

# COMPARISON OF MEASURED AND CALCULATED COUPLING BETWEEN A WAVEGUIDE AND AN RF CAVITY USING CST Microwave Studio®

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## *Abstract*

Accurate predictions of RF coupling between an RF cavity and ports attached to it have been an important study subject for years for RF coupler and higher order modes (HOM) damping design. We report recent progress and a method on the RF coupling simulations between waveguide ports and RF cavities using CST Microwave Studio® in time domain (Transit Solver). Comparisons of the measured and calculated couplings are presented. The simulated couplings and frequencies agree within  $\sim 10\%$  and  $\sim 0.1\%$  with the measurements, respectively. We have simulated couplings with external  $Q$ s ranging from  $\sim 100$  to  $\sim 100,000$ , and verified with experimental measurements. The method also works well for higher  $Q$ s, and can be easily applied in RF power coupler designs and HOM damping for both normal-conducting and superconducting cavities.

## INTRODUCTION

RF coupler design and higher order modes (HOMs) damping studies are critical in cavity designs for high power and high intensity applications. An accurate prediction of the coupling between a cavity structure and ports that attached to it is one of the main tasks in a cavity design. The coupling is described by an external quality factor,  $Q_{\text{ext}}$  or a coupling factor  $\beta$  in a relationship with  $Q_0$ ,

$$\beta = \frac{Q_0}{Q_{\text{ext}}}. \quad (1)$$

Kroll and Yu [1] developed a method in frequency domain to computer  $Q_{\text{ext}}$  indirectly. It requires simulation results of the resonant frequencies of the cavity-waveguide coupling system at different lengths of the waveguide. The induced phase variations are fitted numerically with a theoretical formula to find the coupling. It works very well for a strong cavity and waveguide coupling system. In case of a weak coupling where  $Q$  is high, fitting the phase variation curve demands very high accuracy of simulation of the resonant frequency. This could be difficult for many 3-D codes. P. Balleyguier proposed another frequency domain method [2, 3] that has been widely used for applications in superconducting cavity. It transforms the traveling-wave problem to a standing-wave one. It simulates the cavity-waveguide system by alternating boundary conditions at the waveguide ports between electric and magnetic.

It works well for weak coupling cases where the resonant frequency of the cavity does not change significantly when alternating the boundary conditions.

D. Li proposed a method [4] to calculate the coupling in time domain using MAFIA T module. Ports or beam pipes that coupled to a cavity is simulated by waveguide boundary conditions where electromagnetic fields that satisfy the waveguide modes are propagating out without reflections. The boundary is then equivalent to a matched load or an infinite long transmission line with bandwidth specified by waveguide modes. An excitation source is required to provide initial fields in the cavity-port coupling system. The excitation source can be beam, dipole or by loading  $EM$  fields directly to waveguide ports or inside the cavity. The induced  $EM$  fields are recorded in the cavity as a function of time. Both the resonant frequency (with coupling) and external quality factor,  $Q_{\text{ext}}$  can be obtained from the recorded time varying field signal.

Taking advantage of better mesh technique and more friendly user interface in CST Microwave Studio®, we can now readily simulate RF coupling structures in the transient solver and calculate frequency and  $Q_{\text{ext}}$  using the same time domain method, but with much improved accuracy.

We just noticed the new version of the CST Microwave Studio® has a built-in feature for external quality factor calculations in frequency domain. We do not have any experience on it yet, but would like to learn more and check with existing methods.

## SIMULATION METHOD

The intrinsic and external quality factors of a resonant cavity are defined as

$$Q_0 = \frac{\omega U}{P_w}, \quad Q_{\text{ext}} = \frac{\omega U}{P_{\text{ext}}}, \quad (2)$$

respectively. Where  $\omega = 2\pi f$  with  $f$  as the cavity resonant frequency;  $U$ , the stored energy of the mode in the cavity;  $P_w$ , the RF power dissipation on cavity wall; and  $P_{\text{ext}}$ , the power coupled out of the cavity to matched waveguides. The  $Q_{\text{ext}}$  depends on coupling only while  $Q_0$  depends on cavity geometry. Similarly, a loaded quality factor,  $Q_{\text{load}}$  is defined as,

$$Q_{\text{load}} = \frac{\omega U}{P_w + P_{\text{ext}}}, \quad (3)$$

by summing up all the power dissipations in the cavity. A simple relationship between  $Q_0$ ,  $Q_{\text{ext}}$  and  $Q_{\text{load}}$  can be ob-

tained:

$$\frac{1}{Q_{\text{load}}} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}}. \quad (4)$$

Where  $Q_0$  can be further expressed as  $Q_0 = (1 + \beta)Q_{\text{load}}$ . All these  $Q$ s are measurable parameters of a cavity-port coupling system.

Assuming at time  $t = t_0$ , a cavity has a stored energy of  $U_0$ ; this stored energy will be consumed over time by power dissipations on cavity wall and coupling to waveguides provided there is no excitation source in the cavity. Mathematically this can be described by,

$$\frac{dU}{dt} = -(P_w + P_{\text{ext}}) = -\frac{\omega U}{Q_{\text{load}}}. \quad (5)$$

It is straightforward to solve the above equation and attain,

$$U(t) = U_0 \exp\left(-\frac{\omega t}{Q_{\text{load}}}\right). \quad (6)$$

The stored energy  $U(t)$  decays exponentially with time. Once the exponential decay of the stored energy is obtained, one can easily calculate  $Q_{\text{load}}$  through time decay constant of the field energy. In simulations, we usually set cavity material as perfect conductor (by default). In this case, power dissipation comes from coupling only, therefore  $Q_{\text{ext}} = Q_{\text{load}}$ .

In time domain simulations, the excitation source has to be defined first. The source needs to have a right bandwidth at locations where the *EM* modes that are under study can be excited. Individual mode (including the fundamental one) can be studied separately by selecting an excitation signal and location where only one mode is excited and also by narrowing the signal bandwidth. Once the excitation source is gone, the stored energy decays exponentially, as is described in Eq. 6. Monitors or probes placed in the cavity record the exponential decaying *EM* fields as a function of time with a pre-defined time step. Fig. 1 shows an example of MWS simulations with the excitation pulse (red) and the recorded field (blue) induced in the cavity. The excitation pulse lasts for  $\sim 40$  ns, and the induced fields gradually build up and reach to the maximum right before the pulse leaves the cavity. In MWS, the total stored en-

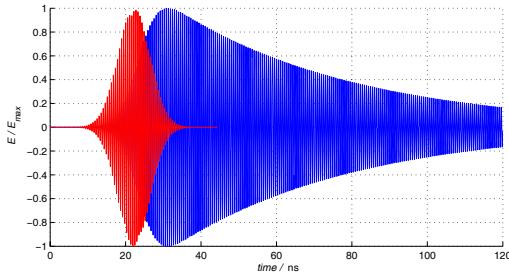


Figure 1: An excitation signal (red) and its induced field (blue) in a cavity.

ergy is normalized to a maximum value; the energy decay

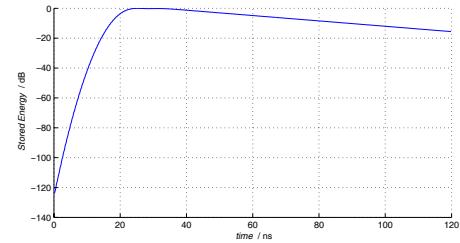


Figure 2: Stored energy decay as a function of time in a logarithmic scale (dB).

is expressed in unit of dB. A typical energy decay curve obtained from MWS simulations is shown in Fig. 2. The decay slope can be calculated by picking up two points in the linear decay section of the curve (log scale) or by linear fitting. The loaded  $Q$  is given by,

$$Q_{\text{load}} = -\frac{10 \lg e \cdot \omega}{k}, \quad (7)$$

where  $k$  is the slope obtained from the linear decay curve.

The recorded time varying field or energy signal can also be transformed to frequency domain by fast fourier transform (FFT). Fig. 3 shows the frequency domain presentation of the induced field signal (blue) in Fig. 1. If a resonant peak is fully resolved in the simulations, its frequency and  $Q$  value could be directly obtained by measuring the bandwidth at 3 dB points of the peak with a good accuracy. Nevertheless, the resonant frequency can also be calculated by counting the number of oscillations over a period using the signal from time domain simulations.

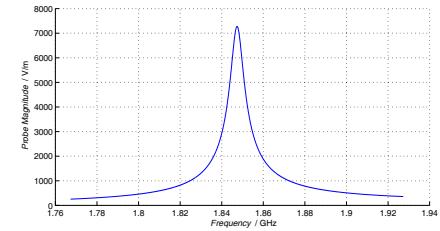


Figure 3: The recorded field signal in Fig. 1 is transformed to the frequency domain.

## THE 3-WAVEGUIDE LOADED CAVITY IN JLAB

The cavity shown in Fig.4 is a one-cell superconducting cavity model in JLab. The cavity has three waveguides at one side of beam pipe to damp HOMs [6]. The fundamental mode,  $\text{TM}_{010}$  is designed to be 1.5 GHz. The three waveguides are oriented to damp most harmful HOMs such as  $\text{TM}_{110}$  and  $\text{TE}_{111}$  and etc.

We simulated this cavity with CST Microwave Studio® where the ends of the waveguides are set as ports with matched load or transmission line with no reflection.

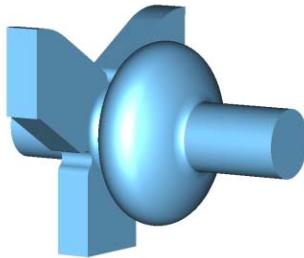


Figure 4: The 3-Waveguide-Loaded cavity in JLab.

The cavity body is set to be perfect conductor for the simulations, therefore the  $Q_{\text{load}} = Q_{\text{ext}}$ . The cavity field is excited by applying a pulse inside the cavity (or at one of the ports).

In order to compare the simulation results with measurements on a cold copper prototype at room temperature, we have included losses on copper cavity wall for the completeness of the simulations although it has neglectable effects.

The structure has a symmetry plane. To study the modes of different polarizations, we set the symmetry plane with either electric boundary condition ( $E_t = 0$ ) or magnetic boundary condition ( $H_t = 0$ ). Tab. 1 summarizes the simulation results with comparison of the measured results at JLab. Good agreements are achieved.

Table 1: Comparison between simulation and measurement of the JLab cavity.

CST Simulation		Measurement	
$f/\text{GHz}$	$Q_{\text{load}}$	$f/\text{GHz}$	$Q_{\text{load}}$
1.84727	276	1.848006	317
1.84764	264	1.848252	227
2.03046	719	2.029628	996
2.03055	746	2.030226	667
2.43190	2750	2.426183	2878

## COUPLER DESIGN FOR A STANDING-WAVE LINAC

Tsinghua University group has been conducting both traveling and standing wave (TW and SW) electron linac R&D for many years. Their linacs have been widely used in industry, hospitals, and more recently for cargo inspection systems. RF coupler design and tuning have been one of the time consuming process. An accurate calculation of the coupling is necessary for future linac design and improvement of existing designs. Fig. 5 shows a waveguide coupler design of Tsinghua University's standing wave linac.

External  $Q$  is simulated as a function of the coupling slot length. Mesh sizes at the coupling slot region are refined for better representation of the complex geometry there, and as consequently give higher accuracy. A cold

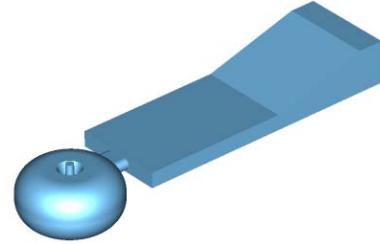


Figure 5: Waveguide coupler by slot.

test model is built for benchmarking the simulations against measurements (Tab.2). The agreement is very good.

Table 2: Results of the coupler for an S-W linac.

$l$ (mm)	CST Simulation		Measurement	
	$f/\text{GHz}$	$Q_{\text{ext}}$	$f/\text{GHz}$	$Q_{\text{ext}}$
18	3.0234	72225	3.0248	75958
20	3.0226	32333	3.0232	30717
22	3.0210	15995	3.0216	16099
24	3.0193	8517	3.0199	8257
26	3.0174	4793	3.0180	4928
28	3.0150	2822	3.0158	2643
30	3.0124	1728	3.0132	1614
32	3.0094	1094	3.0101	1058

## CONCLUSION

Numerical simulation of a cavity-port coupling system by taking advantage of waveguide boundary conditions in time domain provides a natural method for external  $Q$  calculations. Direct computation of the decay time constant from the induced field or energy in a cavity is perhaps one of the simplest ways to get  $Q_{\text{ext}}$  that works for both strong and weak couplings. The application of time domain method becomes very convenient with the transient solver in CST Microwave Studio®. Good agreements are achieved between simulations and measurements, at accuracy of  $\sim 0.1\%$  for frequency and  $\sim 10\%$  for external  $Q$ s. The method can be applied for HOM studies and coupler designs.

## REFERENCES

- [1] N. Kroll, D. Yu, *Computer Determination of the External  $Q$  and Resonant Frequency of Waveguide Loaded Cavities*, Particle Accelerators, Vol 34, p231, 1990.
- [2] P. Balleyguier, *A Straightforward Method for Cavity External  $Q$  Computation*, Particle Accelerators, Vol 57, p113, 1997.
- [3] P. Balleyguier, *External  $Q$  Studies for APT SC-Cavity Couplers*, Proceedings of LINAC-2000, p133.
- [4] Derun Li et al. *Calculations of External Coupling to a Single Cell RF Cavity*, Proceedings of LINAC1998, p977.
- [5] CST Microwave Studio®.
- [6] Private Communication.