# NOVEL METHOD FOR BEAM DYNAMICS STUDY USING AN ALPHA PARTICLE SOURCE 

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#### Abstract

A new method for beam dynamic study has been developed by using alpha particles emitted from a commercially available radioactive source. The precision of the method is estimated in consideration of an event rate in a reasonable measuring time and statistical accuracy. The method could be applied not only for a circular accelerator but also for a unit cell or any type of elements of an accelerator.


## INTRODUCTION

This paper proposes a new method for obtaining a simulated data set of beam dynamics in accelerator. Conventional methods to study the beam dynamics roughly fall into two categories: methods with injection of aimed particles and without injection. Although the former methods are ideal for precise studies, not only construction of an accelerator which to be studied but also an injector machine is necessary to complete the experiments. In the latter methods, a magnetic field is assumed from a field calculation by a computer code such as OPERA-3D or models of the magnets, or field measurements. Then tracking is performed by computer codes with assumed magnetic fields to study the beam dynamics. This is a general method which is performed in the design and construction phase of the accelerators to predict its performance. However, the computer tracking error and imperfection of the assumed field would influence on an accuracy of the prediction, especially nonlinear behavior of the beam. The method described below brings a less expensive, fruitful and comprehensive experimental technique for accelerator study, particularly in low energy secondary particles such as muons, short-lived elementary particles and radio-active nuclei.

## METHOD

The method consists in injecting alpha particles, which are emitted from a sealed alpha source through a degrader foil and a collimator, to an accelerator, and measuring their position and angle by detectors after passing some cells (see Fig.1). The injecting alpha particles must have an equal magnetic rigidity to the designed particles such as electrons, positrons, protons, antiprotons, and muons, in order to simulate particle's orbits in an accelerator. As the magnetic rigidity is defined as a particle's momentum per unit charge, the momentum of alpha particles must be twice

[^0]as large as that of the designed particles. The kinetic energy of alpha particles emitted from typical alpha sources such as ${ }^{241} \mathrm{Am},{ }^{210} \mathrm{Po}$ and ${ }^{148} \mathrm{Gd}$ is $\sim 6 \mathrm{MeV}$. Therefore the method can be applied for accelerators designed for a lower momentum of single charged particles than $\sim 200 \mathrm{MeV} / c$.


Figure 1: Schematic experimental setup.

## APPLICATION TO PRISM-FFAG

We would apply this new method to design and develop PRISM (Phase Rotated Intense Slow Muon source) [1]. PRISM aims to provide a dedicated source of high quality intense muons with a narrow energy-spread and small beam contamination to an experimental of $\mu-e$ conversion search [2]. A narrow energy spread of muons could be obtained by a phase rotation technique with rf acceleration at scaling type fixed-field alternating gradient synchrotron PRISM-FFAG [3]. The present method applied for PRISM-FFAG accelerator would provide a beam dynamics with alpha particles having the same magnetic rigidity as injecting muons. A single particle tracking by using simulated magnetic field for a large aperture sector magnet has demonstrated that PRISM-FFAG would have a transverse acceptance of about $38000 \pi \mathrm{~mm}$ mrad for the horizontal plane $\left(x-x^{\prime}\right), 5700 \pi \mathrm{~mm}$ mrad for the vertical plane ( $y-y^{\prime}$ ) and wide momentum acceptance of 68 $\mathrm{MeV} / \mathrm{c} \pm 20 \%$.

## Apparatus

We design a new device, ALPS (Alpha Particle Source, here in after abbreviated as ALPS) consists of ${ }^{241} \mathrm{Am}$ alpha source that generates alpha particles with an energy of 5.486 MeV (the branching ratio is $85.2 \%$ ), degrader foil and two collimators to meet the present requirement. The alpha particle source is selected from commercially available sources. The source ALPS would be placed at a straight section between $D F D$ sector magnets of the accelerator and could be moved in a radial position between $x=5700 \mathrm{~mm}$ and $x=6700 \mathrm{~mm}$ while in vertical between
$y= \pm 150 \mathrm{~mm}$ corresponding to the horizontal and vertical apertures of the magnets. The position resolution of the actuator for ALPS would be $\pm 0.5 \mathrm{~mm}$. An emitting angle could be adjusted $\pm 1$ radian with respected to the normal to the radial axis.

In the following, we assume that a ${ }^{241} \mathrm{Am}$ alpha source which has an activity of $3.3 \times 10^{6} \mathrm{~Bq}$ is assembled to the ALPS for particle tracking measurements. By placing a degrader foil with a proper thickness just in front of the source we can select momentum to equal to the momentum range of muons for PRISM-FFAG according to the simulation results. A $15.7 \mu \mathrm{~m}$ thick Al foil reduces the average momentum of alpha particle to $136 \mathrm{MeV} / c$, which corresponds to a magnetic rigidity of $68 \mathrm{MeV} / \mathrm{c}$ muons. The typical momentum distribution of alpha particles degraded is estimated as shown in Fig.2. For other momentum, proper thicknesses for aluminum degrader are $19.0 \mu \mathrm{~m}$ and $10.9 \mu \mathrm{~m}$ for muon momentum of $54.4 \mathrm{MeV} / c$ and $81.6 \mathrm{MeV} / c$ respectively.


Figure 2: Momentum distribution of emitted and degraded alpha particles estimated using Geant3.21 [4]. The emission energy and thickness of the Al foil were assumed as 5.486 MeV and $15.7 \mu \mathrm{~m}$ respectively .

In order to precisely measure dynamic properties, such as transverse and longitudinal motion, in a reasonable counting time, it would be desirable for ALPS to generate a fine particle beam like a pencil beam with an enough number of particles. Here, we describe a way to take into account statistical accuracy for the measurements. For a required precision in a limited counting time, we should choose a proper collimator design for ALPS among a various combination in consideration of statistical accuracy. We assume initially uniform distribution of emitting alpha particles from the source surface with 8 mm square (negligible thickness). We compare collimators with three parameters, distance between two collimators $(L)$, diameter $(D)$ and height $(h)$ of holes. For the simplicity, we assumed collimator thickness equal to zero. An example of distributions of $x$ and $x^{\prime}$ with collimators with $D=6 \mathrm{~mm}$, $h=6 \mathrm{~mm}, L=300 \mathrm{~mm}$ is shown in Fig.3.

Since the source ALPS would be located at the center of straight section and the detector would be at a proper position after each sector magnet, particle orbits in PRISMFFAG are described as a movement in an elliptical area of phase spaces, $x-x^{\prime}$ and $y-y^{\prime}$. We would measure the position and angular distribution as well as energy distribution of the particles at straight section in a desired sta-


Figure 3: Simulation results of spatial and angular distribution of alpha particles after collimators which have $D=6$ $\mathrm{mm}, h=6 \mathrm{~mm}, L=300 \mathrm{~mm}$.
tistical accuracy. The collimated particles have spread in energy, position and angle as shown in Fig. 2 and 3. These distributions transform each other by the betatron oscillation. We take means of the distributions of position, angle and energy to obtain a higher accuracy. The achievable accuracies would be dominated by statistical error. In order to estimate the accuracies of the measurement values, here we assume that each initial distribution are transformed maximally to othres. A maximum rms of the angular distribution at the detectors caused by the initial position distribution is described as $\sigma_{x^{\prime} \leftarrow \sigma_{x}}^{\max }=\sqrt{\gamma / \beta} \sigma_{x}$, where $\beta$ and $\gamma$ are Courant-Snyder parameters, and $\sigma_{x}$ is the initial rms of the spatial distribution. Similarly, the angular spread would be as $\sigma_{x \leftarrow \sigma_{x^{\prime}}}^{\max }=\sqrt{\beta / \gamma} \sigma_{x^{\prime}}$. Another influence comes from energy distribution of each particle since a small change of momentum $\Delta p$ causes a change of a radial equilibrium orbit $\Delta r$ as follows $\Delta r / r_{0}=\alpha\left(\Delta p / p_{0}\right)$, where $p_{0}$ is the central momentum, $r_{0}$ the equilibrium orbit, and $\alpha$ the momentum compaction factor. For PRISM-FFAG $\alpha$ is described by using its field index $k$ as $\alpha=1 /(1+k)$. Therefore we take into account rms due to a momentum spread of particle $\sigma_{x \longleftarrow \sigma_{p}}^{\max }=r_{0} \alpha\left(\sigma_{p} / p_{0}\right)$. The parameters $\beta$ and $\gamma$ are deduced from a simulation study of particle tracking for PRISM-FFAG. For this case, $\sqrt{\beta / \gamma}$ is 1.71 ( $\mathrm{mm} / \mathrm{mrad}$ ) for horizontal and $3.85(\mathrm{~mm} / \mathrm{mrad})$ for vertical.

Assuming the aimed accuracy of position measurements as $100 \sim 500$ micron, we estimate necessary counting times to achieve the accuracy. Table 1 shows the required counting time for some aimed accuracy case with collimator configurations, energy cuts, and required number of particles. It is worth noting that among these estimations, the measuring time required would be about a few minutes to 10 minute or less for getting a few hundreds micron precision. It will be an acceptable time as a feasible method for obtaining the dynamic properties for PRISM-FFAG parameters.

Table 1: Estimation of required counting time for the amid precision. Intensity of ${ }^{241} \mathrm{Am}$ source is assumed 3.0 MBq with its size of $8 \mathrm{~mm}^{2}$.

| $\sigma_{x}^{\max }(\mu \mathrm{m})$ | 100 | 100 | 200 | 500 |
| :---: | :---: | :---: | :---: | :---: |
| $D(\mathrm{~mm})$ | 6.0 | 12.0 | 6.0 | 4.0 |
| $h(\mathrm{~mm})$ | 6.0 | 12.0 | 2.0 | 2.0 |
| $L(\mathrm{~mm})$ | 300 | 300 | 300 | 300 |
| $\left\|p-p_{0}\right\|<(\mathrm{MeV} / c)$ | 1.0 | 1.0 | 2.0 | 2.0 |
| $N_{\text {net }}$ | 16476 | 48681 | 5548 | 514 |
| Counting Time (min.) | 6.4 | 2.1 | 3.6 | 1.1 |

## Measurements

This method would be applicable to various kinds of beam dynamics measurements such as closed orbit, tune, and acceptance measurement. These experiments would be helpful and significant for designing and constructing a cell or lattice as well as a total system for PRISM-FFAG accelerator since the secondary muon beam would be obtained solely after a completion of high current proton accelerator and pion decay channel. We describe procedures for measurements of a closed orbit and tune for PRISM-FFAG as examples.

For measurements of closed orbits and betatron tunes of PRISM-FFAG accelerator the source and detector are located on design orbit.The source ALPS would be located slightly deviate from proper equilibrium radius so that alpha particles will exhibit betatron oscillations. We could measure energy and radial distributions of alpha particles through each sector magnet at any lattice position, we could determine a closed equilibrium orbit: for instance the detector would be located at $\theta_{1}$ to $\theta_{9}$ just the middle of the straight section.In an estimated measuring time we could deduce the mean position $x\left(\theta_{1} \sim \theta_{9}\right)$ from their radial distribution at the detectors with the aimed accuracy. Their azimuthally dependence could be fitted with a sinusoidal function with : $x\left(\theta_{i}\right)=A \cos \left(\nu \theta_{i}+\Theta\right)+x_{0}$, where $A$ and $\nu$ denote, respectively, the amplitude and the frequency (tune number) of betatron oscillation, and $\theta_{i} \Theta$, and $x_{0}$ are the detector position in azimuthal angle, the initial azimuthal angle of the source, and the equilibrium radius respectively.

The simulation studies determine the parameters as $x_{0}$ $=6185.8 \mathrm{~mm}$, and $\nu=2.74$ respectively. Therefore, it is worth noting that we could deduce the closed orbit for measured incident alpha particle energy at PRISM-FFAG (scaling type FFAG accelerator) and it enable us to compare the difference between designed and measured value.

## CONCLUSION

The present method could be applied for testing various components and detection devices before commissioning or beam injection of secondary particles in the accelerators


Figure 4: Betatron Oscillation behavior for two different starting positions injection radius; 6148mm (upper) and 6187 mm (lower). The lower simulation shows that the injection radius is close to the equilibrium radial position. The numbers indicated on the horizontal axis show the azimuthal location of the detector.
and storage rings. In addition the precise comparison between an experimental measurement of particle dynamics and a simulation study based on a tracking program open a new field of study for non-linear physics, such as complexities and plasma science. Development of a new simulation program will be helpful for more precise correction to the closed orbit distortion and chromaticity correction, deterioration due to resonances and stop band width.

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