# ANALYSIS OF WAKEFIELDS IN THE ILC CRAB CAVITY

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## Abstract

The large crossing angle schemes of the ILC need a correction of bunch orientation at the interaction point (IP) in order to recover a luminosity loss of up to 80%. The orientation of bunches can be changed using a set of transverse deflecting cavities. The location of these crab cavities would be close to the final focus, and small deflections caused by wake fields in the cavities could cause misalignments of the bunches at the IP. Wake fields in the 3.9GHz deflecting cavities under development at FNAL have been analysed and their effects studied in view of use as the ILC crab cavity. Numerical simulations have been performed to determine the long-range wake potentials of up to quadrupole order modes in this cavity and their effect upon bunches passing through this cavity. Trapped modes within the CKM cavity have been investigated. Short-range wakes have also been a topic of study. The effect of the final focus quadrupole magnets on the deflection given to the bunch have also been calculated and used to calculate luminosity loss due to wake fields.

## **INTRODUCTION**

The baseline design of the ILC has a significant crossing angle between the electron and positron lines at one of the IPs. A consequence of having a crossing angle is the reduction in luminosity due to the geometry of the collision. Such a loss in luminosity can be recovered by rotating both bunches prior to collision with deflecting ("crab") cavities.

A crab cavity is a RF cavity that uses the first dipole mode for its operation instead of the accelerating monopole mode. This dipole mode has zero longitudinal electric field and large transverse fields along its beam axis. The transverse fields provide a Lorentz force perpendicular to the (on-axis) velocity of the bunch.

If the phase of the RF is timed so that the centre of the bunch passes through the cavity when the magnetic field is zero (at "zero-crossing") then the head and tail of the bunch will experience equal and opposite Lorentz forces, causing the bunch to appear as if it has rotated.

A superconducting dipole cavity is currently under development at Fermilab as a time-slicing device for studying bunch structure, which could be used for the ILC crab cavity with some modifications [1, 2]. The cavity, also known as the CKM cavity, is a 13-cell cavity that operates at 3.9GHz in the  $\pi$ -mode of the 1st dipole passband, shown in Figure 1. The separation of modes in the dipole passband between the  $\pi$ -mode and the next mode is ~1MHz for a 13 cell structure. Although methods of field flatness tuning have been developed [3], the dangers of trapped modes that might be excited by the ILC beam are a risk that could be reduced by the use of fewer cells per cavity. At an operating gradient of 6MV/m, four 9-cell versions of this cavity could be used on each side of the interaction region to rotate the bunch on the ILC with 1TeV CM energy.



Figure 1: Electric fields within the FNAL cavity

## **MODAL ANALYSIS**

The frequencies and beam couplings (R/Q) of the monopole and dipole modes for the 9-cell cavity were found using the 2D eigenmodes solver in MAFIA. Long beam pipes (150mm) were used to sufficiently attenuate any non-propagating or trapped modes before the conducting boundaries at the end of the beam pipe. These were checked using additional simulations with 500mm beam pipes. The eigenmode calculations will only be satisfactorily accurate if the fields at the end of the beam pipe are small. In order to verify this is indeed the case, eigenmodes were calculated using firstly, perfect electric boundaries and secondly, perfect magnetic boundaries. The change in boundary conditions will produce a change in the resonant frequency proportional to the fields at the boundary. Field plots are also used to observe if a mode is trapped. If the mode is found to have large fields at the end of the beam pipe, it was disregarded.

The calculations found the operating dipole mode at 3.9GHz had the R/Q of  $230\Omega$  (where the circuit definition, shown in eqn. (1) is used for R/Q)

$$\frac{R}{Q} = \frac{|V(a)|^2}{2\omega U} \left(\frac{c}{\omega r}\right)^{2m}$$
(1)

The rest of the modes in the  $1^{st}$  passband had low (<0.5 $\Omega$ ) R/Q's. As the dipole modes have two

polarisations, the other polarisation of the operating mode is unwanted and will have unfavourable effects on the beam, and as a consequence will need to be damped effectively. There were also dipole modes with high loss factors found at 7GHz (4.9 $\Omega$ ), 8GHz (4.1 $\Omega$ ), 10GHz (1.5 $\Omega$ ), and 13GHz (1.0 $\Omega$ ), as can be seen in Figure 2. Of these modes the mode at 8GHz was found to have very low fields in the end cells and beam pipes, meaning that this mode would likely be trapped.



Figure 2: R/Q of dipole modes in the 9-cell cavity.

In a dipole cavity the fundamental accelerating mode is unwanted but can be excited by the beam. Because the resonant frequency of this mode is far below the cut-off frequency of the beam-pipe, the fundamental mode does not couple strongly to traditional higher order mode (HOM) couplers. If left undamped, this mode would appreciably raise the temperature of the superconducting cavity. In order to prevent this heating effect, the mode is coupled out via a LOM coupler. A 2D eigenmode simulation was performed to find the loss factors of the monopole modes up to 10GHz.



Figure 3: Loss parameter for the 1<sup>st</sup> monopole passband

The fundamental mode passband (shown in Figure 3) at ~2.83GHz had the highest loss factor, and the  $7\pi/9$  mode had the highest loss factor in this passband, at 2.53V/pC. A search was also conducted for quadrupole modes. None that would interact significantly with beam for the

beam to cavity axis alignments that we expect to have were found.

#### **EFFECT OF LONG RANGE WAKES**

The effects of long-range wakes were calculated using the method described in [4].

If the bunch is offset horizontally it will induce the dipole modes with the same polarisation as the operating mode of the cavity. Excitation of the operating mode of the cavity by previous bunches will not deflect the centre of charge of the beam, if the beam arrives at the correct time. However, it will provide extra rotation to the bunch. The deflection for a bunch offset of 4 sigma in the horizontal plane was calculated to be less than 2nrad for all bunches in the train.

For bunches offset vertically the unwanted polarisation of the crabbing mode dominates the deflection and is dependant on the exact frequency of this mode. The energy deposited in the cavity by this mode is minimal when the frequency is given by the  $(n+\frac{1}{2})$  harmonic of the bunch repetition rate, where n is an integer, in this case the maximum deflection in the vertical plane is less than 0.1 nrad.

The presence of wake fields in the crab cavity will readily produce a deflecting kick to the centre of the bunch. This will offset the bunches with respect to each other at the point of collision. The luminosity of the colliding bunches is reduced in a Gaussian manner, according to the offset  $\Delta x$  and  $\Delta y$ , shown in equation 2:

$$S = \exp\left(-\frac{\Delta x^2}{4\sigma_x^2}\frac{\Delta y^2}{4\sigma_y^2}\right)$$
(2)

If we consider a bunch with  $\sigma_x$ =500nm and  $\sigma_y$ =5nm then for 2.5% luminosity loss, the offset must be less than 160nm in the horizontal plane and 1.6nm in the vertical plane. These offset tolerances can be used to calculate the maximum deflecting voltage tolerance at the crab cavity. Assuming a 500GeV beam with R12=16.3 m/rad horizontal and R12=2.4m/rad vertical, we calculate the maximum voltage in the horizontal plane to be 4.9kV, and 0.3kV in the vertical plane. We can also calculate x' to be 9.8nrad horizontally and 0.66nrad vertically.

## SHORT RANGE WAKES

The ILC is designed to have short bunches, ~1ps, which will of course induce high frequency wake fields in the crab cavities and other microwave components. In such a system the broadband high frequency component of the impedance are significant. The modal approach is not sufficient to calculate the broadband part of the impedance hence another approach is required. The transverse wake fields result in deformation of the bunch into a banana-shape, potentially reducing luminosity. Short-range wake fields are dominated by iris diameter, and it will not be possible to expand the iris much over its present 30mm diameter without lowering the frequency of the cavity. However, lowering the frequency of the crab cavity will require tightening an already difficult phase control specification.

Analytic forms [5] permit estimation of wake fields under various assumptions. A well-known technique is to modify these forms so as to have them fit the results of time domain numerical studies [6]. Here we use the analytic forms to reject cases where the 2D time domain MAFIA code has produced unphysical results, due most probably to inadequate meshing. The analytic forms also provide a guide as to the variation of the wake fields with bunch length. As we are unable to use numeric methods at all bunch lengths of interest, and as the numeric methods give systematically lower results than analytic ones, the analytic methods are used to set upper bounds on the impact of short range wakes.

Fig. 4 shows the total longitudinal loss factor,  $k_{\text{tot}} = \Delta E/(q_{\text{bunch}})^2$  as a function of bunch length, for the 13 cell version of the structure, both as calculated analytically and as found with a MAFIA as run with a variety of meshes and flag settings.



Figure 4:  $k_{tot}$  as a function of bunch length, as calculated by a variety of methods.

The analytic form fits well the expression  $b+\alpha\sigma^N$ , albiet without meaningful values for the parameters. From a fit to the MAFIA result with the same form, we expect  $k_{\text{tot}}$  for a structure of four 9-cell cavities to be about 100V/pC.

The transverse wakes are still under study, but the analytic form also appears to be a conservative overestimate. Figure 5 shows the analytic form for a  $\delta$ -function bunch, and after convolution of this form with a Gaussian bunch of 1ps length.



Figure 5: Transverse wake potential and deflecting field.

The case shown is for a 1mm off-axis bunch passing through a 13-cell cavity, but in the analytic form, the wake is proportional to the number of cells and hence the to deflection. Hence, changing the number of cells will not change the fundamental conclusion of Figure 5, namely that the transverse wakes are very small indeed compared to the deflecting field itself. Integration of two colliding Gaussian charge distributions, one for the  $e^-$  and one for the  $e^+$  beams, reveals a negligible luminosity dilution when the bunch shapes are deformed in accordance with figure 5.

## **CONCLUSION**

The wake fields of the 9-cell version of the 3.9GHz crab cavity have been carefully analysed. Deflections due to transverse long-range wakes are considerably lower than the tolerances prescribed for the cavities. Furthermore, the loss parameter for 1ps bunches is  $100 \pm 8 \text{ V/pC}$ , and transverse short range wakes do not appear to have any sizeable impact on the luminosity.

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