RESONANCE TRAPPING, HALO FORMATION AND INCOHERENT EMITTANCE GROWTH DUE TO ELECTRON CLOUD

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Abstract

The pinched electron cloud introduces a tune shift along the bunch, which, together with synchrotron motion, leads to a periodic crossing of resonances. The resonances are excited by the longitudinal distribution of the electron cloud around the storage ring. We benchmark the PIC code HEADTAIL against a simplified weak-strong tracking code based on an analytical field model, obtaining an excellent agreement. The simplified code is then used for exploring the long term evolution of the beam emittance, and for studying more realistic lattice models. Results are presented for the CERN SPS and the LHC.

INTRODUCTION

During the passage of a proton (or positron) bunch through an electron cloud, the cloud electrons are attracted by the beam electric field and their density strongly increases near the beam center ("pinch") [1]. This gives rise to an incoherent particle tune shift, which depends on the longitudinal and radial position within the bunch. The combined effect of synchrotron motion and the variation of the transverse tune shift with longitudinal position can induce the periodic crossing of resonances [2]. With a mechanism similar to that in space charge dominated beam [3], this leads to halo formation and beam emittance increase. Compared with space charge, the electron cloud tune shift is positive, there is no front-back symmetry and the transverse distribution is highly non-uniform. Moreover, in addition to possible lattice errors and non linearities, the electron cloud itself can excite resonances because of its non-uniform distribution around the ring (i.e. it is mainly localized in special elements like dipoles). The incoherent, long-term emittance growth induced by the resonance crossing mechanism takes place also at moderate electron cloud densities, below the threshold of coherent instability. It may explain observations of poor beam lifetime of the LHC proton beam in the CERN SPS and can be a concern for proton machines with long storage times, like the CERN LHC, where synchrotron radiation damping is not very effective.

The HEADTAIL code [4], developed at CERN for electron-cloud effects studies, can be used to investigate the diffusion processes [5]. The interaction between the beam and the electron cloud is computed via a 2D PIC module and is usually concentrated in a finite number (1–100) of interaction points ("kicks") around the ring. This small number of kicks does not properly resolve the ac-

tual betatron motion and it leads to artificial excitation of resonances. HEADTAIL is benchmarked with the MI-CROMAP code, hitherto used only for space-charge simulations [6] and recently modified to model the pinched electron cloud through a simplified analytical model. The short-term agreement between the two codes gives us confidence to use MICROMAP for studying long term emittance growth and beam losses using a realistic model for the SPS and an arbitrarily large number of cloud kicks.

RESONANCE CROSSING AND DIFFUSION PROCESSES

In order to speed up the simulations, the code HEAD-TAIL was run with the weak-strong option. The electron cloud potential, which is z-dependent, is computed only at the first bunch passage and then used for the successive interactions. As discussed in [5, 2], this model is valid for the study of incoherent effects which do not involve a very strong modification of the beam transverse shape. The parameters of the simulations presented in this paragraph refer to the LHC at injection energy [5]. The horizontal and vertical tunes are respectively 64.28 and 59.31, while the synchrotron period is about 170 turns.

During the passage of a bunch, the electrons are strongly pinched at the bunch center and their density can increase locally by up to 2 order of magnitudes, as shown in Figure 1. Assuming an initial average cloud density of 2.8×10^{11} m⁻³, the estimated peak maximum tune shift is $\Delta Q_{max} \approx 0.13$.

The tune shift due to the electron cloud depends on the coordinate z and on the betatron amplitude. As a consequence, resonance islands change their size and location as



Figure 1: Simulated electron density on beam axis during a bunch passage, for an initially uniform electron distribution in a field free region of LHC. The bunch head is for z < 0. Different curves refers to averaging inside a circle of variable radius r, where σ_b denotes the rms beam size.

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a function of z [7]. Beam particles executing synchrotron motion may cross important resonances during their synchrotron motion. They can get trapped inside an island and, as the island position changes along the bunch, they can be transported to larger (smaller) amplitudes [8] or they may "scatter" off the resonance. Also a second effect may arise, namely the crossing of linearly unstable regions [2].

Figure 2 shows the evolution over 50000 turns of the normalized single-particle emittance of two protons at different initial positions inside the bunch and the beam rms emittance growth (green line), induced by an electron cloud, localized at a single point in the ring (pessimistic assumption), for a maximum tune shift $\Delta Q_{max} = 0.13$. After 50000 turns, the rms emittance increases by a factor 4, as a consequence of the two diffusion mechanism discussed above. The emittance of the blue particle varies from $5.4\epsilon_0$ to about $65\epsilon_0$, thus increasing by a factor ~ 12 . The zoom over thousand turns shows periodic jumps of the invariant, clear signature of the resonance scattering mechanism [2]. Also for the particle represented in red (moving at a smaller amplitude) the same phenomenon takes place; its Courant-Snyder invariant increases from 0.25 to $2.5\epsilon_0$. The jumps in the invariant are still present but smaller, since the islands size is smaller closer to the origin.



Figure 2: Vertical beam rms emittance (green line) vs. number of turns, assuming $\Delta Q_{max} = 0.13$, 1 kick/turn, parameters of LHC at injection energy. Courant-Snyder invariant of two particles at different position in the bunch (blue: $z_{max} = 1.29\sigma_z$, red: $z_{max} = 0.365\sigma_z$) and zoom over 1000 turns.

Since the number and the importance of crossed resonances lines depends on the particles' synchrotron amplitudes, emittance increase and beam losses will not occur uniformly inside the bunch. Figure 3 show the number of particles as a function of the longitudinal invariant at the start of the simulation and after 50000 turns, assuming that the proton are lost at a betatron amplitude of $4\sigma_b$. It is possible to see a depletion at $I_z \approx 0.01 \text{ m}^2$.

BENCHMARK

The code MICROMAP has been adapted to studies of incoherent electron cloud effects. It employs an analytical description of the electric field instead of a grid. This



Figure 3: Number of particles (normalized) as function of I_z at the beginning (black) and at the end of the simulation, assuming particles lost at $4\sigma_b$. Simulations for LHC at injection

avoids any inherent noise of the PIC calculation and is much faster, opening up the way to simulations with a larger number of interaction points around the ring. The drawback is that the real electron cloud transverse distribution has to be approximated by a Gaussian.

A benchmark has been done between HEADTAIL and MICROMAP, with the purposes of justifying the use of the latter and of characterizing the effect of the PIC noise on the emittance growth. The benchmark considers an artificial simple model of a round beam with rms size σ_b , a transverse Gaussian electron distribution with constant rms size σ_e and linear increase of the electron density along the bunch, giving a maximum tune shift ΔQ_{max} at the tail of the bunch ($z = 2\sigma_z$). Only one interaction point is assumed, the synchrotron motion is linearized.

Results of the benchmark are excellent for large electron-cloud sizes ($\sigma_e \ge 0.5\sigma_b$), for different values of tune shift [2]. If the cloud size is four times smaller than the beam, which will be closer to the real case with a highly spiked electron distribution, there are some differences in the slope, but the behavior stays qualitatively the same, as shown in Fig. 4 (left). The differences are due to the roughness of the PIC grid, which does not accurately resolve but smoothens the electron density, thereby generating a lower tune shift than expected. In Figure 4 (right) the dependence on the grid size is shown.

In simulations with the analytical code, changing the number of interaction points reproduces the same results as obtained by HEADTAIL, thus confirming once more the physical origin of the slow emittance-growth phenomenon.

As a second step, we implemented in the analytical code the actual longitudinal electron cloud distribution, taken from the PIC simulations. For simplicity, in the transverse plane the cloud is approximated by a bi-Gaussian distribution whose rms size at each z-location is computed assuming $\rho_e \sigma_e^2 = const$. The real transverse cloud profile is much more complicated than a Gaussian and varies along z. Moreover the assumption of charge conservation is not



Figure 4: Left: Vertical emittance vs. number of turns in LHC, for a Gaussian electron cloud, a linearly increasing density and one interaction point. Simulations with HEAD-TAIL (red) and MICROMAP (black). The cloud rms size is $\sigma_e = 0.25\sigma_b$ and the maximum tune shift is $\Delta Q = 0.04$. Right: emittance growth with different PIC grid sizes.



Figure 5: Horizontal (left) and vertical (right) emittance vs. number of turns in LHC, for the HEADTAIL pinched distribution (red) and for the analytical approximation (black). Charge conservation is assumed and the initial cloud rms size is set to $\sigma_{e,0} = 0.65\sigma_b$ (fit in the horizontal plane).

strictly valid since (in the PIC simulations and in reality) the total number of electrons within $1\sigma_b$ increases due to the arrival of electrons from the outer regions. Nevertheless, this approximation allows us to explore the main features of the effect and yields the correct field outside the core of the pinched electron cloud (a fraction of the beam size). Comparison of emittance growth computed with HEADTAIL and with the approximated model, gives qualitatively similar results, but different in absolute growth rates by up to a factor of 2 or 3 (Fig. 5), due to the differences in the electrons distribution.

REAL LATTICE SIMULATIONS

Finally after successful benchmark,the code MI-CROMAP, is used to simulate electron cloud in CERN SPS, assuming a much more accurate model of the accelerator structure. We tracked 1000 proton macroparticles through the full SPS optics (as from a MAD-X file), including 744 beam-electrons interaction points (one per dipole magnet). A maximum tune shift of $\Delta Q_{max} = 0.13$ is assumed. Space charge is not included in the simulations and a large chromaticity $\xi \approx 1$ is used. Simulations for two working points (26.15, 26.18) and (26.18, 26.15) shows a larger emittance growth, beam losses and bunch shortening if the vertical tune is lower, in agreement with observation in SPS [9] where it was necessary to increase Q_y to improve the beam lifetime.



Figure 6: Simulations for SPS with different working point. Top: (26.18, 26.15), Bottom: (26.15, 26.18)

CONCLUSIONS

The incoherent emittance growth for moderate electron cloud densities (i.e. below the threshold of the fast-headtail instability) may be caused by particle diffusion due to resonance crossing and scattering mechanisms, as well as by crossing linearly unstable regions. This phenomenon is a result of the combined effect of the synchrotron motion and longitudinal tune shift modulation, which is induced by the electron pinch at the bunch center. Predicting the exact growth rate requires an accurate modeling of the lattice and the electron distribution in the ring. Preliminary simulations for the SPS may explain the observed beam losses and bunch shortening as a function of working point. More benchmarking experiments in SPS are planned for 2006.

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