THE BEAM-BEAM LIMIT AND FEEDBACK NOISE

K. Ohmi, S. Hiramatsu, M. Tobiyama, Y. Funakoshi and K. Oide
KEK, 1-1 Oho, Tsukuba, 305-0801, Japan

Abstract

Beam-beam interaction is strongly nonlinear, therefore particles in the beam experience chaotic motion. A small noise can be enhanced by the chaotic nature, with the result that unexpected emittance growth may occur. We study the noise of a transverse bunch by bunch feedback system and related luminosity degradation.

INTRODUCTION

Emittance is defined as an average of a half of Courant-Snyder invariant ($\boldsymbol{J}$) over all beam particles. The invariant is given for each particle in each degree of freedom in linearized system. Since the invariant is no longer kept constant in nonlinear system, the emittance growth arises with the result that the luminosity degradation occurs [1].

While the invariant of each particle fluctuate in linear system with the presence of external diffusion $\Delta y(t)$, where $\langle \Delta y(t)\Delta y(t')\rangle = D\delta(t-t')$ for a white noise fluctuation. Evolution of the particle distribution is satisfied to the so-called diffusion equation or thermal conducting equation

$$\frac{\partial}{\partial t}\Psi(y,t) = D\frac{\partial^2}{\partial y^2}\Psi(y,t), \tag{1}$$

where Fokker Plank term, which is related to the betatron oscillation, is neglected, D is called the diffusion rate. Emittance, which is $\langle J_y\rangle$, linearly increases as follows

$$\frac{\varepsilon(t)}{\varepsilon(0)} = 1 + \frac{D}{\sigma_y^2}t. \tag{2}$$

In an electron-positron collider, each particle in the beam experiences the radiation excitation and damping. The radiation excitation is regarded as an external diffusion with a rate D_γ for beam particles. The equilibrium emittance is determined by the balance of the diffusion and the damping with the characteristic time τ.

$$\varepsilon_y = D_\gamma \tau/2. \tag{3}$$

where the factor 2 is due to the fact that the invariant is second order of amplitude, $J_y \sim y^2$. The diffusion rate is given by $D_\gamma/\sigma_y^2 = 2/\tau$ from emittance and damping time, which are popular for us. The fluctuation ($\Delta y = \sqrt{D_\gamma}$) is given by $\sqrt{2/\tau}\sigma_y$ turn by turn. For KEKB ($\tau = 4000$ turn) and DAFNE ($\tau = 10^5$ turn), the fluctuations are $0.022\sigma_y$ and $4.5\times10^{-3}\sigma_y$, respectively.

Nonlinear system with the presence of external diffusion is considered now. If the two diffusions are independent, the total diffusion rate is summation of the two. However the two are not considered to be independent. Everyone has an image in which external fluctuation is enhanced in nonlinear system with chaotic nature [2]. For example, though the system with two degree of freedom does not have the nature of nonlinear diffusion, external diffusion can push out particles into a large amplitude due to the mixing with the nonlinear diffusion. Radiation excitation has a important role for the beam-beam limit in the case without errors nor crossing angle [3].

We next consider another type of fluctuation. Bunch by bunch feedback system and fluctuation of hardware parameter such as voltage or phase give a fluctuation of coherent motion of the beam. Typical case is fluctuation of dipole mode, $\Delta\langle y\rangle$. This fluctuation does not have a diffusion feature for each particle in the beam. For linear system, the fluctuation is observed as a coherent motion, whose frequency component depend on the noise source. For example if the fluctuation is 10% of the beam size, the geometrical luminosity degradation is negligible and the fluctuation is not disaster in linear system.

Coherent fluctuation in nonlinear system is main subject of this paper. Everyone knows Landau damping or nonlinear decoherence, in which coherent amplitude decreases due to tune difference for amplitude of each particle [4]. The coherent amplitude transfers to an increase of the beam size. We can image the coherent fluctuation on the phase space with chaotic nature. In a regime in which external diffusion is strongly coupled to nonlinear diffusion, total diffusion rate does not seems to have large difference between the two cases that the fluctuation is incoherent (diffusion) or coherent.

It is important how large fluctuation affects the luminosity performance. The beam-beam system with a high tune shift is affected by the radiation excitation affects. Therefore coherent fluctuation with a higher level than the radiation excitation affects the luminosity performance. Since the radiation excitation is given $\sqrt{2/\tau}$, the sensitivity for the fluctuation is severe for accelerator with slow damping rate.

This type of diffusion, which is caused by coherent fluctuation in nonlinear system, may be actually observed in some accelerators, KEKB, DAFNE [5], HERA and RHIC [6]. In KEKB, an optimization of the feedback system was essential to get a high luminosity. The gain of the feedback system had to be reduced as long as a coupled bunch instability is cured.

NOISE OF BUNCH BY BUNCH FEEDBACK

Bunch by bunch feedback system can be a source of a fast (turn by turn) coherent fluctuation. The bunch by bunch feedback system is used to suppress a coherent dipole instability [7, 8]. The feedback system kicks the beam to damp its oscillation caused by an instability with the oscillation amplitude measured by position monitor. There is two noise source in the feedback system. One is in position measurement, and another is in the kick strength.

In this section we analyze the transverse bunch-by-bunch beam feedback loop based on z-transform where the feedback loop response is characterized by transfer functions. The stability condition of the feedback loop is given by the characteristic equation of the transfer function, and the solutions of the characteristic equation give the damping time of the beam oscillation [9] and also the beam orbit fluctuation due to the noise component in the feedback loop.

The n-th turn phase space point of the particle at the kicker position s_1 is expressed by

$$\begin{pmatrix} x_n \\ x'_n \end{pmatrix} = e^{-1/\tau} M \begin{pmatrix} x_{n-1} \\ x'_{n-1} \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta x'_n \end{pmatrix} \qquad (4)$$

where

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix} \qquad (5)$$

is the one-turn transfer matrix, and α_1, β_1 and γ_1 are the Twiss parameters at $s = s_1$, and τ is the radiation damping time in the unit of revolution time. $\Delta x'_n$ is the kick angle of the feedback kicker is expressed by

$$\Delta x'_n = \frac{K}{\sqrt{\beta_1\beta_2}} \sum_{m=1}^{N_{tap}} h_m (M_{11}(s_m, s_1) x_{n-N-m} + M_{12}(s_m, s_1) x'_{n-N-m}) + \frac{\Delta k_n}{E/e} \qquad (6)$$

where h_m is the tap weight of the filter, N is the additional delay, K is the kicker strength and $M(s_m, s_1)$ is the transfer matrix from kicker to monitor $s = s_m$.

In the case of the 2-tap FIR filter ($N_{tap} = 2$) employed at the KEKB feedback system with $h_1 = -h_2 = 1$ and without additional delay ($N = 0$), the eigenvalues are given by the solution of the 3rd order equation,

$$\begin{aligned} z^3 &- \left\{2e^{-1/\tau}\cos\mu - K\sin(\mu-\psi)\right\} z^2 \\ &+ \left[e^{-2/\tau} + K\left\{e^{-1/\tau}\sin\psi - \sin(\mu-\psi)\right\}\right] z \\ &- Ke^{-1/\tau}\sin\psi = 0 \end{aligned} \qquad (7)$$

The fluctuation of vertical amplitude is given by

$$\begin{aligned} \langle x^2 \rangle &= \frac{\langle \Delta k^2 \rangle}{E/e} \beta_1^2 e^{-2/\tau} \sin^2\mu \\ &\times \left(\frac{K_1^2}{1-z_1^2} + \frac{K_2^2}{1-z_2^2} + \frac{K_3^2}{1-z_3^2} \right. \\ &+ \left. \frac{2K_1K_2}{1-z_1z_2} + \frac{2K_2K_3}{1-z_2z_3} + \frac{2K_3K_1}{1-z_3z_1} \right) \end{aligned} \qquad (8)$$

where

$$\begin{aligned} K_1 &= \frac{z_1}{(z_1-z_2)(z_1-z_3)} \\ K_2 &= \frac{z_2}{(z_2-z_3)(z_2-z_1)} \\ K_3 &= \frac{z_3}{(z_3-z_1)(z_3-z_2)} \end{aligned} \qquad (9)$$

The expected beam orbit fluctuations at the kicker position calculated from Eq. 8 are shown in Fig. 1. The kick fluctuation is given by the noise voltage $\Delta k = \sqrt{2R_s/Z_0}e$, where $R_s = 8$ kΩ is the kicker shunt impedance and $Z_0 = 50\Omega$ is the line impedance The noise voltage of $\langle e^2\rangle^{1/2}$=7.28 V at $K = 0.1$ and $\langle e^2\rangle^{1/2}$=270 mV are assumed in the calculation based on the circuit noise measurement.

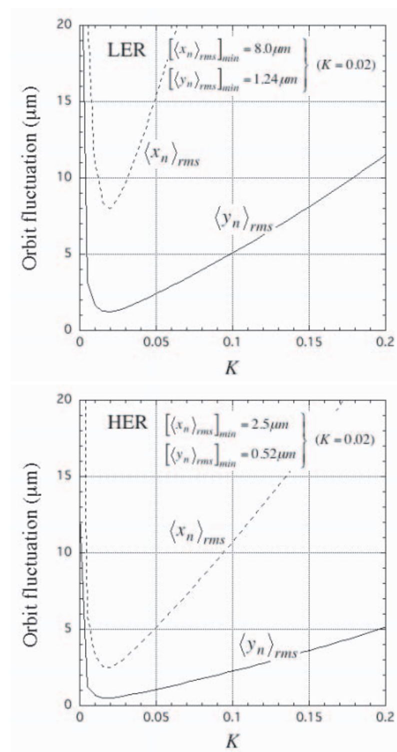

Figure 1: Orbit fluctuation in the LER and HER at kicker position ($\beta_y = 20$ m) for $\nu_x/\nu_y = 0.51/0.57$.

Minimum fluctuations of 8.0 μm (horizontal)/1.2 μm (vertical) for the LER and 2.5 μm (horizontal)/0.5 μm (vertical) for the HER are expected at the loop gain of $K \approx 0.02$ which is considerably smaller than the optimal gain for the minimum damping, where $\beta_1 = \beta_2 = 20$ m, $\psi = -\pi/2$. The expected damping time of about 50 turns from $K = 0.02$ is approximately agrees to the measured damping time in the usual KEKB operation. Since the beam size is $\sigma_x/\sigma_y = 600/60\mu$m at the kicker position, the fluctuation is several % of the beam size.

BEAM-BEAM EFFECT IN THE PRESENCE OF THE FEEDBACK NOISE

Beam-beam simulations including these fluctuations were performed to study the luminosity degradation. Figure 2 shows the luminosity and vertical beam size as a function of amplitude of vertical offset fluctuation. The luminosity is sensitive for the offset fluctuation: 5% degradation for offset fluctuation of $0.01\sigma_y$ is seen in the figure. In the beam-beam limit, radiation excitation played a important role. The diffusion rate of the radiation excitation is $\Delta y \approx \sigma_y\sqrt{2/\tau_y}$, where τ_y is damping time in unit of turn. In our case, since τ_y is 4000-6000 turns, the diffusion rate, which is $0.02\sigma_y$, is comparable with the sensitivity.

This offset fluctuation is the same level as that induced by the bunch by bunch feedback system. The fluctuation of the feedback system can degrade the luinosity performance.

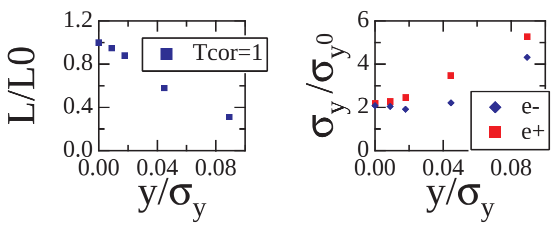

Figure 2: Luminosity and vertical beam size as a function of amplitude of vertical offset fluctuation at the collision point.

CONCLUSION

Bunch by bunch feedback gives fluctuation in vertical orbit with several % of the beam size in KEKB. The magnitude is comparable with fluctuation due to the synchrotron radiation. Stronger gain, which is essential for suppression of an coupled bunch instability, induces larger fluctuation. The fluctuation can affect beam-beam performance, and luminosity dependence on the feedback gain is actually observed in usual operation of KEKB.

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REFERENCES

[1] K. Ohmi et al., in this proceedings, MOPLS032.

[2] J.L. Tennyson, AIP Conference Proceedings **87**, 345 (1982)

[3] K. Ohmi et al., Phys. Rev. Lett. **92**, 214801 (2004).

[4] T. Sen and J.A. Ellison, Phys. Rev. Lett. **77**, 1051 (1996).

[5] P. Raimondi, private communication at super B workshop in HAWAII (2005).

[6] C. Montag, private communication at super B workshop in HAWAII (2005).

[7] M. Tobiyama and E. Kikutani, Phys. Rev. ST-AB **3**, 012801 (2000).

[8] M. Tobiyama, Phys. Rev. ST-AB **9**, 012801 (2006).

[9] V. M. Zhabitsky, NIM A391, 96 (1997).