

COMPARISON OF THREE CSR RADIATION POWERS FOR PARTICLE BUNCHES AND LINE CHARGES*

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Abstract

We are outlining here some general aspects of coherent synchrotron radiation (CSR). Our emphasis is the comparison of the radiation power with the Schwinger power and with the power recently introduced by Saldin, Schneidmiller and Yurkov. The latter power formula applies to a one-dimensional bunch of particles moving on the same spatial curve hence a line charge treatment is possible. The Schwinger power and the Saldin-Schneidmiller-Yurkov power are closely related since both are defined in terms of mechanical work per unit time. Originating from the Lorentz-Dirac theory, the Schwinger power is the negative mechanical work per unit time done by half the retarded minus half the advanced electromagnetic field. One can view the Saldin-Schneidmiller-Yurkov power as a modified Schwinger power which involves an electric field tailored to cope with one-dimensional bunches. A merit of the Saldin-Schneidmiller-Yurkov power is that it does not involve the advanced field while a merit of the Schwinger power is that it even can be applied to bunches which are not one-dimensional. Nevertheless one-dimensional bunch models are important since they are used in various CSR codes and since they serve to some extent as role models for higher-dimensional models as for example the Maxwell-Vlasov approach of G.Bassi et al [1].

INTRODUCTION

Our topic is the synchrotron radiation (SR) emitted by a one-dimensional bunch of particles or by a line charge. Our main issue is the comparison of the radiation power with the Schwinger power and with the Saldin-Schneidmiller-Yurkov power. The radiation power is basically the flux of the Poynting vector of the retarded electromagnetic field. A bunch of N particles of charge q has charge density ρ and current density \mathbf{j} of the form:

$$\rho(t, \mathbf{x}) = q \sum_{i=1}^N \delta(\mathbf{x} - \mathbf{x}_i(t)),$$

$$\mathbf{j}(t, \mathbf{x}) = q \sum_{i=1}^N \dot{\mathbf{x}}_i(t) \delta(\mathbf{x} - \mathbf{x}_i(t)),$$

where $\mathbf{x}_i(t)$ is the position of the i -th particle at time t and q is the particle charge. Since we have to deal with the Saldin-Schneidmiller-Yurkov power we have to assume

that all particles of the bunch move with the same speed and on the same spatial curve and so the backreaction of the radiation on the particles is neglected. The line charges are explained in a special section. Our emphasis is on analytical results - however numerical simulations are planned, too.

ABOUT SR IN GENERAL

Definition: Synchrotron radiation (SR) is the electromagnetic radiation emitted from one or more relativistic charged particles.

We thus deal with classical SR hence with Maxwell-Lorentz theory. We confine, in this paper, to free space so the effect of metallic boundaries is not taken into account. We use SI-units - for a recent textbook on SR with SI-units, see [2].

RADIATION POWER FOR A BUNCH OF PARTICLES

The radiation power \mathcal{P}_{true} of a bunch of N charged particles is defined for example in [2],[3]. It is a linear functional of the Poynting vector \mathbf{S}_{true} of the bunch's retarded field: $\mathbf{S}_{true} := \frac{1}{\mu_0} (\mathbf{E}_{ret} \times \mathbf{B}_{ret})$, where μ_0 is the vacuum magnetic permeability. For the comparison of the three powers, the following quantities are useful as well [2],[3]:

- Fourier transform of \mathcal{P}_{true} in time
- Total radiated energy $\int_{-\infty}^{\infty} \mathcal{P}_{true}(t) dt$.

The subscript *true* serves to remind that the 'true' bunch consists of particles. Due to the linearity of the Maxwell equations the retarded electromagnetic field $(\mathbf{E}_{ret}, \mathbf{B}_{ret})$ is the sum of the retarded Lienard-Wiechert fields of the N particles hence

$$\mathbf{S}_{true} = \mathbf{S}_{true}^{ISR} + \mathbf{S}_{true}^{CSR},$$

where

$$\mathbf{S}_{true}^{ISR} := \sum_{i=1}^N \mathbf{S}_{i,i}, \quad \mathbf{S}_{true}^{CSR} := \sum_{\substack{i,k=1 \\ i \neq k}}^N \mathbf{S}_{i,k},$$

$$\mathbf{S}_{i,k} := \frac{1}{\mu_0} (\mathbf{E}_{ret}^i \times \mathbf{B}_{ret}^k),$$

and where $(\mathbf{E}_{ret}^i, \mathbf{B}_{ret}^i)$ is the retarded Lienard-Wiechert field of the i -th particle. If $N = 1$, i.e., for a bunch with only one particle, the radiation power equals the relativistic Larmor rate [3],[4]:

$$\mathcal{P}_{true}(t) = \frac{q^2}{6\pi c^3 \epsilon_0} \gamma^4 |\ddot{\mathbf{x}}(t)|^2,$$

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where ε_0 is the vacuum electric permeability, γ is the (constant) Lorentz factor of the particle and $c = \sqrt{\frac{1}{\varepsilon_0\mu_0}}$. Note that we used the fact that $\dot{\mathbf{x}} \cdot \ddot{\mathbf{x}} = 0$. Since \mathcal{P}_{true} is a linear functional of the Poynting vector \mathbf{S}_{true} and since $\mathbf{S}_{true} = \mathbf{S}_{true}^{ISR} + \mathbf{S}_{true}^{CSR}$ one can write, for an arbitrary particle number N :

$$\mathcal{P}_{true} = \mathcal{P}_{true}^{ISR} + \mathcal{P}_{true}^{CSR},$$

where the definitions of \mathcal{P}_{true}^{ISR} and \mathcal{P}_{true}^{CSR} are obvious from above. Clearly, \mathcal{P}_{true}^{ISR} is the sum of the relativistic Larmor rates of the particles:

$$\mathcal{P}_{true}^{ISR}(t) = \frac{q^2\gamma_1^4}{6\pi c^3\varepsilon_0} \sum_{i=1}^N |\ddot{\mathbf{x}}_i(t)|^2.$$

Thus \mathcal{P}_{true}^{ISR} is completely understood whereas, in general, $\mathcal{P}_{true}^{CSR}(t)$ is a very complicated function of t and only partially understood. Hence \mathcal{P}_{true} is an object of intense research. A basic law of SR is the N^2 -theorem: if the bunch is concentrated at a point, i.e., $\mathbf{x}_1(t) = \dots = \mathbf{x}_N(t)$ then \mathcal{P}_{true} is N^2 times the relativistic Larmor rate of a single particle: $\mathcal{P}_{true}(t) = \frac{N^2 q^2}{6\pi c^3\varepsilon_0} \gamma_1^4 |\ddot{\mathbf{x}}_1(t)|^2$.

DEFINITION OF CSR

Definition: SR is called incoherent synchrotron radiation (ISR) iff $\mathcal{P}_{true} \approx \mathcal{P}_{true}^{ISR}$. Otherwise SR is called coherent synchrotron radiation (CSR).

If the particles are widely separated, $\mathcal{P}_{true} \approx \mathcal{P}_{true}^{ISR}$ hence SR=ISR. If $N = 1$ then $\mathcal{P}_{true} = \mathcal{P}_{true}^{ISR}$ hence SR=ISR. In the situation of the N^2 -theorem one has: $\mathcal{P}_{true}^{CSR} = (N-1)\mathcal{P}_{true}^{ISR}$ hence, if $N \geq 2$, SR=CSR.

The Fourier transform in time of \mathcal{P}_{true} reveals that the CSR-amount of SR is largely frequency dependent. One has the rule of thumb: if the SR wavelength is comparable to the bunch length then the ISR is negligible for that wavelength. Thus, and due to the N^2 -theorem, in early synchrotrons the CSR was a concern at radio/microwave frequencies [5],[6].

The double-sum structure of \mathcal{P}_{true}^{CSR} is a crucial feature of CSR - it is responsible for interference effects between the retarded Lienard-Wiechert fields of different particles and this is the origin of the terminology: 'coherent synchrotron radiation'. CSR is an important effect for FEL's, linear colliders, bunch compressors etc. For a review of CSR in accelerators see [7]. In astrophysics CSR is studied in the context of radiopulsars.

SCHWINGER POWER FOR A BUNCH OF PARTICLES

The Schwinger power for SR was studied in detail first in 1949 [8]. The Schwinger power \mathcal{P}_{Sch} of N charged particles is defined by:

$$\mathcal{P}_{Sch}(t) := - \int \mathbf{j}(t, \mathbf{x}) \cdot \mathbf{E}_{Sch}(t, \mathbf{x}) d^3 \mathbf{x},$$

where $\mathbf{E}_{Sch} := \frac{1}{2}(\mathbf{E}_{ret} - \mathbf{E}_{adv})$ and where \mathbf{E}_{adv} is the advanced electric field of the bunch. Note that $-\mathcal{P}_{Sch}$ is the mechanical work per unit time done by \mathbf{E}_{Sch} . By the linearity of the Maxwell equations we have

$$\mathcal{P}_{Sch}(t) = - \sum_{i,k=1}^N \int \mathbf{j}_k(t, \mathbf{x}) \cdot \mathbf{E}_{Sch}^i(t, \mathbf{x}) d^3 \mathbf{x},$$

where $\mathbf{j}_k(t, \mathbf{x}) := q\dot{\mathbf{x}}_k(t)\delta(\mathbf{x} - \mathbf{x}_k(t))$ and where $\mathbf{E}_{Sch}^i := \frac{1}{2}(\mathbf{E}_{ret}^i - \mathbf{E}_{adv}^i)$ with \mathbf{E}_{adv}^i being the advanced electric Lienard-Wiechert field of the i -th particle. Thus

$$\mathcal{P}_{Sch}(t) = -q \sum_{i,k=1}^N \dot{\mathbf{x}}_k(t) \cdot \mathbf{E}_{Sch}^i(t, \mathbf{x}_k(t)).$$

The Schwinger power is closely related with the Lorentz-Dirac equation. In fact the Lorentz-Dirac theory gives $\mathcal{P}_{Sch} = \mathcal{P}_{true}^{ISR} + \mathcal{P}_{Sch}^{CSR}$ [4],[9] where

$$\begin{aligned} \mathcal{P}_{Sch}^{CSR} &:= - \sum_{\substack{i,k=1 \\ i \neq k}}^N \int \mathbf{j}_k(t, \mathbf{x}) \cdot \mathbf{E}_{Sch}^i(t, \mathbf{x}) d^3 \mathbf{x} \\ &= -q \sum_{\substack{i,k=1 \\ i \neq k}}^N \dot{\mathbf{x}}_k(t) \cdot \mathbf{E}_{Sch}^i(t, \mathbf{x}_k(t)). \end{aligned}$$

The underlying idea of the Schwinger power is sketched in the section on heuristics.

SALDIN-SCHNEIDMILLER-YURKOV POWER FOR A BUNCH OF PARTICLES

The Saldin-Schneidmiller-Yurkov (SSY) power is an important tool introduced in 1997 [10]. The SSY power \mathcal{P}_{SSY} of N charged particles is defined by

$$\mathcal{P}_{SSY}(t) := \mathcal{P}_{true}^{ISR} + \mathcal{P}_{SSY}^{CSR},$$

where \mathcal{P}_{SSY}^{CSR} is explained in more detail in the section on heuristics.

- Simulation programs using the SSY power are [11]:
- ELEGANT by M.Borland at Argonne Lab
- CSRtrack by M.Dohlus and T.Limberg at DESY
- simulation program by P.Emma at SLAC

LINE CHARGE

Recall that we assume that all bunch particles move with the same speed v and on the same spatial curve $\mathbf{X} = \mathbf{X}(s)$ labeled by Euclidean arc length s . A line charge is obtained by smoothing out the bunch of particles with a line charge distribution function $\lambda = \lambda(t, s)$. Thus $\lambda(t, s)ds =$ is the number of particles between s and $s + ds$ at time t . The function λ allows to transform particle observables into line charge observables via the replacement: $\sum_{i=1}^N \rightarrow \int \lambda(t, s)ds$. For example, one transforms \mathcal{P}_{Sch} as follows: since the trajectory of the i -th particle is of the

form $\mathbf{x}_i(t) = \mathbf{X}(S_i(t))$ with $S_i(t) = S_i(0) + vt$ one has, due to the double-sum structure of \mathcal{P}_{Sch} , a function $\tilde{\mathcal{P}}_{Sch} = \tilde{\mathcal{P}}_{Sch}(s, s')$ such that $\mathcal{P}_{Sch}(t) = \sum_{i,k=1}^N \tilde{\mathcal{P}}_{Sch}(S_i(t), S_k(t))$. Thus one transforms \mathcal{P}_{Sch} via $\mathcal{P}_{Sch} \rightarrow \mathcal{P}_{Sch,line}$, where

$$\mathcal{P}_{Sch,line}(t) := \int \lambda(t, s) \lambda(t, s') \tilde{\mathcal{P}}_{Sch}(s, s') ds' ds .$$

Analogously one performs the transformations $\mathcal{P}_{SSY} \rightarrow \mathcal{P}_{SSY,line}$ and $\mathcal{P}_{true} \rightarrow \mathcal{P}_{line}$. Note that $\mathcal{P}_{SSY,line}$ is studied in detail in [10],[12] and \mathcal{P}_{line} in [13],[14],[15].

HEURISTIC COMPARISON OF THE THREE POWERS

Since radiation is an essentially irreversible phenomenon one may adopt the following heuristic model of radiation: because

$$\mathbf{E}_{ret} = \frac{1}{2}(\mathbf{E}_{ret} - \mathbf{E}_{adv}) + \frac{1}{2}(\mathbf{E}_{ret} + \mathbf{E}_{adv}) ,$$

and since $\frac{1}{2}(\mathbf{E}_{ret} + \mathbf{E}_{adv})$ and $\frac{1}{2}(\mathbf{E}_{ret} - \mathbf{E}_{adv})$ have contrary behaviors under time reversal [4] one is led to the belief [8] that

$$\mathcal{P}_{true}(t) \approx -\frac{1}{2} \int \mathbf{j}(t, \mathbf{x}) \cdot (\mathbf{E}_{ret}(t, \mathbf{x}) - \mathbf{E}_{adv}(t, \mathbf{x})) d^3 \mathbf{x} ,$$

i.e., $\mathcal{P}_{true} \approx \mathcal{P}_{Sch}$. Note that the Schwinger power is finite for particles (and even for a line charge, a surface charge and a volume charge).

However, the mechanical work per unit time done by \mathbf{E}_{ret} :

$$\frac{dE_{mech}}{dt}(t) := \int \mathbf{j}(t, \mathbf{x}) \cdot \mathbf{E}_{ret}(t, \mathbf{x}) d^3 \mathbf{x} ,$$

is infinite for a line charge and for particles. Nevertheless $\frac{dE_{mech}}{dt}$ is finite for a volume charge and a surface charge and numerical simulations of $\frac{dE_{mech}}{dt}$ are an important enterprise in the accelerator community - for a surface charge, see for example the Maxwell-Vlasov approach in [1],[16].

The same idea as for the Schwinger power may be applied to the SSY power: \mathcal{P}_{SSY}^{CSR} can be obtained by replacing in \mathcal{P}_{Sch}^{CSR} the field \mathbf{E}_{Sch}^i by subtracting from \mathbf{E}_{ret}^i a Coulomb term (this procedure is called 'renormalization' in [10],[12]). The SSY power is finite for line charge and particles (and undefined for volume charge and surface charge). An advantage of the SSY power over the Schwinger power is that the SSY power does not involve advanced fields. An advantage of the Schwinger power over the SSY power is that the Schwinger power has a wider scope since it even works for bunches whose particles do not move on the same spatial curve.

WHAT NEXT

We will compare \mathcal{P}_{true} with \mathcal{P}_{Sch} and \mathcal{P}_{SSY} and we will compare \mathcal{P}_{line} with $\mathcal{P}_{Sch,line}$ and $\mathcal{P}_{SSY,line}$. We will study various geometries of the orbit $X(s)$ (arc, piecewise linear etc.) and various distribution functions λ of the line charge (tophat, Gaussian etc.).

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