CORRECTION OF VERTICAL DISPERSION AND BETATRON COUPLING FOR THE TPS STORAGE RING

H.J. Tsai, H.P. Chang, P.J. Chou, C.C. Kuo, G.H. Luo, M.H. Wang
National Synchrotron Radiation Research Center
101 Hsin-Ann Road, Hsinchu Science Park, Hsinchu, Taiwan

Abstract

A proposed 3 GeV Taiwan Photon Source (TPS) is a low emittance (1.7 nm-rad) medium energy storage ring with 24 DBA cells. The vertical emittance due to betatron coupling and spurious vertical dispersion generated by the magnet errors and off-center orbits in sextupoles and quadrupoles are analyzed. The sensitivities due to magnetic alignment errors are estimated. Using the SVD method, the result of global vertical dispersion and betatron coupling correction is presented.

INTRODUCTION

The lattice TPS24P18K1 [1] of TPS, which consists of 24 double-bend cells with 6-fold symmetry and a circumference of 518.4m is shown in Figure 1. The designed natural emittance with slightly positive dispersion in the straight sections is 1.7 nm-rad. It is designed with small beam emittance to produce very high brilliance beams of synchrotron radiation. This low emittance lattice structure uses strong quadrupoles and sexturpoles. Therefore, imperfections in the magnet alignments and magnetic fields will lead to an increase of vertical emittance. Major sources causing the increase of vertical emittance are spurious vertical dispersion and betatron oscillation coupling. Spuirous vertical dispersion is caused by vertical bending error, quadrupole rotation error, vertical off-center errors in quadrupoles and vertical off-center errors in sextupoles in the dispersive region. Betatron coupling is due quadrupole rotation errors and vertical off-center errors in the sextupoles in the dispersive region. All these errors need to be eliminated or minimized in order to achieve a small vertical emittance of the stored beam.

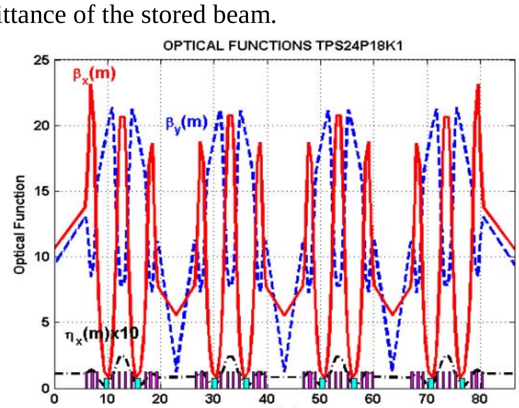

Figure 1: Optical functions of the TPS (distributed dispersion type, 24P18K1).

#jacky@nsrrc.org.tw

The emittance ratio of both planes is defined as $\kappa = \varepsilon_y / \varepsilon_x$. The photon brilliance is inversely proportional to the emittance product of both planes. Therefore, it is required to have small emittance ratio, say 1%, in the routine operation to get high photon brilliance. In this report, the generating sources and their contributions to the vertical emittance are analyzed. The correction method and the skew positions and required strengths are also described.

SPURIOUS VERTICAL DISPERSION

Vertical dispersion directly causes the increase of vertical emittance by quantum excitation. The vertical emittance can be calculated as following:

$$\varepsilon_y = \frac{c_q\gamma^2\langle I_y\rangle}{\rho} = \frac{c_q\gamma^2}{\rho}\left\langle[\eta_y^2+(\eta_y\alpha_y+\eta'_y\beta_y)^2]/\beta_y\right\rangle_{dipole}$$
$$\approx 2\frac{c_q\gamma^2}{\rho}\left\langle\eta_y^2/\beta_y\right\rangle_{dipole}$$

where the quantum constant $C_q = 3.84\times10^{-13}m$, the relativistic factor $\gamma = 5871$ at 3 GeV, ρ = bending radius. Hence,

$$\left\langle\eta_y^2/\beta\right\rangle_{dipole} = \frac{1}{8\sin^2\pi\nu_y}\left(\sum_i\frac{\beta_{yi}L_i^2}{\rho^2}\Delta\vartheta^2_{i,dipoles}+\sum_i\beta_{yi}k_{1i}L_i^2\Delta y^2_{co,i,quad}\right.$$
$$\left.+\sum_i\beta_{yi}k_{1i}\eta_{xi}^2L_i^2\Delta\vartheta^2_{i,quad}+\sum_i\beta_{yi}k_{2i}\eta_{xi}^2L_i^2\Delta y^2_{co,i,sext}\right),$$

where

$\Delta\vartheta_{i,dipoles}$: vertical dipole error due to dipole rotation errors.

$\Delta y_{co,i,quad}$: vertical off-center in quadurpoles .

$\Delta\vartheta_{i,quad}$: quadrupole rotation errors in the dispersive region.

$\Delta y_{co,i,sext}$: vertical off-center in sextupoles located in the dispersive region.

The contributions to the vertical dispersion from each driving term can be analyzed. Vertical emittance can be calculated as following:

$$\varepsilon_y(nm-rad) = 1.2\times10^{-2}\,\Delta\vartheta^2_{dipole}+6.2\times10^{-1}\Delta y^2_{co,quad}$$
$$+1.1\times10^{-2}\Delta\vartheta^2_{quad}+2.4\times10^{-1}\Delta y^2_{co,sext}$$

where magnet roll ϑ is in units of mrad and off-center Δy_{co} is in units of mm. A corrected orbit with zero-COD still generate vertical dispersion because the orbit is off the magnet center by an amount equivalent to the alignment error. We assume that the rotation errors are 0.2 mrad rms in dipole, 0.1 mrad rms in quadrupole and sextupole. The orbit offset with respect to the quadrupole and sextupole center are varied from 0.01 mm rms to 0.25 mm rms in the calculations. Figure 2 shows the vertical emittance generated from the those assumed conditions.

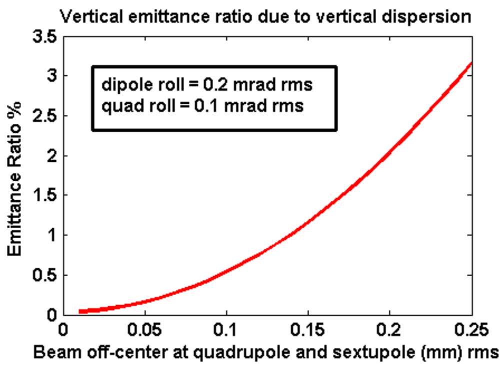

Figure 2: Vertical emittance from spurious vertical dispersion generated from dipole roll, quadrupole roll, and beam off-center at quadrupole and sextupole.

LINEAR BETATRON COUPLING

For betatron coupling, the skew quadrupole compoments k_s can be generated from the quadrupole rotation errors $k_s = 2k_1\Delta\vartheta$ and vertical off-center distortions in sextupoles $k_s = k_2\Delta y_{co}$. Considering the single linear coupling resonance $\nu_x - \nu_y = \ell$ and assuming only the skew quadrupole components k_s, the coupling driving strength is

$$G_{1,-1,l}e^{i\chi} = \frac{1}{2\pi}\oint\sqrt{\beta_x\beta_y}\,\frac{k_s}{B\rho}e^{i[\varphi_x-\varphi_y-(\nu_x-\nu_y-l)\theta]}ds\,.$$

where G is coupling driving strength. Assuming that different errors are uncorrelated and the phase advance between errors is random distrubution, this last equation can be rewritten as:

$$\langle G^2\rangle = \frac{1}{(2\pi)^2}\left[\Delta\vartheta_{quad}\sum_i(2k_1 l)^2\beta_{x,i}\beta_{y,i} + \Delta y_{co,set,}\sum_i(k_2 l)^2\beta_{x,i}\beta_{y,i}\right]$$

where $\Delta\vartheta_{quad}$ is quadrupole rotation errors and $\Delta y_{co,sex}$ is vertical off-center in sextupoles. We can calculate the vertical emittance due to betatron coupling analytically. The linear coupling driving strength is expressed as:

$$G^2 = 1.1\times10^{-3}\Delta\vartheta_{quad}^{\ \ 2} + 4.1\times10^{-3}\,\Delta y^2_{co,sex}\,,$$

The emittance ratio is defined as $\kappa = \frac{G^2}{G^2+2\Delta^2}$, $\Delta = \nu_x - \nu_y - \ell$.

The contributions from off-centered beam at sextupole are important. Therefore, the alignment of sextupole magnets needs to be well controlled. The COD should be corrected to an acceptable value. Assuming that the quadrupole rotation is 0.1 mrad rms, we can calculate the emittance ratio due to betatron coupling as a function of orbit offset at the sextupole location as shown in Figure 3.

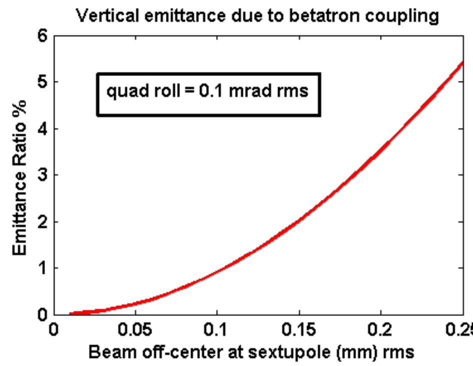

Figure 3: Vertical emittance due to betatron coupling as a function of off-centered beam position at sextupoles. A quadrupole roll error of 0.1 mrad rms is assumed.

ESTIMATES FOR TPS

The typical alignment errors which are significant for betatron coupling and vertical dispersion are:

- Dipole rotation errors : 0.2 mrad (rms).
- Quadrupole rotation errors in the dispersive region : 0.1 mrad (rms).
- Quadurpole transverse displacement error: 0.1 mm (rms).
- Sextupoles transverse displacement error: 0.1 mm (rms).

The analytical results of driving terms including vertical dispersion, vertical emittance and vertical emittance ratio, are shown in Table 1. The total vertical dispersion generated is estimated to be 11.5 mm rms without residual COD distribution, which corresponds to an emittance ratio of 0.54%. The RMS vertical dispersion after orbit correction [2] for 200 sampling machines around the storage ring was simulated by AT [3] using the same set of misalignment errors. The simulated mean value is 10.57 mm rms with residual COD distribution as shown in Figure 4.

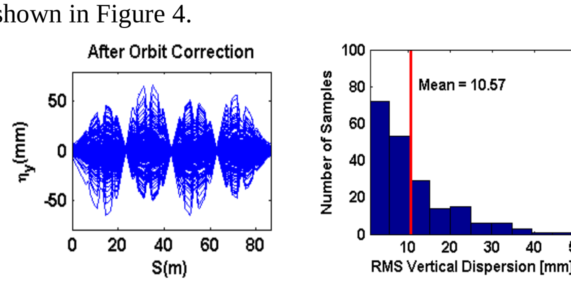

Figure 4: Simulation of residual RMS vertical dispersion after orbit correction for 200 sampling machines. The mean value is 10.57mm rms.

Table1: The analytic estimates of alignment sensitivities.

Error Type (rms) ($<\beta_{bend}> = 19.1$[m])	Quantity	F(Driving Term)		$<\eta^2/\beta>$[m]	$<\eta_y>$[m]	ε_y[m-rad]	$\varepsilon_y/\varepsilon_x$
Dipole Rotation: 0.2mrad	$\Delta\theta$	$\Delta\theta/\rho$	2.76E-05	1.33E-07	1.60E-03	4.85E-13	0.03%
Quadrupole Rotation: 0.1mrad	$\Delta\theta$	$\kappa 1\ \eta x\ \Delta\theta$	1.91E-05	2.91E-08	7.46E-04	1.06E-13	0.01%
Vertical Quadrupole Position: 0.1mm	Δy	$\kappa 1\ \Delta y$	1.38E-04	1.69E-06	5.69E-03	6.17E-12	0.36%
Vertical Sextupole Position: 0.1mm	Δy	$\kappa 2\ \eta x\ \Delta y$	2.45E-04	6.42E-07	3.51E-03	2.34E-12	0.14%
				Total:	1.15E-02	9.10E-12	0.54%

The analytical results of vertical emittance due to betatron coupling from quadurpole rotation and sextupole position error are shown in Table 2. The betatron coupling from sextupole position errors dominates the overall emittance ratio, with an estimated contribution of 0.73% compared to 0.2% from quadurpole rotations. The overall betatron coupling is estimated to be 0.93%. The total value of the emittance ratio that we find is 1.47% without residual COD contribution. However, after COD correction, the residual COD in quadruplole and sextupole are 0.041 mm rms and 0.042 mm rms respectively. Take residual COD into account, the emittamce ratio becomes 2.77%, with the contributions of 1.01% from vertical dispersion coupling and 1.76% due to betatron coupling respectively. The 3rd-generation light sources have to reduce the emittance ratio below 1%. It is required that skew quadrupoles be installed for coupling correction in the TPS storage ring.

Table 2: Betatron coupling from quads and sextupoles.

Errot Type (rms)	Δ	G	k
Quadrupole Rotation: 0.1 mrad	5.3E-2	3.32E-03	0.2%
Vertical Sextupole Position: 0.1 mm	5.3E-2	6.40E-03	0.73%

COUPLING CORRECTION

In the TLS, the vertical dispersion and betatron coupling strength were corrected using a set of skew quadrupoles with the cross orbit response method [4]. The singular value decomposition (SVD) algorithm is employed in the simulations. In this study, the skew quadrupoles are placed at location with high dispersion and high $\beta_x \beta_y$. So we place one skew in SF for one cell and 24 skew correctors for the whole ring.

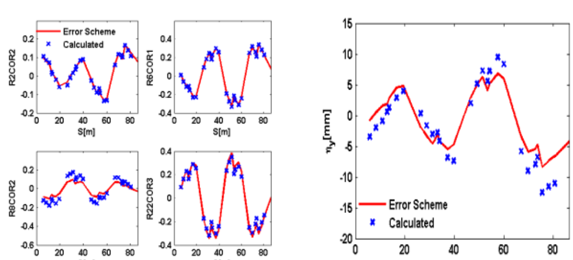

Figure 5: Comparison of one sampling machine and calculated in vertical orbit response and vertical dispersion.

Figure 5 shows the vertical orbit response of one sampling machine with respect to the corresponding horizontal kick. It's compared with the calculated response in 24 section, with 3 horizontal steerers in each section. The vertical dispersions of one sampling machine and calculation are also presented.

The maximun integrated skew strength is calculated around 5x10^{-3} m^{-1}. In Figure 6, the left side gives the mean rms value of skew strengths was 1.87x10^{-3} m^{-1} for 52 sampling machines, the right side gives the mean emittance ratio of 52 scheme of errors was 2.8% calculated by PATPET [5] with residual COD contribution. This value is consistent with results of the analytical calculation 2.77%.

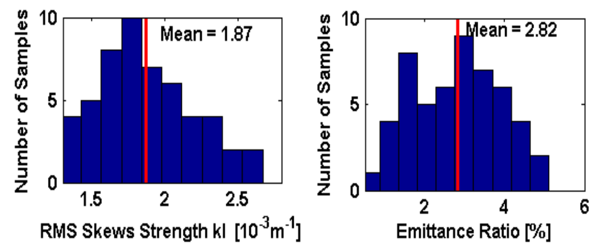

Figure 6: The left plot shows the rms value of skew strengths for 52 sampling machines and the right one is the mean emittance ratio of 52 sampling machines calculated by PATPET.

CONCLUSION

We can use cross plane response and SVD methods for the correction of the betatron coupling and vertical dispersion simultaneously with a set of skew quadrupoles as we have applied in the TLS. The skew quadrupoles alignment error need to be carefully and controlled, which are built inside the sextupoles with additional coils.

REFERENCES

[1] C.C. Kuo, et al, "Lattice Study for Taiwan Photon Source", PAC'05, Tennessee, May 2005, p. 2989.

[2] H.J. Tsai, et al, "Closed Orbit Correction of TPS Storage Ring", this procedings.

[3] A. Terebilo, "Accelerator Toolbox for MATLAB" , SLAC-PUB-8732, May 2001.

[4] C.C. Kuo, et al, "Coupling Correction Study at NSRRC", PAC 2003,pp.890-892.

[5] H. Wiedemann, el al, "User's Guide for PATPET version 88.2" unpublished, 1997, SLAC.