of the DA for the LHC was carried out. As far as the dependence of the DA on the number of phase space angles is concerned, the optimal value lies between 50 and 100. As far as the dependence on the number of realizations are concerned, the optimal value is around 300. These figures have to be compared with the ones used for the standard protocol, namely 5 angles and 60 realizations. Even with the increased computing power presently available, the optimal values derived in this paper seem to be out of reach. However, it is clear that at least a moderate increase to about 20 angles and 100 realizations seems feasible in terms of CPU-time and necessary to improve the reliability of the DA estimate.

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