MATCHED AND EQUIPARTITIONED METHOD FOR HIGH-INTENSITY RFQ ACCELERATORS[*]

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Abstract

To prevent the emittance growth and halo formation in high intensity linacs[1~3], a design method has been proposed for high-intensity RFQ dynamics design by keeping beam envelope matched, confining energy balance within the beam and avoiding structure resonances. Results are given for a test RFQ designed by MATCHDESIGN - a new code based on the method. Comparisons of simulation results between this RFQ and a conventional RFQ had proved the feasibilities and merits of the new method.

INTRODUCTION

The Radio Frequency Quadruple (RFQ) is an indispensable unit for low-energy and high - intensity linac, the beam dynamics of which had been extensively studied by LANL and a generalized method had been proposed [4]. Overall RFQ Linac is divided into four sections: Radial Matching (RM), Shaper (SH), Gentle Buncher (GB) and Acceleration section (AC). Some extensions for this method had been explored by Yamada [5], Schempp[6] and Jameson[7]. As the beam parameters are not taken into consideration in these conventional methods, the matched conditions of beam are not satisfied intrinsically along the RFQ. In fact, the beam is not brought to and held in an equilibrium state along the structure, so space charge forces may couple the longitudinal and transverse motions and drive the beam towards an equipartitioned (EP) state, and consequently with some consequent emittance growth and related beam loss. Moreover, except for the envelope resonance at 90° phase advance, structure resonances are not systematically avoided. As a result, a dynamics design and optimization have to be carried out normally by simulation codes like PARMTEQM [4], TOUTATIS[8], LIDOS[9] and pteqHI[10] The design method that does not involve the beam features lacks sufficient physical sense.

A partially EP design was first realized in IFMIF RFQ, which showed that the beam could be brought to a matched and EP condition in a RFQ; however the RFQ was hardly energy balanced in this initial work and emittance growth occurred [11]. Later Jameson developed methods for linac design using matching plus EP condition and care in crossing resonances which EP condition aids significantly [12].

There are only three equations we know so far (two envelope equations and an EP equation) representing the physics of the beams with space charge in a linac, which relates the beam physics and can be used to design the structural parameters of the accelerator around the beam. However, more than three parameters should be determined for a linear accelerator, so additional rules or some simplifications are necessary. The beam radius will be varied very slowly in a RFQ if the transverse phase advance with beam current σ_t is kept constant [14], which can be used as the fourth equation to determine these parameters. The four equations above were combined in this new design method and all design procedures can be realized by a code called MATCHDESIGN. Therefore a beam dynamics design for a matched and equipartitioned beam can be done automatically.

MATCHED AND EP DESIGN

In the shaper (SH) the input transverse beam internal energy is much higher than longitudinal one, so the beam is not in equilibrium and it is also gradually compressed by a bunching voltage. As mismatch is a major source of emittance growth and halo formation, the transverse matched conditions should be kept throughout the whole RFQ, even in the SH section [10]. Controlling the beam length and the longitudinal bunching to reach an EP condition at the end of SH is an important aspect, and the beam may be held in equilibrium in GB/ACC section. Assuming there is a bunched ellipsoidal beam in the GB/ACC section, according to the smooth approximation the radius of the matched beam envelope follows formula (1) and (2)[1]:

$$\varepsilon_{tn} = \frac{a^2 \gamma \sigma_t}{\lambda} \tag{1}$$

$$\varepsilon_{ln} = \frac{b^2 \gamma^3 \sigma_l}{\lambda} \tag{2}$$

where $\sigma_t = \sqrt{B^2/8\pi + \Delta - SCHx}$, $\sigma_l = \sqrt{-2\Delta - SCHz}$. σ is the phase advance with beam current, t denotes transverse and l longitudinal, ε is the normalized rms (root mean square) emittance, a and b are transverse and longitudinal rms beam radii respectively (assuming an ellipsoidal distribution after SH section). B is the focusing parameter for RFQ accelerator, λ is the rf wavelength, I is the beam current; space charge $SCHx = \frac{I\lambda^3(1-ff)}{a^3 b \gamma^3}k$, $SCHz = \frac{2I\lambda^3 ff}{a^3 b \gamma^3}k$, $ff \approx a/3b$,

γ and β are the relativistic gamma and beta, and $k = \frac{3}{8\pi}\frac{z_o q 10^{-6}}{mc^2}$. $Z_0 = 376.73\Omega$, Δ is rf defocusing parameter

$\Delta = \frac{\pi^2 q V_0 A \sin\phi_s}{2mc^2\beta^2}$, A, V_0, ϕ_s and β are accelerating parameter, intervane voltage, synchronous phase and relative velocity respectively.

If there is an imbalance of oscillation energy between the degrees of freedom in the beam, free energy is available which, by driving resonances, can cause emittance increase. In order to prevent this type of emittance growth, the energies in the degrees of freedom should be balanced. Then the beam will be in equilibrium and follow the equipartition condition (3)[1-2]:

$$\frac{\sigma_t}{\sigma_l} = \frac{\varepsilon_{\ln}}{\varepsilon_{tn}} = \frac{\gamma b}{a} \qquad (3)$$

Only three controlling equations (1~3) are really available to solve the four beam paramters (ε_{tn}, $\varepsilon_{\ln}$, a, b), the four design parameters for each cell of a RFQ accelerator: $B(n), \phi_s(n), M(n)$ and $V_0(n)$, where n denotes cell number and M is the modulation. Therefore, the excess parameters must be specified initially: e.g. $\varepsilon_{tn} = 0.2$ mm mrad, $\varepsilon_{\ln}/\varepsilon_{tn} =1.4$ (in GB/ACC section). The voltage V_0 is chosen and fixed as a constant. In order to have a quasi-constant beam size, equation (4) should be satisfied and the focusing parameter B should be changed along accelerating channel instead of having invariant B and σ_{t0} (phase advance with zero current) in the generalized method [4]

$$\sigma_t = \text{Constant} \qquad \text{or}$$

$$B(n+1) = B(n) - [\Delta(n+1) - SCHx(n+1) - \Delta(n) + SCHx(n)] 4\pi / B(n) \quad (4)$$

The beam radius a is varied very slowly and is a quasi-constant when the transverse matching condition is always satisfied. Therefore after simplifications there are only four parameters ($B(n)$, $\phi_s(n)$, $M(n), b(n)$) should be solved in each cell of a RFQ accelerator. This procedure results in a rapid decrease of the RFQ aperture in the end of SH section. After decreasing to a minimum, the aperture is slowly increased in ACC section. The synchronous phase is also kept nearly constant. Now only eqs(1) and (4) are available to keep the transverse size constant and matched.

A TEST DESIGN BY MATCHDESIGN

A test design of a proton RFQ operated at 402.5MHz is generated by MATCHDESIGN. The basic parameters are listed in Table.1.

Table.1 Basic parameters of the test RFQ

Frequency (MHz)	402.5
Input energy (MeV)	0.065
Output energy (MeV)	2.50
Peak beam current (mA)	55
Input emittance [Trans.,Norm,rms](π mm mrad)	0.2

Firstly MATCHDESIGN finds out the proper aperture and modulations of the vanes for the RFQ accelerator according to the longitudinal and transverse current limits. Then the parameters of matched beam in the SH section are accordingly determined through matching procedures to depress the large amplitude envelope oscillation [13]. Secondly the elliptical parameters of matched beam in the entrance of RFQ can be determined

backward by inverse transformation. Afterwards the focusing parameter B will be tuned as described in section II, while the accelerating field is rising, to maintain matched and EP conditions until the minimum aperture point. Finally the transverse beam size and match will be maintained in the ACC section. The dynamics simulation of the matched design above was carried out by PARMTEQM [4]. The rms beam sizes, emittances and oscillation energies T ($T = \varepsilon_{\ln}/\gamma b$ or ε_{tn}/a) versus cell number are plotted in Fig.1~3. To see clearly the improvement that the new design method gives, comparisons are made in these figs between the test RFQ and a conventional high intensity RFQ (C-RFQ) [16].
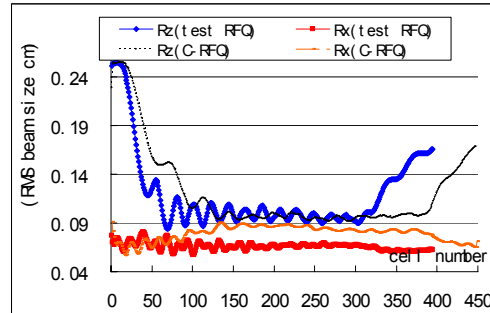

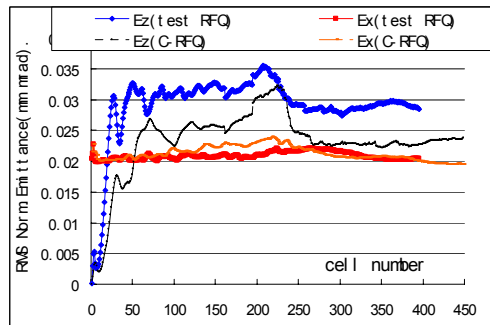Fig.1 Beam radius (RMS) versus cell number
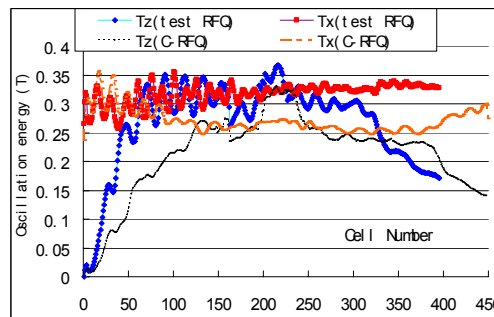

Fig.2 Emittance versus cell number

Fig.3 Oscillation energies versus cell number

The main dynamics parameters for both designs are given in Table.2.

The design trajectory of the test design is plotted on the Hofmann Chart calculated for the nominal emittance ratios ($\varepsilon_z/\varepsilon_x \approx 1.4$)[15] in Fig.4. It shows there are many integer resonances in the chart, e.g. 1/2, 1/3 and so on. Although equipartitioning is not strictly necessary for bunch stability and emittance conservation, there is no free energy in an EP beam to drive a resonance; hence the growth rate of the resonance near 1/1.4 in Fig. 4 is zero. A

long trajectory crossing the clear region may still show small accumulating emittance growth from the infinite number of minor rational number resonances, but only with low growth rates. This has been already testified by dynamics simulation results of the given test design.

Table.2 Main dynamics parameters of both designs

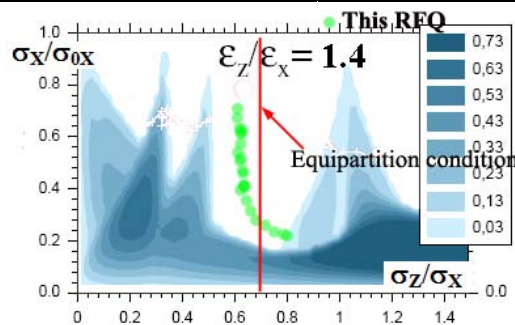|		This RFQ	C-RFQ
Frequency (MHz)		402.5	402.5
Synchronous phase (°)		-90~-33	-90~-30
Design current(mA)		55	55
Vane modulation		1~1.77	1~1.73
Intervane voltage (kV)		82	83
Output emittance [N,rms]	X (π mm mrad)	0.204	0.194
	Y (π mm mrad)	0.212	0.195
	Z (π mm mrad)	0.284	0.237
Minimum aperture(mm)		2.1	2.4
Cell numbers		394	448
Length(m)		3.40	3.73
Beam transmission (%)		99	96

Fig.4 Design trajectory of the test design

The behaviour of beam in the case of high intensity is very complicated. First, the rms properties of the beam in the RFQ should be matched to the accelerating channel as well as possible. So the beam sizes in the test RFQ are kept nearly constant, which validates the design requirement of constant beam size. Second, in the SH a large part of the increase in the longitudinal rms emittance comes just from the bunching action (see Fig.2). Third, in the SH the longitudinal space-charge forces also increase the emittance, but we are very careful not to bunch too quickly. Fourth, because the transverse energy is larger than the longitudinal energy until the shaper end as shown in Fig.2 and Fig.3, where EP is achieved (at the 79th cell), there is also an emittance growth from energy exchange between planes - the equipartitioning process. Fifth, space charge driven resonances in the GB/AC section need to be avoided in spite of the relatively short length of the linacs [12], given matching condition is fully satisfied. So it is helpful for RFQ dynamics designs to plug the analytical results for given (initial) emittance ratio into a resonance chart (Hofmann chart) with contour levels for the analytically calculated resonance growth rates shown in the plane of tune depression in one direction (here chosen as x) versus the focusing ratio σ_z/σ_x.

The C-RFQ is a successful conventional design with high transmission through many trial and error procedures[16]. However, the Eq.(4) is not satisfied and the rms beam size increased from 0.06 cm to 0.09 cm, later decreasing to 0.06 cm (Fig.1). Fig.2 shows that both designs are EP from the end of the shaper to the minimum aperture point. In the test design matching equations, the transverse beam sizes were kept throughout the RFQ and EP was satisfied in GB section, therefore the five mixed-up effects above were carefully disposed, which results higher transmission efficiency in spite of the fast bunching process and less cell numbers (see Table.2).

CONCLUSION

In this paper matched design methods for high intensity RFQ are proposed. Matching equations, EP condition and constant transverse beam size are used to solve the design parameters. Based on the new method, we developed a code – MATCHDESIGN that can directly work out a good design (the input file for simulation codes) without unnecessary trial and error procedures. Although RFQ linacs are relatively short and particles pass only once, the beam evolution is still mainly controlled by space charge driven resonances, especially in heavy space charge regime. Therefore the Hofmann chart is useful to guide the dynamics design of RFQ and to avoid space charge driven resonances. It turned out that the design trajectory of the test design appears near EP line and crosses clear region, so the output beam has small accumulating emittance growth in the simulation. The given test RFQ with very low emittance growth and almost no beam-loss has proved the advantages of this new method.

ACKNOWLEDGEMENTS

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