EFFECT OF NONLINEAR SYNCHROTRON MOTION ON TPS ENERGY ACCEPTANCE

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Abstract

For a synchrotron light source of low emittance design the first order momentum compaction factor is usually small. The contribution of second order momentum compaction factor has to be considered. The contour of longitudinal phase space shows significant deformation due to the nonlinear effect. This will affect the energy acceptance of the particles and reduce the Touschek lifetime. In this paper we analyze the effect of the nonlinear synchrotron motion of Taiwan Photon Source (TPS) lattice design [1]. The reduction of energy acceptance is estimated. Touschek lifetime is calculated including this nonlinear effect.

INTRODUCTION

In the TPS design, the peak RF voltage is chosen as 3.5 MV. The peak RF voltage and the lattice design determine the first order momentum compaction factor α_1 and higher order momentum compaction factor α_2, etc. The longitudinal energy acceptance of TPS design determined by the first order momentum compaction factor can be as large as $\pm 5\%$. However, this result does not include the contribution of higher-order momentum compaction factor which is large in TPS design. Therefore the second order momentum compaction factor α_2 has to be included in the longitudinal equation of motion. The longitudinal equation of motion becomes nonlinear and the RF bucket is deformed. The energy acceptance is reduced and the consequence is the reduction of Touschek life-time.

NONLINEAR LONGITUDINAL MOTION DUE TO HIGHER ORDER MOMENTUM COMPACTION FACTOR

The momentum compaction factor α is defined as the ratio of the relative change of orbit path length $\frac{\Delta C}{C_0}$ to the relative change of particle energy $\delta = \frac{\Delta p}{p_0}$, where C_0 is the circumference of the reference particle with reference momentum p_0.

$$\alpha = \frac{d(\frac{\Delta C}{C_0})}{d\delta} = \alpha_1 + 2\alpha_2\delta \tag{1}$$

$$\Delta C = \int_0^{C_0} ds \left\{ \sqrt{(1+\frac{x}{\rho})^2 + x'^2 + y'^2} - 1 \right\} \tag{2}$$

$$\begin{aligned} x &= x_\beta + x_{co} + D_{x0}\delta + D_{x1}\delta^2 + \cdots \\ y &= y_\beta \\ x' &= x'_\beta + D'_{x0}\delta + D'_{x1}\delta^2 + \cdots \\ y' &= y'_\beta \end{aligned} \tag{3}$$

where ρ is the bending radius, x_β is the betatron oscillation, x_{co} is the closed orbit distortion, D_{x0}, D_{x1} are the first and second order dispersion function and D'_{x0}, D'_{x1} are derivative of the first and second order dispersion function respectively. We do not consider the vertical dispersion function due to errors. Because the change of path length due to betatron motion is small and the closed orbit is well controlled in modern machine, we will not consider path length difference due to these two effects. The first and second order dispersion functions are given by [2]

$$D''_{x0} + (\frac{1}{\rho^2} - k)D_{x0} = \frac{1}{\rho} \tag{4}$$

$$D''_{x1} + (\frac{1}{\rho^2} - k)D_{x1} = -\frac{1}{\rho} + (\frac{2}{\rho^2} - k)D_{x0} + (\frac{2}{\rho}k - \frac{1}{\rho^3} + \frac{m}{2})D^2{}_{x0} + \frac{1}{2\rho}D'_{x0}{}^2 \tag{5}$$

where *k* and *m* are the quadrupole and sextupole strength respectively.

$$\alpha_1 = \frac{1}{C_0}\int_0^{C_0} ds \frac{D_{x,0}}{\rho} \tag{6}$$

$$\alpha_2 = \frac{1}{C_0}\int_0^{C_0} ds (\frac{D_{x1}}{\rho} + \frac{D'_{x0}{}^2}{2}) \tag{7}$$

The longitudinal equation of motion including the nonlinear momentum compaction factor are

$$\dot\phi = -\omega_{rf}(\alpha_1\delta + \alpha_2\delta) \tag{8}$$

$$\dot\delta = \frac{eV_{rf}}{E_0T_0}(\sin\phi - \sin\phi_s) \tag{9}$$

where V_{rf} is the RF voltage, ω_{rf} is the RF angular frequency, ϕ_s and ϕ are the RF phase of the reference and test particle respectively, E_0 is the particle energy and T_0 the revolution period.

The longitudinal Hamiltonian is

$$H(\phi,\delta) = \omega_{rf}(\alpha_1\frac{\delta^2}{2} + \alpha_2\frac{\delta^3}{3}) + \frac{eV_{rf}}{E_0T_0}[(\cos\phi - \cos\phi_s) + (\phi - \phi_s)\sin\phi_s] \tag{10}$$

with a non zero value of α_2 there are now two stable fix points and two unstable fix points. The stable fixed points are ($\phi = \phi_s$, $\delta = 0$) and ($\phi = \pi - \phi_s$, $\delta = -\alpha_{1/}\alpha_2$). The unstable fixed points are ($\phi = \pi - \phi_s$, $\delta = 0$) and ($\phi = \phi_s$

$\delta = -\alpha_1/\alpha_2$). The RF bucket acceptance is deformed in contrast to the linear case. When $\frac{\alpha_1}{\alpha_2}$ larger than the linear RF bucket acceptance, the two stable phase space area lie on top of each other in the phase space. When the ratio of α_1, α_2 smaller than the linear RF bucket acceptance, the two stable phase space area lie next to each other in the phase space [3]. The contour of RF bucket for this Hamiltonian is no longer symmetric with respect to the ϕ axis in the phase space.

TPS NONLINEAR LONGITUDINAL MOTION

The nominal energy of TPS is 3.0 Gev. The emittance of the lattice is 1.7 nm·rad with dispersion in the long straight section. In order to achieve small emittance the strength of quadrupole magnet is large. The dispersion function in the dipole is small. The sextupole strength required for chromaticity compensation and dynamic aperture optimization is also large. These lead to a small α_1. The first order momentum compaction factor α_1 is 2.012E-04 and the second order momentum compaction factor α_2 is 2.351E-03 for TPS design. The α_2 is an order of magnitude large than α_1. The longitudinal phase space derived from Eq. (10) is shown in Figure 1. The RF bucket centered around $\phi = \pi - \phi_s$, $\delta = 0$ is asymmetric in energy, the upper limit is at 4.28%($\left|\frac{\alpha_1}{2\alpha_2}\right|$) and the lower limit is at -8.55%($-\left|\frac{\alpha_1}{\alpha_2}\right|$). Comparing to the linear case the RF bucket height is $\pm 5\%$.

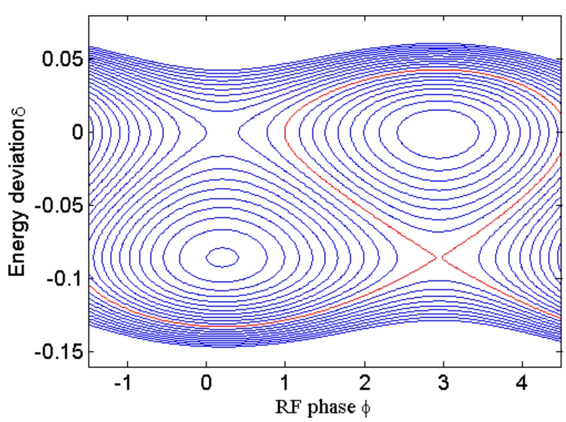

Figure 1. Longitudinal phase space of TPS lattice. The α_1 is 2.012E-04 and α_2 is 2.351E-03.

MOMENTUM ACCEPTANCE AND TOUSCHEK LIFETIME

The momentum acceptance calculated fro particle tracking should include the nonlinear terms in the synchrotron motion. The asymmetry in the RF bucket shape shown in Figure 1 generates an asymmetry in the momentum acceptance as shown in Figure 2. Figure 2 depicts the tracking results from the energy acceptance/Touschek lifetime module in the BETA code [4]. The module has been modified to take into the non-linear synchrotron motion. The algorithm of the modification is described in [5]. Horizontal half chamber size of 35 mm and 30 mm and minimum vertical half chamber size of 5 mm for insertion device are used for physical acceptance tracking. For vacuum chamber less than 30 mm the physic aperture is smaller than the dynamic aperture in the energy deviation region from -5% to +10%. The energy acceptance along the ring with and without second order momentum compaction is shown in Figure 3. The horizontal half vacuum chamber size of 35 mm is imposed in the particle tracking.

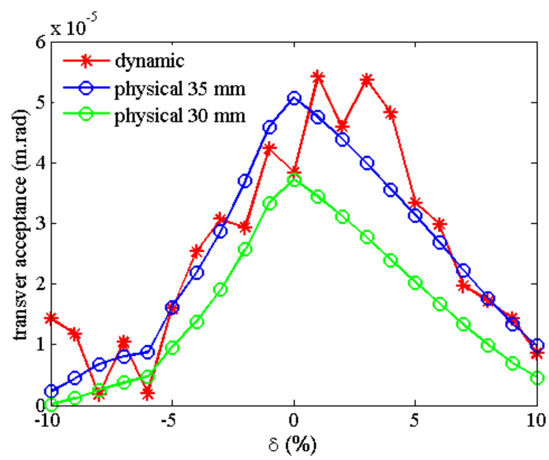

Figure 2. Horizontal dynamic and physical acceptance as a function of δ.

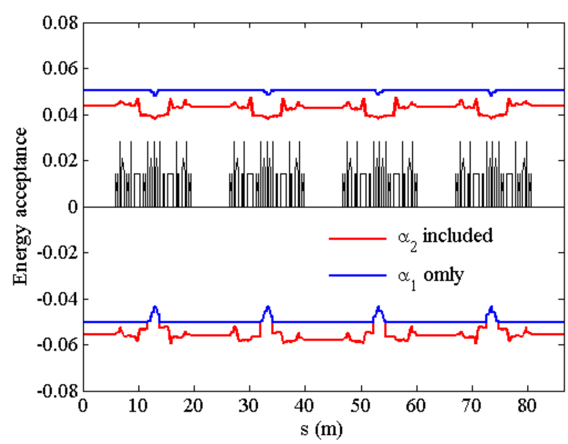

Figure 3. Energy acceptance along the ring with and without second order momentum compaction. A horizontal half vacuum chamber 35 mm is assumed.

The Touschek lifetime is simulated by assuming 0.8 mA per bunch, bunch length 9.29 ps (3.5 MV RF voltage) and coupling coefficient 0.01. The Touschek lifetime is reduced from 30 hour to 24.25 hour due to the nonlinearity of RF energy acceptance.

Because of strong sextupoles required for lattice optimization. The optical function is affected by energy deviation and the chromatic orbit distortion is no longer proportional to δ. The calculation of energy acceptance due to energy deviation has to take these into consideration [5]. Figure 4 shows the energy acceptance along the ring with consideration of optic function as function of energy and chromatic orbit distortion. With this modification the Touschek lifetime is further reduced to 22.56 hour. The Touschek lifetime for different chamber size, linear and nonlinear synchrotron motion, and optic function with modification is shown in table 1.

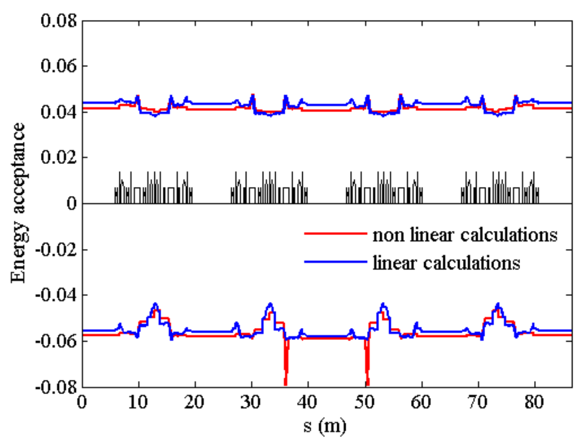

Figure 4. Energy acceptance along the ring with and without recalculation of chromatic orbit distortion and energy dependent optical functions. The horizontal half vacuum chamber size is 35 mm.

Table 1: Touschek lifetime calculated for different conditions.

Half vacuum chamber (mm)	**Without** α_2 (hour)	**With** α_2 (hour)	**With non-linear optic function** (hour)
35	30.03	24.25	22.56
30	29.91	23.94	21.71

DISCUSSION

Because the emittance of the TPS design is small, the second order momentum compaction factor is large. The effect of longitudinal nonlinearity can not be neglected. The nonlinearity results in the asymmetry of RF energy acceptance. This will reduce Touschek lifetime. In our case the Touschek lifetime is reduced from 30 hours to 24 hours. If the nonlinearity of optic function is considered the Touschek lifetime is further reduced to 22.5 hours. In order to increase Touschek lifetime the nonlinearity of the lattice design has to be minimized. The efforts to correct the nonlinear property of TPS design is reported in this conference [6]. The Touschek lifetime is improved to 28.78 hours in this lattice[6] instead of 22.5 hours.

REFERENCES

[1] C.C. Kuo, et al., “Design of Taiwan Future Synchrotron Light Source”, these proceedings.

[2] J. P. Delahaye and J. Jäger , “Variation of the Dispersion Function Momentum Compaction Factor and Damping Partition Number with Particle Energy Deviation”, Particle Accelerator, 1986, Vol. 18, p. 183.

[3] David Robin, et al., “Quasi-isochronous Storage Ring”, PRE E, Vol 48, p. 2149 (1993).

[4] J. Payet, et al, “BETA-LNS code”, ver. 7.30.

[5] A. Nadji, et al., “Energy Acceptance and Touschek Lifetime Calculation for the SOLEIL Project”. EPAC 98, (1998).

[6] H.P. Chang, et al., “Nonlinear beam Dynamics of TPS”, these proceedings.