

DIAGNOSIS OF COUPLING AND BETA FUNCTION ERRORS IN THE PEP-II B-FACTORY

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Abstract

The SLAC Control program has an automatic betatron phase advance measuring system whereby the beta functions of the two storage rings are measured. This facility has recently been extended to measure coupling between the horizontal and vertical motion and to fit the measured values of β functions and coupling to their modes of propagation. This facility aids the diagnosis and correction of coupling and focusing errors.

INTRODUCTION

A technique has been developed at CERN[1] to calculate the lattice beta functions using the betatron phase advance information measured by a multi-turn BPM system. A similar system has been used for the PEP B-Factory control system[2] and has recently been extended to:

- Work with 90°/cell lattices with just one Beam Position Monitor (BPM) per cell per plane of beam oscillation.
- Fit a $d(\beta)/\beta$ wave over a portion of the ring.
- Calculate and display a measure of the betatron coupling and fit this over a portion of the ring.
- Estimate the beta function at the beam waist and the offset of this waist from the Interaction Point (IP).

The beam in the storage ring is excited in each of the orthogonal transverse modes of oscillation and beam position information is recorded for 1024 turns around the ring. The amplitude and phase of the oscillation of each mode is extracted from this data.

THE 90°/BPM PROBLEM

Knowing the Twiss parameters (α , β and μ) of the lattice model, we may use the phase information from the model and from the measurements at three BPMs to find the beta function at those BPMs. For the middle BPM of a group of three

$$\frac{\beta(\text{meas})}{\beta(\text{model})} = \frac{\sin(\mu_{23}) \sin(\mu_{12}) \sin(\mu_{13m})}{\sin(\mu_{23m}) \sin(\mu_{12m}) \sin(\mu_{13})}$$

where:

- μ_{13} = measured phase between BPMs 1 and 3
- μ_{12} = measured phase between BPMs 1 and 2
- μ_{23} = measured phase between BPMs 2 and 3

- μ_{13m} = model phase between BPMs 1 and 3
- μ_{12m} = model phase between BPMs 1 and 2
- μ_{23m} = model phase between BPMs 2 and 3

There are similar expressions for finding the β functions at the first and last BPMs of the group.

These formulae assume that the lattice is error free between those BPMs.

The PEP 90°/cell arcs

Most of the arcs of the PEP rings have just “single-view” BPMs, i.e. they measure horizontal or vertical orbit displacements only, not both. The Low Energy Ring (LER) has 90°/cell phase shift so $\sin(\mu_{13m})$ will be equal to zero and $\sin(\mu_{13})$ will be very close to zero leading to divide by zero errors, or to great inaccuracy if the model is not exactly 90°/cell.

It is possible to overcome this problem if we use both phase and amplitude information. The amplitude information depends on the calibration of the BPMs, whereas the phase measurement does not, but since the BPMs in these 90°/cell sections are similar and only the relative amplitudes of the two BPMs is required, calibration errors are not important.

Measuring β by amplitude alone has the same problem as the phase measurement. Since the $d\beta/\beta$ error propagates at 2μ the 90°/cell BPMs could coincide with the nodes of the error wave. Since the deviations of the amplitude and phase are propagating in quadrature we expect to be able to measure β by combining the two measures.

At each of the those “single-view” BPMs, that only measure in the opposite plane, we interpolate the amplitude and phase using measurements from the adjacent BPMs and record a dummy reading there. We then use that dummy reading in the phase equation shown above to calculate β .

The motion at BPMs 1, 2 and 3 can be written

$$x_1 = \widehat{x}_1 \cos(n\mu + \delta_1)$$

$$x_2 = \widehat{x}_2 \cos(n\mu + \delta_2)$$

$$x_3 = \widehat{x}_3 \cos(n\mu + \delta_3)$$

\widehat{x}_2 and δ_2 are calculated from \widehat{x}_1 , \widehat{x}_3 , δ_1 and δ_3 as follows

$$\widehat{x}_2 = \sqrt{a_1^2 + a_3^2 + 2a_1a_3 \cos(\delta_3 - \delta_1)}$$

$$\delta_2 = \delta_1 + \sin^{-1} \left[\frac{a_3}{\widehat{x}_2} \sin(\delta_3 - \delta_1) \right]$$

$$a_1 = \left(m_{11}^{12} - \frac{m_{12}^{12}}{m_{13}^{12}} m_{11}^{13} \right) \widehat{x}_1$$

$$a_3 = \frac{m_{12}^{12}}{m_{13}^{12}} \widehat{x}_3$$

where

m_{mn}^{ij} is the m_{mn} element of the transport matrix from i to j .

FITTING OF $d\beta/\beta$ WAVE OVER A PORTION OF THE RING

A beta function perturbation ($d\beta/\beta$, $d\mu$) propagates in amplitude and phase as 2μ where μ is the betatron phase advance. If R is the ratio $R = \beta(\text{measured})/\beta(\text{model})$ we get ($d\beta/\beta$) by the expression

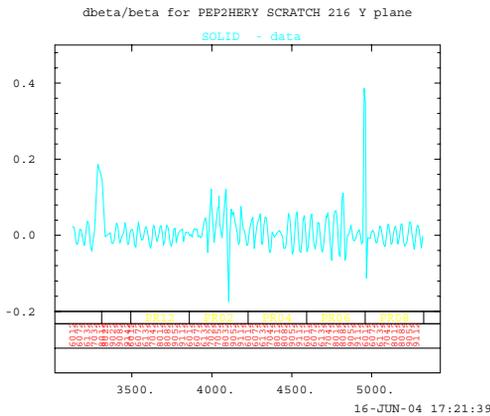


Figure 1: Function $d\beta/\beta$, for the vertical plane of the High Energy Ring (HER), plotted against distance around the ring. (The interaction Point is just past 400m.)

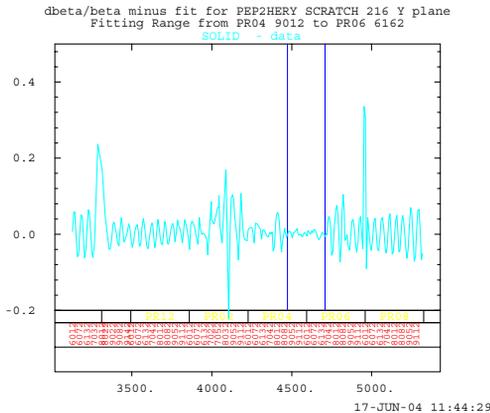


Figure 2: A beta-beat wave ($d\beta/\beta$) is fitted to the measured data. The figure shows that fit subtracted from the measured data. The fit is in the arc between the two tune correction straights. A mismatch appears to the right of the downstream tune straight.

$$\frac{d\beta}{\beta} = 2 \frac{R-1}{R+1}$$

By letting β and α vary at the entrance to the lattice we can fit a wave propagating at phase ($\mu(\text{model}) + \mu(\text{measured})$) to the measured value of ($d\beta/\beta$) over a portion of the ring. [We also have options to fit using $2\mu(\text{model})$ or $2\mu(\text{measured})$.]

CALCULATE AND DISPLAY A MEASURE OF BETATRON COUPLING

Following David Sagan[3, 4] of the CESR project at Cornell we use \overline{C}_{12} as one measure of the coupling, where \overline{C}_{12} is the ratio of the out of phase component of the “out of plane” amplitude to “in plane” amplitude, normalized to the square root of the ratio of the “out of plane” and “in plane” amplitudes.

$$\overline{C}_{12} = \sqrt{\frac{\beta_y}{\beta_x}} \frac{\hat{x}}{\hat{y}} \sin(\phi_x - \phi_y)$$

for the mode-2 excitation

$$\overline{C}_{12} = \sqrt{\frac{\beta_x}{\beta_y}} \frac{\hat{y}}{\hat{x}} \sin(\phi_y - \phi_x)$$

for the mode-1 excitation

\overline{C}_{12} may be calculated by using either driven mode. For the chosen mode both “in plane” and “out of plane” data are recorded.

\overline{C}_{12} can be fitted as the sum of two waves, one propagating at the difference resonance ($\mu_1 - \mu_2$) and the other at the sum resonance ($\mu_1 + \mu_2$). By introducing two such waves at the entrance of the lattice, with variable amplitude and phase, we may fit to the measured value of \overline{C}_{12} over a region of the ring.

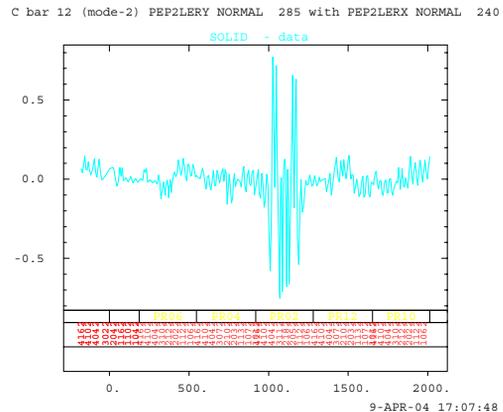


Figure 3: The out-of-phase component of the coupling between horizontal and vertical motion, \overline{C}_{12} for the Low Energy Ring (LER).

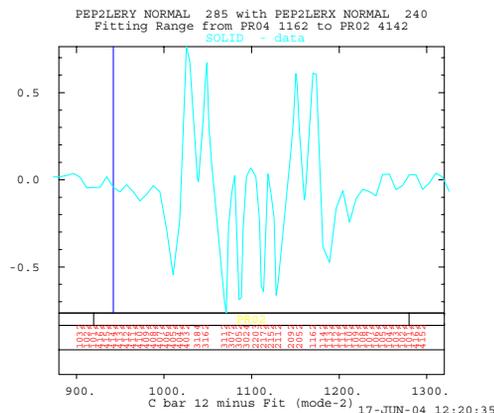


Figure 4: \overline{C}_{12} close to the Interaction Point of the LER. The measured value is very close to the model value shown in figure 5

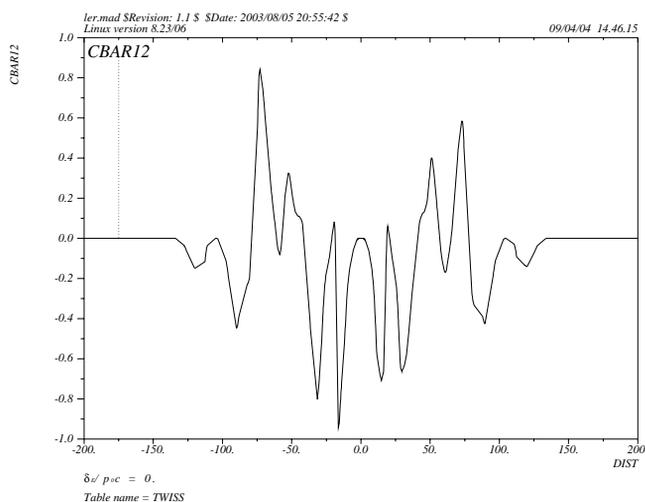


Figure 5: Plot from the MAD [5] program of the function “ $\overline{C}_{12} = -R(1,2) \cdot \sqrt{\text{BETX} \cdot \text{BETY}}$ ”

ESTIMATE THE BETA FUNCTION AT THE BEAM WAIST

Surrounding the Interaction point is some free space (in the field of the solenoid) and two BPMs situated 0.716 m from the IP. Using the phase and amplitude of the betatron motion at these BPMs enables us to fit the beam waist to find β_{minimum} and its offset from the IP. The β function at the waist is mainly sensitive to the phase measurement so is quite believable (at least for the horizontal plane) while the offset is more sensitive to errors in calibration of the BPMs. A model of the machine is not required, just the distance between the BPMs is needed.

We solve these equations

$$\frac{\beta_1}{\beta_2} = \frac{\beta_0^2 + l_1^2}{\beta_0^2 + l_2^2}$$

$$d\phi = \arctan(l_1/\beta_0) + \arctan(l_2/\beta_0)$$

$$l_1 + l_2 = 1.432$$

where the subscript 1 refers to the upstream BPM and subscript 2 refers to the downstream BPM. The ratio β_1/β_2 is found from the square of the amplitudes at those BPMs. $d\phi$ is the measured phase advance between the BPMs.

For the vertical plane the phase shift between the BPMs is getting close to π radians and the method used above becomes very sensitive to small errors in phase. For this case we simply use the calculated values of the beta functions at those BPMs to estimate the waist parameters by solving the expressions

$$\beta_1 = \beta_0 + l_1^2/\beta_0$$

$$\beta_2 = \beta_0 + l_2^2/\beta_0$$

$$l_1 + l_2 = 1.432$$

for β_0 and $\delta l = (l_2 - l_1)/2$

SUMMARY

What we have is working well but it has limitations.

- In highly coupled regions we are using formulae relevant to the uncoupled case.
- The results are almost model independent. We need to incorporate these results with the on-line model in order to estimate and apply corrections.
- As a first step we need to be able to subtract out the model value of \overline{C}_{12} .

REFERENCES

- [1] P. Castro-Garcia, “Luminosity and beta function measurement at the electron-positron collider ring LEP”, Doctoral Thesis, Universitat de València, May 17, 1996.
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- [5] H. Grote and F.C. Iselin, “The MAD Program, User’s Reference Manual”, CERN/SL/90-13 (AP) (1996).