DEPENDENCE OF TRANSVERSE INSTABILITIES ON AMPLITUDE DEPENDENT TUNE SHIFTS

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Abstract

In the Photon Factory electron storage ring (PF ring), transverse instabilities have been observed in multi-bunch operation mode. The instabilities can be suppressed by octupole magnets which produce amplitude dependent tune shifts. We have measured the response of the instabilities to the tune shifts with varying the field strengths of the octupole magnets to extend our knowledge on the instabilities. The thresholds of the instabilities were observed at a certain field strength, and they depended on the bunch filling pattern of the beam and the beam current per bunch. In addition, we have observed that the present behavior of the instabilities is clearly different from that before the reconstruction of the ring.

INTRODUCTION

In the PF ring, a vertical instability has been strongly generated in comparison with a horizontal one, and has produced the blowup of the vertical beam size. We suppressed the instability using octupole magnets through the Landau damping which is related with the amplitude dependent tune shifts [1]. The tune shifts can be generated by the sextupole, octupole and higher order magnetic field. The octupole magnets were convenient for us since we could effectively control the amplitude dependent tune shifts without changing the chromaticities.

We have guessed that the instability is caused by the ion trapping since it is related to the vacuum condition of the ring [2]. In the multi-bunch mode, the PF ring is usually operated with a partial filling of the rf buckets; the bunched beam is filled in continuous 280 rf buckets and remained 32 rf buckets are empty to eliminate the ions from the surrounding of the beam. Since this operation was quite successful, there is no room for doubt about that the vertical instability is caused by the ion-trapping. However, we have not understand why only the vertical instability was strongly produced and the horizontal instability was not generated. We have tried to conduct another experiment due to active controls of the amplitude dependent tune shifts to extend our knowledge about the instabilities. If the instabilities are caused by the ion trapping, we may observe some phenomena related to the number of empty buckets. If the instabilities are produced by the head-tail effects, we may observe some phenomena depended on a beam current per bunch.

In this paper, we will simply formulate the amplitude dependent tune shifts produced by the octupole magnets at the PF ring and then describe the measurements on the instability and the results. In addition, we will demonstrate the results on the present instabilities compared with those before the reconstruction for the straight-sections upgrade project at the PF ring [3], and the effect of a transverse bunch-by-bunch feedback system [4] on the instabilities.

AMPLITUDE DEPENDENT TUNE SHIFT

The amplitude dependent tune shifts are generally described by

$$\begin{pmatrix} \Delta\nu_x \\ \Delta\nu_y \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}, \tag{1}$$

where $\Delta\nu_x$ and $\Delta\nu_y$ are horizontal and vertical tune shifts, and J_x and J_y are action variables of betatron oscillations. The parameters, α_{xx}, α_{xy}, α_{yx} and α_{yy}, are described as follows. Based on perturbation theory for the betatron oscillation, these parameters, which are induced by octupole field, are calculated by

$$\alpha_{xx} = \frac{1}{16\pi}\int_s^{s+C} ds S_4(s)\beta_x^2(s), \tag{2}$$

$$\alpha_{xy} = \alpha_{yx} = -\frac{1}{8\pi}\int_s^{s+C} ds S_4(s)\beta_x(s)\beta_y(s), \tag{3}$$

$$\alpha_{yy} = \frac{1}{16\pi}\int_s^{s+C} ds S_4(s)\beta_y^2(s), \tag{4}$$

where s is a longitudinal coordinate in a circular accelerator, C is a circumference, and $\beta_x(s)$ and $\beta_y(s)$ are horizontal and vertical betatron functions, respectively. Using a vertical component of magnetic field, B_y, and a magnetic rigidity of the beam, $B_0\rho$, the effective octupole strength is described by $S_4(s) = (1/B_0\rho)\partial^3 B_y/\partial x^3$. Assuming that the octupole field is uniform in the octupole magnet, the effective field strength can be described by $K_3 = S_4 L_{\mathrm{oct}}$, where L_{oct} is the magnetic effective length of the octupole magnet. In addition, since we change the magnetic fields of four octupole magnets to the same value during the measurements, the parameters are approximated by

$$\alpha_{xx} = \frac{1}{16\pi}\frac{K_3}{L_{\mathrm{oct}}}\sum_{i=1}^{4}\int_{s_{i,0}}^{s_{i,1}} ds\beta_x^2(s), \tag{5}$$

$$\alpha_{xy} = \alpha_{yx} = -\frac{1}{8\pi}\frac{K_3}{L_{\mathrm{oct}}}\sum_{i=1}^{4}\int_{s_{i,0}}^{s_{i,1}} ds\beta_x(s)\beta_y(s), \tag{6}$$

$$\alpha_{yy} = \frac{1}{16\pi}\frac{K_3}{L_{\mathrm{oct}}}\sum_{i=1}^{4}\int_{s_{i,0}}^{s_{i,1}} ds\beta_y^2(s). \tag{7}$$

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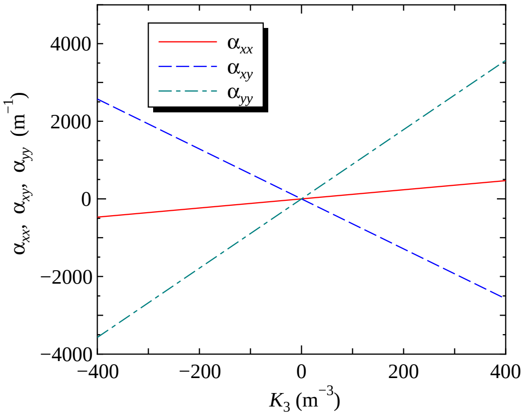


Figure 1: Parameters of amplitude dependent tune shifts as a function of the octupole field strength, K_3, in the PF ring. They are calculated based on Eqs. (5)-(7).

They are directly proportional to the strength, K_3. Here, $s_{i,0}$ and $s_{i,1}$ are longitudinal coordinates at an entrance and an exit of i-th octupole magnet, and the distance, $s_{i,1} - s_{i,0}$, is $L_{\rm oct}$. The parameters under the current optics of the PF ring are calculated to be $\alpha_{xx} = 1.175 \times K_3$ $(\rm m^{-1})$, $\alpha_{xy} = \alpha_{yx} = -6.435 \times K_3$ $(\rm m^{-1})$ and $\alpha_{yy} = 8.923 \times K_3$ $(\rm m^{-1})$. The parameters as a function of K_3 are shown in Fig. 1. Since a beam has a finite spatial spread, the amplitude dependent tune shifts provide the beam with tune spreads. The tune spreads allow us to suppress the instabilities. This is called Landau damping. During the measurements, we changed K_3 from -390 $(\rm m^{-3})$ to 390 $(\rm m^{-3})$.

MEASUREMENTS

In order to observe the instabilities, we measured beam spectrums from a button-type electrode using a real-time spectrum analyzer, RSA-230 (Tektronix). A span and a center frequency of the measurements were 1.0002 GHz and 5 MHz. In the frequency span, the following twelve betatron sidebands are included. Horizontal betatron sidebands are $2f_{\rm rf} - f_{\rm rev} - f_{\beta_x}$, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_x}$, $2f_{\rm rf} - f_{\beta_x}$, $2f_{\rm rf} + f_{\beta_x}$, $2f_{\rm rf} + f_{\rm rev} - f_{\beta_x}$ and $2f_{\rm rf} + f_{\rm rev} + f_{\beta_x}$, and vertical betatron sidebands are $2f_{\rm rf} - f_{\rm rev} - f_{\beta_y}$, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_y}$, $2f_{\rm rf} - f_{\beta_y}$, $2f_{\rm rf} + f_{\beta_y}$, $2f_{\rm rf} + f_{\rm rev} - f_{\beta_y}$ and $2f_{\rm rf} + f_{\rm rev} + f_{\beta_y}$. Here, $f_{\rm rf} = 500.1$ MHz and $f_{\rm rev} = 1.6$ MHz are rf frequency and revolution frequency, f_{β_x} and f_{β_y} are frequencies of horizontal and vertical betatron oscillations. In the measurements, we assume that amplitudes of $2f_{\rm rf} - f_{\rm rev} + f_{\beta_x}$ and $2f_{\rm rf} - f_{\rm rev} + f_{\beta_y}$ represent the horizontal and vertical instabilities.

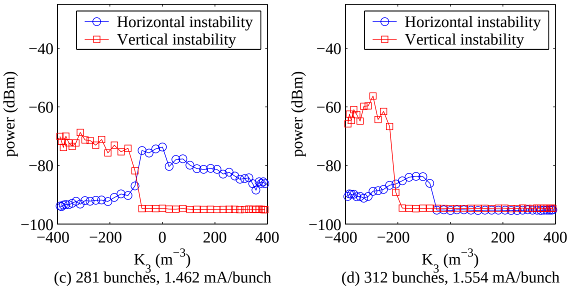
RESULTS

Fig. 2 shows the measured results with a fixed bunch current of about 1.6 mA; (a) represents the result with a filling pattern of 100 bunches, (b) 200 bunches, (c) 281 bunches and (d) 312 bunches. The conditions in Fig. 2 (c) are similar to those of the user operation in the multibunch mode; the initial total current is 450 mA. As shown in Figs. 2 (a)-(c), the vertical instability appeared when K_3 is less than -100 $(\rm m^{-3})$, -200 $(\rm m^{-3})$, or -150 $(\rm m^{-3})$, but they disappeared when K_3 is more than -100 $(\rm m^{-3})$, -200 $(\rm m^{-3})$, or -150 $(\rm m^{-3})$. On the other hand, for the uniform filling with 312 bunches, the vertical instability always appeared in the measured range of K_3 as shown in Fig. 2 (d). This indicates that we can not suppress the vertical instability by exciting our octupole magnets in the uniform filling.

Next, we measured the behavior of the instabilities when the bunch current is about 0.8 mA. The results are displayed in Fig. 3. The horizontal instability was hardly observed in the measured range of K_3 for any filling patterns, while the vertical instability was observed. The vertical instabilities for the bunch numbers of 100 and 200 were suppressed when K_3 are less than -300 $(\rm m^{-3})$ and -200 $(\rm m^{-3})$, respectively.

Fig. 4 shows the measured results before and after the straight-sections upgrade project in the PF-ring [3]. Before the upgrade, only the vertical instability was observed in the range of $-200 < K_3 < 200$ $(\rm m^{-3})$. On the other hand, the vertical instability was observed in the range of $K_3 < -100$ $(\rm m^{-3})$, and the horizontal instability was observed in the range of $K_3 < 100$ $(\rm m^{-3})$ after the upgrade. Since vacuum chambers in the straight-sections were renewed and the cross sections of the vacuum chambers for

(a) 100 bunches, 1.575 mA/bunch (b) 200 bunches, 1.564 mA/bunch



(c) 281 bunches, 1.462 mA/bunch (d) 312 bunches, 1.554 mA/bunch

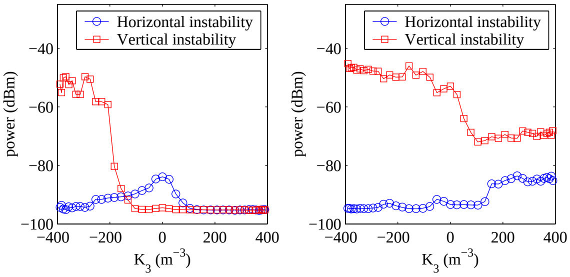
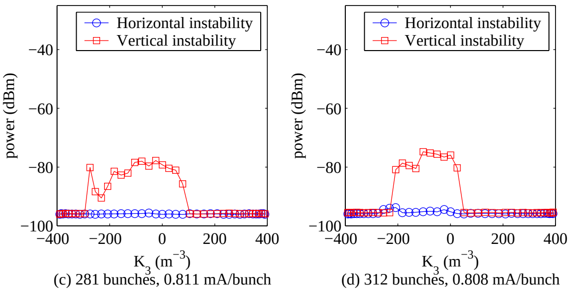
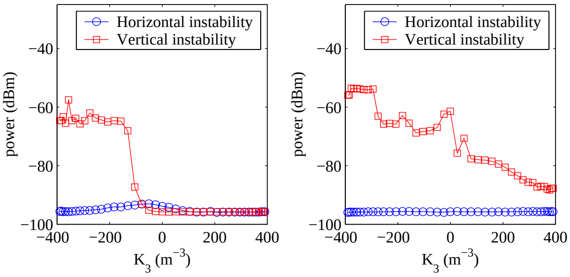


Figure 2: Measured results with a fixed bunch current of approximately 1.6 mA. Blue circles indicate power of horizontal instabilities, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_x}$ and red squares indicate power of vertical ones, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_y}$. The measurements were carried out following four conditions: partial filling patterns with (a) 100 bunches, (b) 200 bunches, (c) 281 bunches, and uniform filling pattern with (d) 312 bunches.



(a) 100 bunches, 0.815 mA/bunch

(b) 200 bunches, 0.803 mA/bunch



(c) 281 bunches, 0.811 mA/bunch

(d) 312 bunches, 0.808 mA/bunch

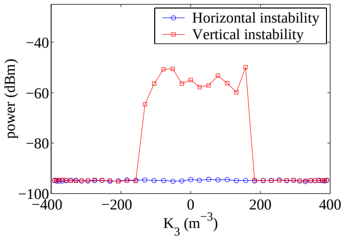
Figure 3: Measured results with a fixed bunch current of about 0.8 mA. Blue circles indicate power of horizontal instabilities, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_x}$ and red squares indicate power of vertical ones, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_y}$. The measurements were carried out under same condition except for beam current per bunch.

the quadrupole magnets were reduced, the impedance of the ring may be changed due to the upgrade. The change of the cross section may bring any change to the ion distribution in the surrounding of the beam.

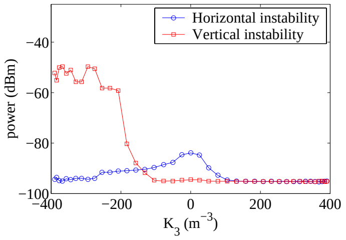
In addition, Fig. 5 shows the effect of a transverse bunch-by-bunch feedback [4]. For the partial filling with 200 bunches, both of the horizontal and vertical instabilities were suppressed by the bunch-by-bunch feedback. On the other hand, the horizontal instability was suppressed by the feedback, however, the vertical instability could not be suppressed for the uniform filling with 312 bunches.

SUMMARY

We have measured the response of the instabilities to the amplitude dependent tune shifts with varying the field strengths of the octupole magnets to extend our knowledge on the instabilities. The measurements were conducted under several conditions with a fixed bunch current of approximately 1.6 mA and 0.8 mA. We found that the horizontal instability depended on the beam current per bunch, while the vertical instability was strongly generated as increasing the number of stored bunch. Before and after the straight-sections upgrade in the PF ring, the different response of the instabilities were observed. We guess that the different response of the vertical instabilities is caused by any change of impedance or ion distribution in the surrounding of the beam. In addition, we measured the effect of the transverse bunch-by-bunch feedback, and we confirmed that it can suppress the instabilities for the partial filling.

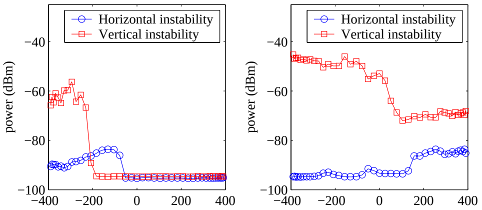


(a) Before Straight−sections Upgrade



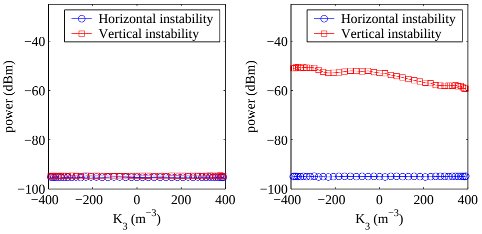
(b) After Straight−sections Upgrade

Figure 4: Measured results before and after the straight-sections upgrade in the PF ring. Stored numbers of bunch before and after the upgrade are 280 and 281, respectively. Blue circles indicate power of horizontal instabilities, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_x}$ and red squares indicate power of vertical ones, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_y}$.



(a−1) 200 bunches, FB OFF

(b−1) 312 bunches, FB OFF



(a−2) 200 bunches, FB ON

(b−2) 312 bunches, FB ON

Figure 5: Measured results with and without a transverse bunch-by-bunch feedback. Blue circles indicate power of horizontal instabilities, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_x}$ and red squares indicate power of vertical ones, $2f_{\rm rf} - f_{\rm rev} + f_{\beta_y}$. (a) partial filling with 200 bunches, (b) uniform filling with 312 bunches.

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