A NEW ANALYTICAL METHOD TO EVALUATE TRANSIENT THERMAL STRESSES IN CYLINDRICAL RODS HIT BY PROTON BEAMS

A. Bertarelli, A. Dallocchio, T. Kurtyka, CERN, Geneva, Switzerland.

Abstract

This paper presents an analytical solution for the thermo-mechanical problem of CNGS target rods rapidly heated by fast extracted high energy proton beams. The method allows the computation of the dynamic transient elastic stresses induced by a proton beam hitting off-axis the target.

The studies of such dynamic thermo-mechanical problems are usually made via numerical methods. However, an analytical approach is also needed to quickly provide reference solutions for the numerical results.

An exact solution for the temperature field is first obtained, using Fourier-Bessel series expansion. Quasi-static thermal stresses are then computed as a function of the calculated temperature distribution, making use of the thermoelastic displacement potential for the equivalent isothermal two-dimensional stress problem. Finally, the contribution of dynamic stresses due to longitudinal and bending stress waves is determined by means of the modal summation method.

This method can be effectively applied to any solid having cylindrical shape, made out of isotropic elastic material.

INTRODUCTION

The interaction of high energy particle beams with solids may lead to critical problems in the structural design of such devices like collimators and targets.

This paper deals with the study performed on the cylindrical graphite targets used in the CNGS experiment [1]. The CNGS project consists in producing a neutrino beam at CERN and sending it towards the Gran Sasso INFN laboratory in Italy. A beam of this type is generated from collisions of a proton beam with protons and neutrons in a graphite target.

If a target is hit off-axis, which is usually the case due to mechanical misalignment, both longitudinal and bending thermo-elastic vibrations are excited, thus developing dynamic stresses which could reach high values.

The method presented in this paper has been also applied to the preliminary study of an LHC collimator submitted to a robustness test simulating the injection error at 450 GeV [2].

In fact, the analytical model presented, has a general validity: it can be directly applied to any cylindrical rod and simply adapted to the case of rectangular bar. For studies of more complex geometries, the analytical method is useful to perform preliminary analyses and obtaining reference solutions for further numerical calculation.

The last section of the paper is dedicated to the comparison of the analytical model with a numerical one developed at CRS4 research centre, Sardinia, Italy and solved with Spectral Element Method [3].

MODEL PARAMETERS AND MAIN ASSUMPTIONS

The system has been modelled as a simply supported rod of cylindrical cross section. As it is shown in Figure 1, the rod is considered axially free to expand (z direction), while the displacements at the ends of the bar are inhibited for the transversal x and y directions. The rod length L is 100 mm and the radius R is equal to 2 mm.

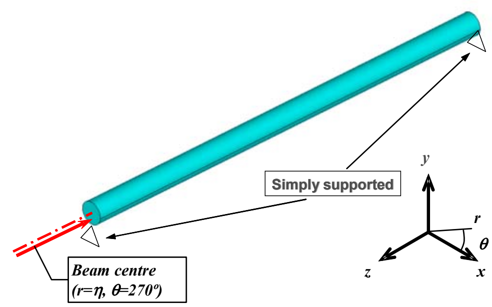

Figure 1: Target rod scheme and reference system; simply supported cylindrical rod hit off–axis by a fast extracted beam of 3.5 x 1013 p at 400 GeV over an extraction time τ of 10 μs.

In order to simplify the analytical approach, thermal and mechanical properties of graphite are assumed to be constant with temperature (Table 1).

Table 1: Graphite Properties assumed for the analysis

Density	ρ	1758 kg/m^3
Mean Specific Heat	c	1350 J/kgK
Mean Thermal Conductivity	K	70 W/mK
Mean Thermal Expansion Coefficient	α	3.9 10^{-6} K^{-1}
Young's Modulus	E	9.3 MPa
Poisson's Ratio	ν	0.032
Thermal diffusivity	κ	K/ρc

The energy distribution is supposed to be constant over the length of the cylinder making the problem two-dimensional; this assumption is confirmed by Monte-Carlo simulation performed with FLUKA® code.

The energy deposition due to the beam impact has been modelled over the cross-section of the rod with a Gaussian distribution with a standard deviation σ equal to 0.63 mm and an eccentricity η equal to 1.5 mm with respect to the axis of the rod. The beam centre is at R=η and θ=270°C, considering the circular cross-section of the rod in polar coordinates.

Equation (1) represents the Gaussian distribution of deposited energy density in polar coordinates; U_{MAX} is the maximum specific energy equal to 8.61 x 10^5 *J/kg*.

$$U(r,\theta) = U_{Max} e^{-\frac{r^2+\eta^2+2r\eta\sin\theta}{2\sigma^2}} \quad (1)$$

THERMAL ANALYSIS

During the 10 µs beam pulse, the energy deposited in the rod is assumed to increase linearly. Due to the rapidity of the phenomenon, the system can be considered adiabatic, thus the initial temperature distribution at the end of the beam pulse can be simply evaluated dividing the specific energy U(r, θ) by the specific heat c. At the end of the beam impact, the heat conduction equation assumes the following form:

$$\nabla^2 T(r,\theta,t) = \frac{1}{\kappa}\frac{\partial T(r,\theta,t)}{\partial t} \quad (2)$$

Equation (2) has been solved in cylindrical coordinates applying the separation of variables method and obtaining a solution in terms of Fourier-Bessel expansion. The adiabatic hypothesis provides the boundary conditions for equation (2), while the temperature distribution at the end of the beam pulse is assumed as initial condition. The solution for (2) is:

$$T(r,\theta,t) = \sum_n \sum_s C_{n,s} J_n(\lambda_{n,s} r) \cdot e^{-\kappa \cdot \lambda_{n,s}^2 \cdot t} \cdot H_n(\theta) \quad (3)$$

In the expression (3) J_n is a Bessel function of the first order, while $C_{n,s}$ and $\lambda_{n,s}$ are the coefficient of the expansion and the eigenvalues of the problem respectively; $H_n(\theta)$ is the Fourier expansion with which the initial temperature distribution has been approximated.

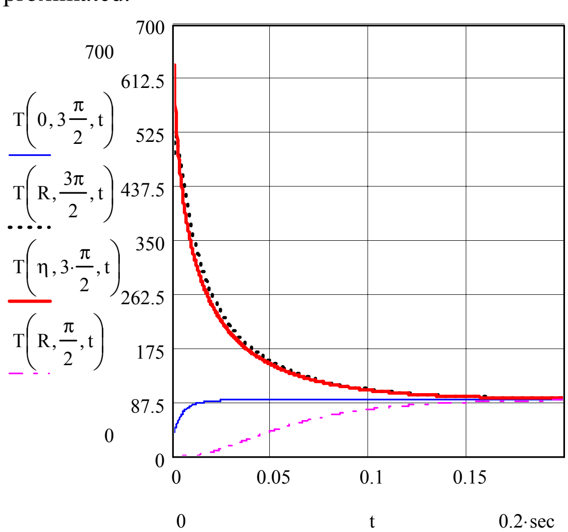

Figure 2: Temperature distribution as a function of time at different location along the radius of the cylinder.

STRESS ANALYSIS

Quasi-static Stresses

Once the temperature distribution is known, quasi-static stresses can be evaluated applying the Goodier's method [4] [5]. According to this procedure, stress components are calculated from the superposition of two effects: a stress field deriving from the application of a displacement potential ψ(r,θ,t) which satisfies the thermo-elastic equation, but not the mechanical boundary conditions (free-boundary), and a stress field due to isothermal pressure loads applied on the outer surface to restore the free-boundary condition.

The relationship between the temperature distribution and the displacement potential for a two-dimensional plain-strain thermo-elastic problem is given by:

$$\frac{\partial \psi(r,\theta,t)}{\partial t} = \frac{1+\nu}{1-\nu}\alpha\kappa T(r,\theta,t) \quad (4)$$

The stress components have been obtained using the classic stress-strain relationship in cylindrical coordinates [5]. Figure 3 shows the distribution of quasi-static stress components, the axial stress has been calculated assuming no axial strain.

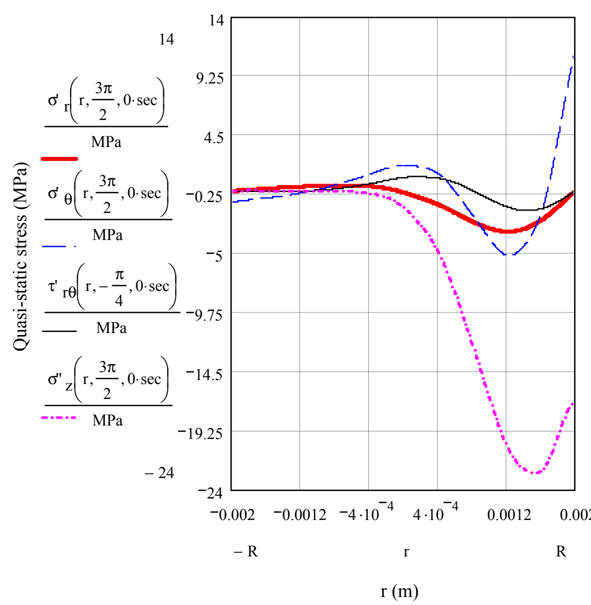

Figure 3: Quasi-static stress distribution – radial, tangential, shear and axial stress components (εz = 0).

Dynamic Stresses

The effects of a fast extracted proton beam on a target provoke a rapid non-uniform temperature rise.

Thermal expansion of the rod is prevented because of inertia effect, thus exciting thermo-elastic stress wave propagation. The evaluation of the thermally induced axial and flexural vibrations has been done making use of the modal summation method, [6]. Assuming that transverse displacements are small, axial and flexural behaviour could be studied separately.

The non-uniform eccentric temperature distribution has been modelled with an equivalent axial force and a bending moment applied at the extremities of the rod.

The equivalent loads raise up with a ramp from zero to a constant value during the beam pulse τ; studying the time response of the system to these dynamic loads it is possible to obtain axial and bending dynamic stresses [7].

Total axial stress acting on the rod can be simply calculated by summing the quasi-static and dynamic stress components. Figure 4 shows the dynamic longitudinal and flexural stresses as a function of time.

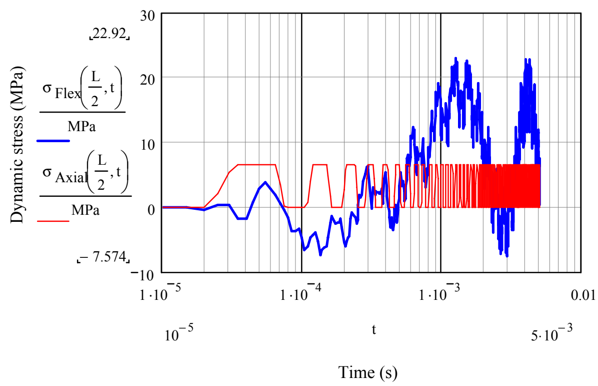

Figure 4: Dynamic stress due to longitudinal and flexural vibrations; first longitudinal and flexural frequencies of vibrations are respectively 11500 *Hz* and 361 *Hz*.

COMPARISON WITH NUMERICAL RESULTS

The presented analytical model has been compared with a more detailed and complex numerical analysis performed at CRS4 [3] on the same target rod.

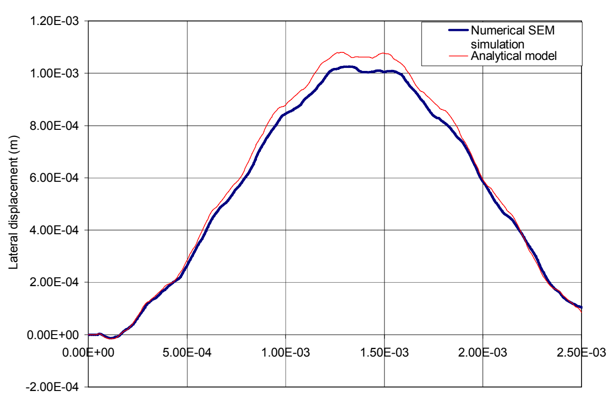

Figure 5: Lateral displacement as a function of time at the rod centre – comparison between numerical and analytical models.

Though the analytical method makes use of constant material properties independent of the temperature and of a simplified energy distribution, the results are in very good agreement with the numerical Spectral Element Method simulation as shown in Figure 5.

CONCLUSIONS

An analytical method to calculate transient dynamic thermal stresses in a graphite cylindrical target rod due to a fast extracted proton beam has been developed.

Temperature distribution, quasi-static thermal stresses and thermally induced vibrations have been evaluated.

Results from the analytical model have been compared with a very detailed numerical simulation. The two approaches are in very good agreement concerning stresses and displacements, in spite of the simplifications adopted in the analytical method.

On account of this, it is possible to conclude that the presented analytical model could be effectively used to simply and quickly provide good reference solutions for the mechanical design of devices submitted to high energy particle beam impact.

ACKNOWLEDGMENTS

Useful discussions with L. Bruno (CERN AB/ATB) are gratefully acknowledged.

REFERENCES

[1] K. Elsener et al., General Description of the CERN Project for a Neutrino Beam to Gran Sasso (CNGS), CERN AC note 2000-03, CERN, Geneva, 2000.

[2] A. Bertarelli, A. Dallocchio et al., Permanent Deformation of the LHC Collimator Jaws Induced by Shock Beam Impact: an Analytical and Numerical Interpretation, These Proceedings, Edinburgh, 2006.

[3] L. Massidda, Dynamic Structural Analysis of the CNGS Target Rods – Graphite CL2020PT Additional Analyses, CRS4 TECH REP, Centre for Advanced Studies Research and Development in Sardinia, Italy, 2005.

[4] J.N. Goodier, On the Integration of the Thermo-elastic Equations, Phil. Mag. (seventh series), 23, 1017-1032, 1937.

[5] S.P. Timoshenko, J.N. Goodier, Theory of Elasticity, Chapter 13, McGraw-Hill, New York, 3rd ed. 1970.

[6] W.T. Thomson, Theory of Vibration with Applications, Chapter 11, Chapman & Hall, London, 4th ed. 1993.

[7] A. Bertarelli and T. Kurtyka, Dynamic Thermo-Mechanical Phenomena Induced in Isotropic Cylinders Impacted by High Energy Particle Beam, Proceedings of the 8th International Conference on Structures Under Shock Impact - SUSI VIII, Crete, Greece, 2004.