PECULIARITIES OF INFLUENCE OF COHERENCY PROCESSES AT CHARGED PARTICLES CHANNELING ON PARTICLE BEAMS CHARACTERISTICS

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Abstract

Peculiarities of influence of coherency processes at relativistic and nonrelativistic charged particles channeling on spatial and angular characteristics of particle beam that has passed through a thin crystal are discussed. In was shown, that the influence of different particle states interference within the area of coherent channeling leads to very strong periodic dependence of final beam angular width from the crystal length. This effect allows to control beam parameters (e.g., to form narrower beam, that it was before falling on the crystal). Influence of coherency of particle states in a single channel and several channels on the angular distribution is also studied.

INTRODUCTION

The process of charged particles channelling in crystals is described by the laws of a quantum mechanics. Such description leads to the existence of region of mutual coherency z < $L_{coh(\beta n,\alpha k)}$ of different channeling states with wave functions $\psi_{\beta n}(\vec{r}_\perp, z \ge 0, t)$ and energies $\varepsilon_{\beta n}$ as within the bounds of one channel (states in one channel are denoted by Latin indices), as in different channels (Greek indices) near the entering crystal surface. In the work [1] it was shown that $L_{coh(\beta n,\alpha k)}$ value lies in the interval from 1μ for heavy nonrelativistic and light subrelativistic particles up to 20μ for superrelativistic electrons and positrons. The existence of region of mutual coherency leads to the number of new effects, that have no analogue in systems, where only noncoherent channeling is examined (at such channeling different quantum states are not connected by determine phases).

The quantification of transverse motion leads to the quantification of particle transversal momentum $p_{\beta nz}$, wave vector $k_{\beta nz}$

$$p_{\beta nz} \equiv \hbar k_{\beta nz} = \frac{1}{c}\sqrt{E^2 - (m_0c^2 + \varepsilon_{\beta n})^2} \approx \frac{1}{c}\sqrt{E^2 - m_e{}^2c^4} - \varepsilon_{\beta n} m_0 c / \sqrt{E^2 - m_0{}^2c^4} \quad (1)$$

and to the different phases of different wave functions in channel at different longitudinal coordinate z values.

$$\varphi_{\beta n}(\vec{r}_\perp, z \ge 0) e^{-iEt/\hbar} \equiv \varphi_{\beta n}(\vec{r}_\perp) e^{-i(Et - p_{\beta nz})/\hbar} \quad (2)$$

The full wave function of channeling particle in the region of mutual coherency is described by the superposition

$$\sum_{\beta}^{N} \sum_{n}^{N_\beta} C_{\beta n} \varphi_{\beta n}(\vec{r}_\perp) \exp\{-i(Et - p_{\beta nz} z)/\hbar\} \quad (3)$$

In the limits of mutual coherency range z < $L_{coh(\beta n,\alpha k)}$ different wave functions can interfere between each other. This fact leads to the existence of longitudinal coordinate intervals where several functions $\psi_{\beta n}(\vec{r}_\perp, z \ge 0, t)$ have the same or different by 2kπ phase (constructive interference), and intervals where phase their phase differs on (2k+1)π (destructive interference). In the regions of constructive interference possibility of channeling particles localization increases, destructive - decreases.

The influence of interference effects on particle beam, which has passed through a thin crystal in the plane channeling mode, is examined below.

Peculiarities of transparent beam motion in the coherent channeling mode it thin crystals

Let's consider particle beam, which has passed through thin crystal with the length $l < L_{coh(\beta n,\alpha k)}$, characteristics for the case, when l exactly corresponds to the length of constructive of destructive particle wave functions interference. Calculations are made for the plane channeling mode, for which $r_\perp = x$.

Coherent channeling in the bounds of one channel

The particle, which is falling on crystal, wave function in the range z ≤ 0 has the following form

$$\varphi_1(x, z \le 0) = \int_{-\infty}^{+\infty} f_0(k_x) e^{-i(k_x x + k_z z)} dk_x \quad (4)$$

Here $f_0(k_x)$ characterizes initial angular beam distribution at the channel entrance. Particle wave function in the limits of crystal channel is determined by the expression

$$\varphi_2(x, l \ge z \ge 0) = \sum_n C_n \varphi_n(x) e^{ik_{nz}z} \quad (5)$$

where $$C_n = \int_{-a/2}^{a/2} \varphi_1(x)\varphi_n(x) dx \quad (6)$$

- amplitudes of levels population in the channel with the width a.

Particle wave function after leaving crystal (behind the crystal back surface) has the form

$$\varphi_3(x, z \ge l) = \int_{-\infty}^{+\infty} f(k_x) \exp\{-i(k_x x + k_z z)\} dk_x \quad (7)$$

From the continuity condition of functions (5) and (7) at $z=l$ we can find function $f(k_x)$

$$f(k_x) = \sum_n C_n f_n(k_x),$$

$$f_n(k_x) \equiv e^{i(k_{zn}+k_z)l} \int\limits_{-a/2}^{a/2} \varphi_n(x) e^{ik_x x} dx \,, \qquad (8)$$

which determines directional diagram $|f(k_x)|^2$ of the beam, that has passed through the crystal.

At incoherent channeling mode phase correlations between different coefficients C_n are not determined and

$$\left|f_{incoh}(k_x)\right|^2 = \sum_n \left|C_n f_n(k_x)\right|^2 = \sum_n \left| C_n \int\limits_{-a/2}^{a/2} \varphi_n(x) e^{-ik_x x} dx \right|^2 \qquad (9)$$

In the case of coherent channeling directional diagram has fundamentally different form

$$\left|f_{coh}(k_x)\right|^2 = \sum_n \left|C_n f_n(k_x)\right|^2 + \sum_{n\neq m}\sum_m C_n^* C_m f_n^*(k_x) f_m(k_x) =$$

$$\sum_n \left| C_n \int\limits_{-a/2}^{a/2} \varphi_n(x) \exp\{-ik_x x\} dx \right|^2 + \qquad (10)$$

$$+ \sum_{n\neq m}\sum_m C_n^* C_m e^{-i(k_{nz}-k_{mz})l} \int\limits_{-a/2n}^{a/2} \varphi_n^*(x) e^{ik_x x} dx \int\limits_{-a/2}^{a/2} \varphi_m(x') e^{-ik_x x'} dx'$$

and depends on interference of different own functions.

The maximal influence of channeling processes on beam directional diagram is in the case of parabolic channel usage. The spectrum of transverse motion energy levels ε_n and spectrum of transverse wave numbers k_{nz} are equidistant for such channels.

In nonrelativistic case we have

$$k_{nz} = \sqrt{2m(E - \hbar\omega(n+1/2))}/\hbar \approx$$
$$k_z(1 - n\hbar\omega/2E) = k_z - n\delta k_z \qquad (11)$$

In relativistic case

$$k_{nz} \approx \frac{\sqrt{E^2 - m_e^{\,2} c^4}}{c\hbar} - \frac{\hbar\omega(n+1/2) m_e c}{\hbar\sqrt{E^2 - m_e^{\,2} c^4}} = k_z - n\delta k_z \qquad (12)$$

where $\varepsilon_n = \hbar\omega(n+1/2)$, $\omega = \sqrt{8V_0/m_e a^2}$, V_0 - the height of channel walls.

From (10) follows that in a general case coherent cophased and antiphased modes of coherent channeling correspond to the conditions

$$(k_{nz} - k_{mz})l \equiv (n-m)\delta k_z l = 2k\pi; k = 0,1,2,...,$$
$$(k_{nz} - k_{mz})l \equiv (n-m)\delta k_z l = (2k+1)\pi; k = 0,1,2,... \qquad (13)$$

Let's consider peculiarities of nonrelativistic protons and relativistic positrons passing through the thin crystal. For calculations the initial angular beam distribution of each of the beams is taken in the form of symmetrical relative to the crystal axis Gaussian function

$$f_0(k_x) = \exp\{-k_x^2 / dk_x^2\} \qquad (14)$$

a) Peculiarities of nonrelativistic protons passing through the thin crystal.

For estimations the following typical system parameters were chosen: a = 1 A, E = 10^5 eV, V_0 = 20 eV, beam divergence $\Delta\theta_0 = 0.5'$, that corresponds to $dk_x = 10^8 sm^{-1}$.

On fig. 1 and 2 beam angular distributions calculated on the basis of formula (10) are presented. Fig. 1 corresponds to the coherent mode through the crystal with the length l, which corresponds to the cophased mode, fig. 2 - antiphased. Angular distribution on fig. 3 corresponds to the noncoherent channeling mode, formula (9).

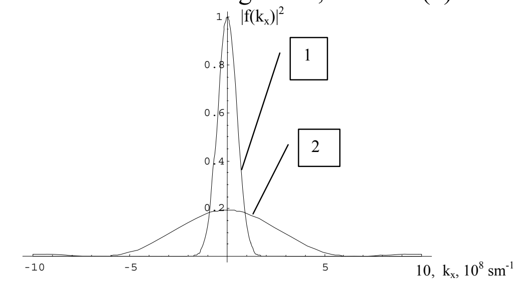

Figure 1: Angular distribution of proton beam, initial (1) and final in cophased coherent channeling mode (2)

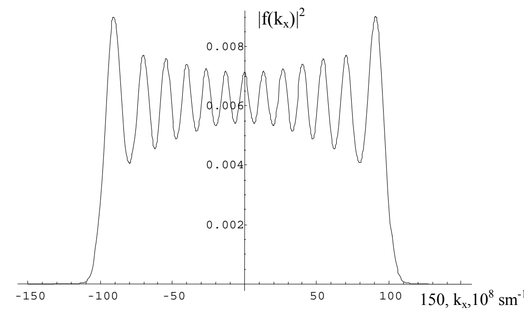

Figure 2: Angular final distribution of proton beam in antiphased coherent channeling mode

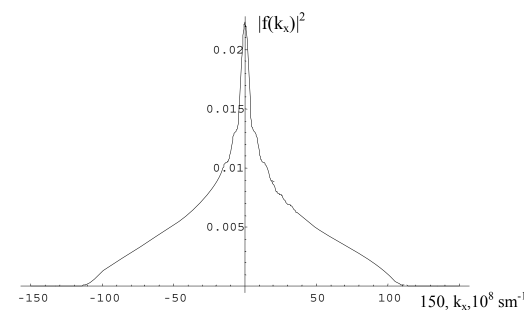

Figure 3: Angular final distribution of proton beam in noncoherent channeling mode

Dependencies presented demonstrate that at initial bean divergence $\Delta\theta_0 = 0.5'$ for cophased mode at l = l_{phased} final divergence is nearly $\Delta\theta \approx 2'$, for noncoherent channeling mode and antiphased at (at l = $l_{antiphased}$) $\Delta\theta \approx 1{,}5^o$. So simple crystal length change (periodical) can lead to the change width of passing beam in 45 times, at the condition of coherent channeling on the total crystal length.

b) Peculiarities of relativistic positrons passing through the thin crystal.

For the estimations of influence of channeling on characteristics of passing beam another set of typical parameters has been chosen - a = 1 A, E = 10^7 eV, V_0 =50 eV, $\Delta\theta = 0.65'$, that corresponds to $dk_x = 10^8 см^{-1}$.

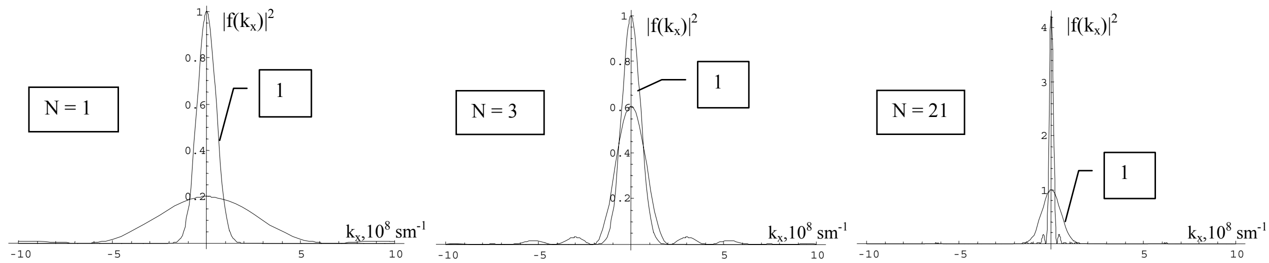

Figure 7: Angular distribution of relativistic positrons beam after channeling in cophased mode for coherency between N=1, 3, 21 channels. Curve 1 describes initial angular distribution.

In fig. 4. angular distributions of initial beam and beam, that has passed through crystal in coherent cophased mode are shown. On fig. 5 beam angular distribution of beam, that has passed through crystal in coherent cophased, antiphased and noncoherent channeling modes.

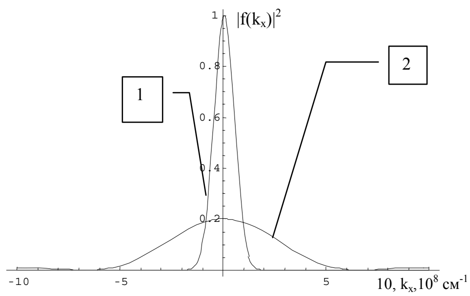

Figure 4: Angular distribution of positron beam, initial (1) and final in cophased coherent mode (2).

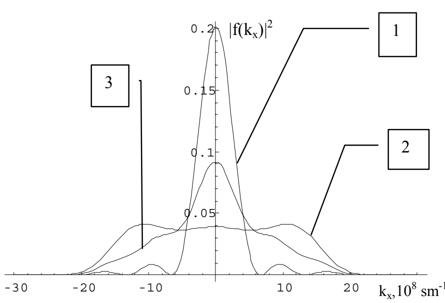

Figure 5: Angular distribution of positron beam, in cophased coherent (1) antiphased coherent (2) and noncoherent (3) modes.

At initial beam divergence $\Delta\theta_0 = 0.65'$ final divergence for cophased mode is nearly $\Delta\theta \approx 3'$, and for antiphased and noncoherent modes $\Delta\theta \approx 12'$.

Coherent particle channeling in the system of several channels

The process of channeling states excitation is connected with the continuity of particle wave function in the system of channels also.

The obvious generalization of a common solution (8) - (11) on a case of several channels allows to find expression for a directional diagram of particle beam that has passed through N parallel channels in coherent channeling mode.

$$\left|f_{coh}(k_x)\right|^2 = \frac{1}{N}\left|\sum_{\beta=1}^{N}\sum_{n} C_n e^{i(k_{nz}-k_z)l} \int_{-a/2}^{a/2} \varphi_{1m}(x) e^{-ik_x(x+\beta a)} dx\right|^2 \quad (17)$$

On fig. 6 directional diagrams of initial falling beam and beam after the crystal in antiphased own functions interference mode at coherent channeling with 1 and 7 channels are presented.

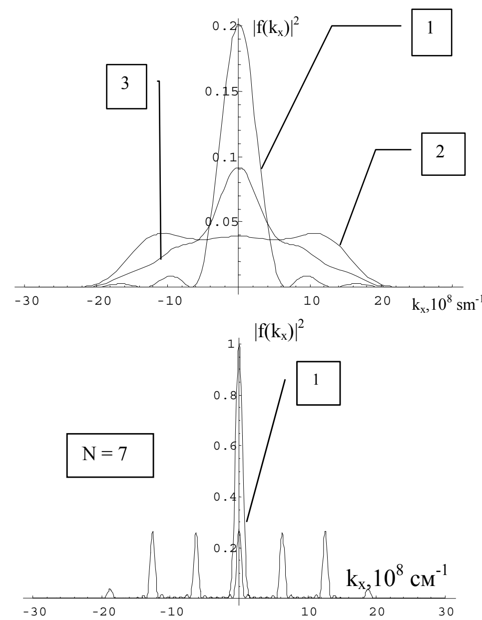

Figure 6: Angular distribution of relativistic positron beam after channeling in antiphased mode for coherency between N=1 and 7 channels. Curve 1 describes initial angular distribution.

It can be seen that the total beam divergence on channel exit in antiphased mode for the case of coherency between several channels is the same, as for the single channel, only angular structure differs.

On fig. 7 directional diagrams of the same initial falling beam and beam after the crystal in cophased mode at existence of coherency between 1, 3, 21 channels are presented. From the results presented very important conclusion can be made - at the existence of coherency between several channels angular beam width with the increase on channels number decrease, and beam can be narrower, than initial one! At N=21 angular width of final beam is in 5 times less that at the entrance of the crystal.

REFERENCES

[1]. V.I. Vysotskii and M.V.Vysotskyy, Surface #3 (2006) 93 (In Russian).