COST OPTIMIZATION OF NON-SCALING FFAG LATTICES FOR MUON ACCELERATION*

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Abstract

Fixed Field Alternating Gradient (FFAG) accelerators are a promising idea for reducing the cost of acceleration for muon accelerators as well as other machines. This paper presents an automated method for designing these machines to certain specifications, and uses that method to find a minimum cost design. The dependence of this minimum cost on various input parameters to the system is given. The impact of the result on an FFAG design for muon acceleration is discussed.

INTRODUCTION

In the design of particle accelerators, one needs a method to choose a machine design from a spectrum of possible designs. Ideally, one finds some cost function which depends on the parameters of the design, and uses numerical optimization techniques to minimize that cost. The difficulty often lies in creating a working design in an automated fashion, which is required if one is to computationally find the cost as a function of machine design parameters.

The design of linear non-scaling fixed field alternating gradient accelerators (FFAGs) [1] is particularly amenable to this technique. The machines consist of identical cells with a simple cell structure. We assume the parameters given in Tab. 1. The normalized transverse acceptance in this table is, at a given point in the ring for a given energy, the product of the maximum horizontal (vertical) position deviation from the energy-dependent closed orbit horizontal (vertical) position and the maximum horizontal (vertical) momentum deviation from the closed orbit horizontal (vertical) momentum, divided by the particle mass times the speed of light. We look at three consecutive energy ranges, each covering a factor of 2 in energy. There is a relationship between the longitudinal acceptance (50 eV s desired) accelerated and the voltage required [2]. The voltage V required depends on the lattice design: it is proportional to the difference ΔT between the minimum and maximum times of flight for the ring (the total height of the parabola shown in Fig. 1). It is characterized by the quantity $V/\omega\Delta T\Delta E$, where ω is the angular RF frequency and ΔE is the difference between the minimum and maximum energies in the machine. ΔT increases quadratically with ΔE, which is the reason for only accelerating by a factor of 2 in these machines. Furthermore, as the central energy decreases, $V/\omega\Delta T\Delta E$ must increase [3]. We look at three different kinds of lattices: a doublet (FD), a triplet (FDF), and a FODO lattice.

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Table 1: Design parameters for FFAG lattices.

Drift for cavity	2 m
Drift between magnets	0.5 m
RF Frequency	201.25 MHz
Voltage per cavity	7.5 MV
Normalized transverse acceptance	30 mm

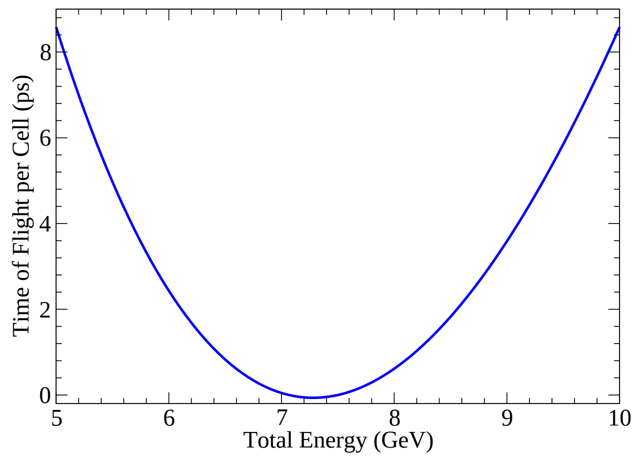

Figure 1: Time of flight deviation per cell as a function of energy deviation for a typical linear non-scaling FFAG.

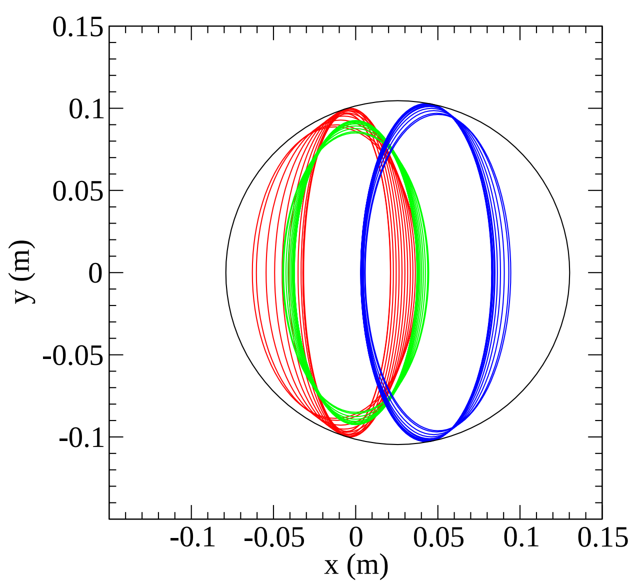

Figure 2: Ellipses in the D magnet for an optimized doublet. Colored groups are at the same energy: red (left) is low, green (middle) is central, and blue (right) is high. Within a group, different ellipses are different positions.

Table 2: Cost-optimum lattices. Decay computations are based on an approximate model.

Minimum total energy (GeV)	2.5			5			10		
Maximum total energy (GeV)	5			10			20		
$V/(\omega\Delta T\Delta E)$	1/6			1/8			1/12		
Type	FD	FDF	FODO	FD	FDF	FODO	FD	FDF	FODO
No. of cells	89	70	91	140	111	145	191	153	197
D length (cm)	58	93	55	71	109	67	87	128	81
D radius (cm)	13.0	16.0	15.8	9.5	11.5	11.3	7.1	8.6	8.3
D pole tip field (T)	3.2	3.1	2.7	3.6	3.6	3.1	4.4	4.4	3.8
F length (cm)	86	46	87	98	55	96	115	67	110
F radius (cm)	16.8	15.1	21.3	11.7	10.6	14.4	8.6	8.09	10.5
F pole tip field (T)	2.1	2.3	1.6	2.6	2.7	2.1	3.2	3.3	2.7
No. of cavities	47	49	63	45	47	57	43	46	55
RF voltage (MV)	350	363	465	332	350	426	321	343	405
$\Delta E/V$	7.1	6.9	5.4	15.1	14.3	11.7	31.1	29.2	24.7
Circumference (m)	351	340	493	587	575	855	863	859	1164
Decay (%)	10.7	10.0	11.3	18.2	17.0	19.6	27.0	25.5	28.6
Magnet cost (PB)	35.7	41.5	49.2	29.6	34.7	37.5	27.7	32.4	32.6
RF cost (PB)	22.7	23.6	30.3	21.7	22.7	27.6	20.8	22.2	26.4
Linear cost (PB)	8.8	8.5	12.3	14.7	14.4	20.4	21.6	21.5	29.1
Total cost (PB)	67.2	73.6	91.8	65.9	71.8	85.5	70.1	76.1	88.1
Cost per GeV (PB/GeV)	26.9	29.4	36.7	13.2	14.4	17.1	7.0	7.6	8.8

We use a cost formula described elsewhere in these proceedings [4]. To determine magnet apertures, we find the closed orbit and beta functions at the minimum and maximum energies at several points in the magnet, and find a circular aperture which encloses all the ellipse determined by the acceptance, these beta functions, and the closed orbit [5] (see Fig. 2). The integrated quadrupole strengths, magnet lengths, and number of cells are then varied to minimize the cost. The RF voltage is computed based on $V/\omega\Delta T\Delta E$. At the end, the design is re-optimized with an integer number of cells and RF cavities.

Making a good initial guess is important for this optimization. The first guess is obtained by experience-based guesses for the number of cells and the magnet lengths. The lattice is then designed assuming tunes at the lowest energy of 0.35 [6].

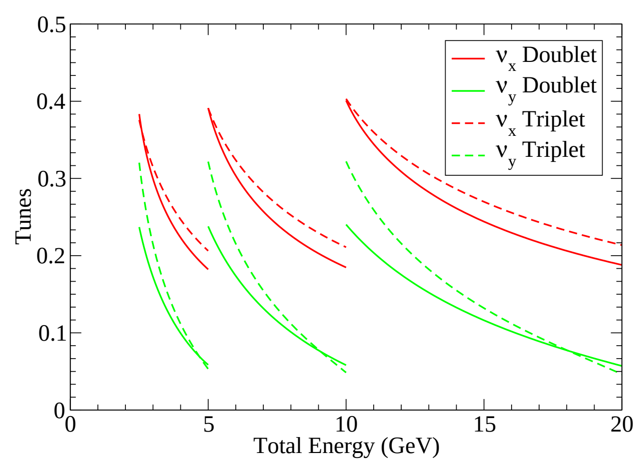

Figure 3: Tunes for filled optimized lattices for different energy ranges. Horizontal tunes are higher.

OPTIMIZATION RESULTS

The results of optimizing the cost are shown in Tab. 2. The resulting lattices have unacceptable circumferences and levels of decay. This is the result of not assigning a cost per muon; this must be done to produce a good optimum. An alternative is to constrain the system to nearly every cell filled with RF cavities, except for 8 cells left open for injection and extraction hardware; the results of this optimization are shown in Tab. 3. These lattices have a much more reasonable amount of decay.

The doublet lattice is always the most cost-effective, while the triplet lattice requires the lowest voltage. For almost all of the lattices, the magnet cost is decreasing with number of cells at the optimum, despite the increase in the number of magnets, since the magnet apertures are decreasing. The voltage decreases for more cells when one optimizes for cost because ΔT is inversely proportional to the number of cells. It is the linear cost that finally gives an optimum cost in Tab. 2, whereas in Tab. 3 more cells leads to more cavities and thus greater cost. The tune as a function of energy is roughly independent of energy for a given lattice type, as shown in Fig. 3. Vertically, this is because at a given tune, the D magnet pipe is determined by having the low energy and high energy ellipses roughly the same height (see Fig. 2), and particular tunes lead to this. Horizontally, a higher tune leads to a smaller ΔT [6], but if it gets too high the beta functions increase at the low en-

Table 3: Lattices optimized with all but 8 filled cells.

Minimum total energy (GeV)	2.5			5			10		
Maximum total energy (GeV)	5			10			20		
$V/(\omega \Delta T \Delta E)$	1/6			1/8			1/12		
Type	FD	FDF	FODO	FD	FDF	FODO	FD	FDF	FODO
No. of cells	65	60	76	79	72	91	93	85	105
D length (cm)	62	96	56	82	119	77	105	143	98
D radius (cm)	13.6	16.5	16.0	10.2	12.7	11.7	7.8	9.7	8.7
D pole tip field (T)	3.7	3.3	1.9	4.6	4.2	3.8	5.8	5.5	5.0
F length (cm)	99	48	93	126	64	119	162	85	151
F radius (cm)	19.1	15.8	22.8	15.3	12.8	17.8	12.7	10.9	14.6
F pole tip field (T)	2.2	2.4	1.7	2.8	3.1	2.2	3.5	3.7	2.8
No. of cavities	57	52	68	71	64	83	85	77	97
RF voltage (MV)	428	390	510	533	480	623	638	578	728
$\Delta E/V$	5.8	6.4	4.9	9.4	10.4	8.0	15.7	17.3	13.7
Circumference (m)	268	295	418	362	393	543	481	521	681
Decay (%)	6.8	8.2	8.8	7.4	8.9	9.4	8.5	10.1	10.4
Magnet cost (PB)	36.4	41.6	49.6	32.8	37.4	40.0	34.1	39.2	38.4
RF cost (PB)	27.7	25.3	33.0	34.5	31.1	40.3	41.3	37.4	47.1
Linear cost (PB)	6.7	7.4	10.4	9.1	9.8	13.6	12.0	13.0	17.0
Total cost (PB)	70.8	74.3	93.1	76.3	78.3	93.8	87.4	89.6	102.5
Cost per GeV (PB/GeV)	28.3	29.7	37.2	15.3	15.7	18.8	8.7	9.0	10.2

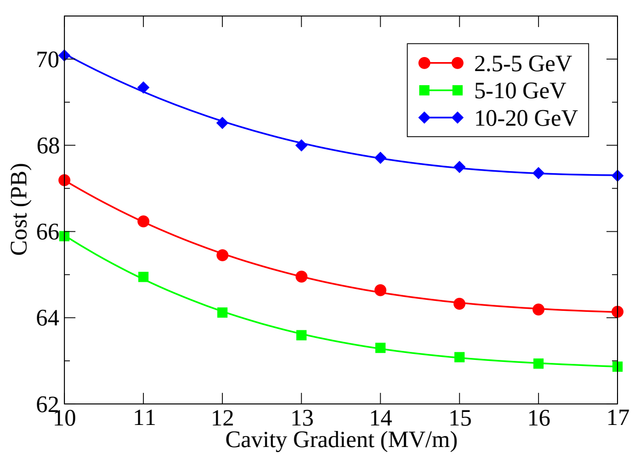

Figure 4: Optimal cost as a function of cavity gradient.

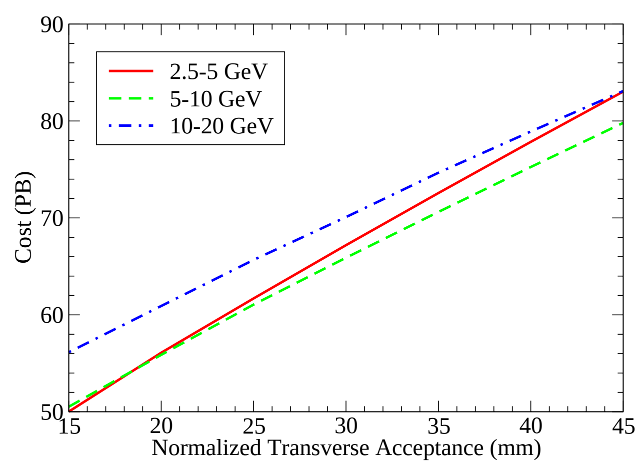

Figure 5: Optimal cost as a function of transverse normalized acceptance.

ergy. For superconducting RF cavities, increasing the cavity gradient from 10 MV/m (here) reduces the cost slightly (Fig. 3). The transverse acceptance has a strong effect on the cost, as shown in Fig. 5).

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