EFFECTS OF POSITRONS ON RELATIVISTIC SOLITONS IN LASER-PLASMA INTERACTIONS

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Abstract

An extended 1D kinetic model is used to investigate the effects of positrons on relativistic solitons in electron-positron-ion plasmas. The soliton existence domains were specified in the plane of normalized frequency(ω)-temperature(λ) with varying relative density of the positrons, α_m. The domain becomes larger when the parameter α_m increases because of the increase of Langmuir frequency. The soliton form does not depend significantly on the value of the ratio α_m even in the limit of high temperature.

INTRODUCTION

Relativistic solitons have been investigated because of its importance for basic nonlinear science[1, 2]. Relativistic solitons are self-trapped, finite size, electromagnetic waves that propagate without diffraction spreading.

With peta-watt lasers currently available, $e^+ - e^-$ pair-production by laser-plasma interaction has been suggested[3, 4]. By the quiver motions of electrons over energy of $2m_ec^2$ due to the intense field of the laser, electron-positron pairs can be created[4]. A three-species plasma can be made by this pair creation. The effects of the created positrons on the nonlinear structures, including relativistic solitons, is not known well at present.

Investigations of relativistic solitons in electron-positron-ion plasmas can give us an useful insight about the roles of positrons and ions on the relativistic solitons. In addition, hot electrons by the intense fields of laser can change the relativistic soliton and the cold plasma models must be modified to model the relativistic soliton with hot particles.

In this report, we briefly report our recent study about physical conditions of relativistic soliton existence in electron-positron-ion plasmas with emphasis on kinetic effects. The conditions in $\alpha_m - \lambda$ plane are discussed in connection with role of ions for the soliton which was briefly noted in Ref. [1].

GOVERNING EQUATIONS OF THE RELATIVISTIC SOLITON IN THE ELECTRON-POSITRON-ION PLASMA

In the interaction of high power lasers and a plasma, the transverse thermal motion of the charged particles is negligible compared with the motion caused by the strong transverse field of the electromagnetic wave. We can model this with an anisotropic distribution function for the j-th species of the charged particles[2]:

$$f_j(W_j, \mathbf{P}_j) = \frac{N_j}{2m_jK_1(\beta_j^{-1})}\delta(\mathbf{P}_{\perp j})\exp\left[-\frac{W_j}{T_j}\right] \quad (1)$$

where the total energy of the particles, W_j, of the j-th species is $W_j = m_j\gamma_j + q_j\phi(\mathbf{r}, t)$. We normalize the equations and all of the following physical quantities by c with $c = 1$. γ_j is the relativistic factor, $\gamma_j = (1 + (p_j/m_j)^2)^{1/2}$. The particle generalized momentum is $\mathbf{P}_j(\mathbf{r}, t) = \mathbf{p}_j + q_j\mathbf{A}(\mathbf{r}, t)$. Here $\phi(\mathbf{r}, t)$ and $\mathbf{A}(\mathbf{r}, t)$ are the scalar and the vector potential of the electromagnetic field. $\beta_j = T_j/m_j$ is the ratio of the thermal energy to the rest energy of the j-th species particles. $m_j, q_j, N_{0j}, \mathbf{p}_j$, and T_j are the mass, electric charge, unperturbed density, momentum, (constant) temperature of the j-th species. $K_i(\xi)$ is the modified Bessel function of the second kind of the i-th order with argument ξ. (See Ref. [2] for details.)

The quasineutrality condition gives us the relations between particle densities in the form $N_{0i}Z_i = (1 - \alpha_m)N_{0e}, N_{0p} = \alpha_mN_{0e}, Z_iN_{0i} + N_{0p} = N_{0e}$ where Z_ie is the ion charge, α_m is the ratio of the unperturbed positron density to the electron density, $\alpha_m = N_{0p}/N_{0e}$. The subscript i is for ion, p for positron, e for electron.

With the distribution function (1), we obtain the particle density and the current density. From Maxwell equations with the electric charge density and the current density, we can obtain equations for the vector and scalar potentials. They can be written in the form:

$$\begin{aligned} a_{xx} + \omega^2 a = a\Bigg[&\frac{K_0(\gamma_{\perp e}\lambda_e^{-1})}{K_1(\lambda_e^{-1})}\exp\left[\frac{\varphi}{\lambda_e}\right] \\ &+\alpha_m\frac{K_0(\gamma_{\perp p}\,\lambda_p^{-1})}{K_1(\lambda_p^{-1})}\exp\left[-\frac{\varphi}{\lambda_p}\right] \\ &+(1-\alpha_m)\rho Z\frac{K_0(\gamma_{\perp i}(\rho\lambda_i)^{-1})}{K_1((\rho\lambda_i)^{-1})}\exp\left[-\frac{Z\varphi}{\lambda_i}\right]\Bigg] \quad (2) \end{aligned}$$

$$\begin{aligned} \varphi_{xx} = \Bigg[&\gamma_{\perp e}\frac{K_1(\gamma_{\perp e}\lambda_e^{-1})}{K_1(\lambda_e^{-1})}\exp\left[\frac{\varphi}{\lambda_e}\right] \\ &-\alpha_m\gamma_{\perp p}\frac{K_1(\gamma_{\perp p}\lambda_p^{-1})}{K_1(\lambda_p^{-1})}\exp\left[-\frac{\varphi}{\lambda_p}\right] \\ &-(1-\alpha_m)\gamma_{\perp i}\frac{K_1(\gamma_{\perp i}(\rho\lambda_i)^{-1})}{K_1((\rho\lambda_i)^{-1})}\exp\left[-\frac{Z\varphi}{\lambda_i}\right]\Bigg] \quad (3) \end{aligned}$$

Here $\gamma_{\perp i} = \sqrt{1+\rho^2 Z^2 a^2}$, $\gamma_{\perp p} = \gamma_{\perp e} = \sqrt{1+a^2}$, and φ and $\mathbf{a}_\perp$ are new dimensionless variables defined as $\varphi = e\phi/m_e$ and $\mathbf{a}_\perp = e\mathbf{A}_\perp/m_e$. Time and space coordinates t and x have been normalized by the Langmuir frequency ω_{pe}, thus $\omega_{pe}t \rightarrow t$ and $\omega_{pe}x \rightarrow x$. β_j are rewritten as $\lambda_e = T_e/m_e = \beta_e, \lambda_p = T_p/m_e$, and $\lambda_i = T_i/m_e$ with a new normalization by m_e. $\rho = m_e/m_i = 1/1836$ is the ratio of the mass of the electron to the ion and Z is the ion charge number. With the assumptions of circularly polarized electromagnetic fields and nondrift localized solutions, the vector potential can be written as $a(x,t)$ as $a(x,t) = a(x)e^{i\omega t}$, where ω is frequency of the electromagnetic wave.

The solution must satisfy the boundary condition $a, a_x, \varphi, \varphi_x \rightarrow 0$ as $\mid x \mid \rightarrow \infty$ for localized solutions. The solution behaves as $\exp(-\sigma \mid x \mid)$ with large x, and σ must be positive for a soliton solution. With small amplitude equation, $a_{xx} - \Delta\omega^2 a(x) \approx 0$, $\Delta\omega^2 > 0$ for a soliton solution. The condition for the soliton solution is given as:

$$(1-\alpha_m)\rho Z \frac{K_0((\rho\lambda_i)^{-1})}{K_1((\rho\lambda_i)^{-1})} + \alpha_m \frac{K_0(\lambda_p^{-1})}{K_1(\lambda_p^{-1})} + \frac{K_0(\lambda_e^{-1})}{K_1(\lambda_e^{-1})} - \omega^2 > 0 \qquad (4)$$

Solitons exist in the domain satisfying the condition in $\omega-\lambda$ plane. The boundary of the domain is given from $\Delta\omega^2 = 0$.

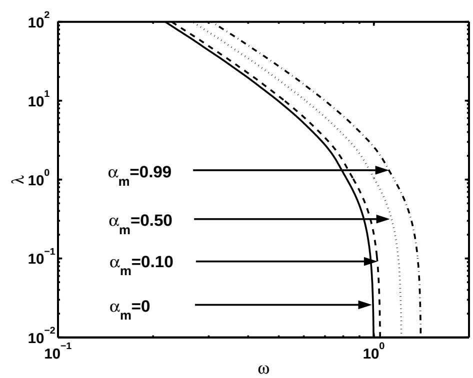
We consider isothermal plasmas, for which $\lambda_e = \lambda_p = \lambda_i = \lambda$. In the case considered here, $Z = 1$ and $\rho = 1/1836$, which correspond to H^+(proton) ion component. The electric charge quasineutrality is assumed, which can be expressed as $\varphi_{xx} \cong 0$. As a result we obtain:

$$a_{xx} + \omega^2 a = a\left[(1-\alpha_m)\rho\frac{K_0(\gamma_{\perp i}(\rho\lambda)^{-1})}{K_1((\rho\lambda)^{-1})}F(a;\rho,\alpha_m)^{-\frac{1}{2}} + \alpha_m\frac{K_0(\gamma_{\perp p}\lambda^{-1})}{K_1(\lambda^{-1})}F(a;\rho,\alpha_m)^{-\frac{1}{2}} + \frac{K_0(\gamma_{\perp e}\lambda^{-1})}{K_1(\lambda^{-1})}F(a;\rho,\alpha_m)^{\frac{1}{2}}\right] \qquad (5)$$

where a function $F(a;\rho,\alpha_m)$ is given by:

$$F(a;\rho,\alpha_m) = \alpha_m + (1-\alpha_m)\frac{\gamma_{\perp i}}{\gamma_{\perp e}}\frac{K_1(\lambda)}{K_1((\rho\lambda)^{-1})}\frac{K_1(\gamma_{\perp i}(\rho\lambda)^{-1})}{K_1(\gamma_{\perp e}\lambda^{-1})} \qquad (6)$$

The simplified equation (5) was solved to obtain the soliton solutions of the vector potential a, the scalar potential φ, and the longitudinal electric field E. in the next section. We investigated in the parameter ranges of $0.03 \leq \omega \leq 1.50$, $10^{-3} \leq \lambda \leq 10^3$, $\alpha_m = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$, 0.1 to 0.9, 0.99, 0.999, 0.9999, 0.99999.



(a)

Figure 1: Changes of the existence domain are shown with ω_{pe}. $10^{-2} \leq \lambda \leq 10^2$ and $10^{-1} \leq \omega \leq 2$.

EFFECTS OF POSITRONS ON RELATIVISTIC SOLITONS IN ELECTRON-POSITRON-ION PLASMAS

The region of localized solutions from Eq. (4) is specified in Fig. 1 and the variation of the region with increasing α_m is shown in Fig. 1. The soliton exists in the region under the boundary curve. ω is the frequency of incident electromagnetic wave normalized by the Langmuir frequency $\omega_{pe} = (4\pi N_{0e}e^2/m_e)^{1/2}$.

The existence domain is divided to two regions by $\lambda = 1$. With $\lambda < 1$, the maximum frequency becomes gradually saturated and converges to $\omega_{pe}^* = (4(1+\alpha_m)\pi N_{0e}e^2/m_e)^{1/2}$. With $\lambda \geq 1$, the maximum frequency decreases exponentially by increase of λ. As α_m increases to 1.00, the boundary of the domain shifts to higher ω and λ. This is explained by the increase of the Langmuir frequency because of increased proportion of light particles in the plasma and larger mobility of positrons compared to ions.

The left side of Fig. 2 shows changes of the soliton with increasing α_m and $\lambda = 0.01$, which represents the soliton in temperatures under $m_e c^2$, and the right side with $\lambda = 30$, which represents the soliton in high temperatures above $m_e c^2$. The soliton does not show any change with increasing α_m up to very high proportion of positrons in low temperature as seen in the left side. Even with $\alpha_m = 0.99$, where the proportion of the ions is only 1 percent of the electrons, the soliton shows little change. Compared with the left side, the soliton is not changed much as α_m increases, but with $\alpha_m = 0.99$, the scalar potential almost vanishes.

A function $M(\psi;\lambda,\alpha_m)$ was introduced to investigate the dependency of the soliton form on λ and α_m quantitatively. The function $M(\psi;\lambda,\alpha_m)$ is defined as:

$$M(\psi;\lambda,\alpha_m) = \frac{\max|\psi(x;\lambda,\alpha_m)|}{\max|\psi(x;\lambda,0)|} \qquad (7)$$

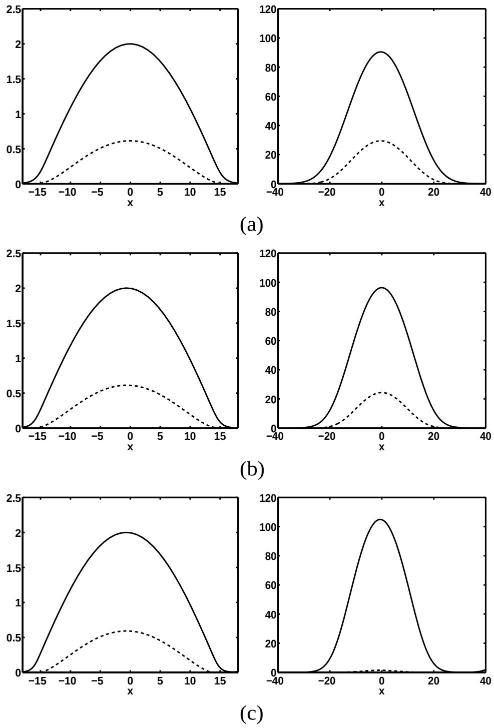


Figure 2: Comparison of soliton solutions with varying $\alpha_m = 0.1, 0.5, 0.99$ in a low(left side, $\lambda = 0.01$) and a high temperature(right side, $\lambda = 30$). The vector potential a is drawn with solid line and the scalar potential φ with dashed line. (a) $\alpha_m = 0.1$ (b) $\alpha_m = 0.5$ (c) $\alpha_m = 0.99$

where $\psi(x; \lambda, \alpha_m)$ is each field distribution a, φ, E with given λ and α_m and $\max |\psi(x; \lambda, \alpha_m)|$ is the maximum of absolute value of the field amplitude.

The region is divided to two regions by $\lambda = 1$. We can see that $M(\psi; \lambda, \alpha_m)$ is 1 with $\lambda \leq 1$ in Fig. 3. The condition $M(\psi; \lambda, \alpha_m) \geq 1$ specifies the region of soliton existence without influences of positrons in $\lambda - \alpha_m$ plane.

This role of ions is shown explicitly in Fig. 3 extending the discussion in Ref. [1] to the plasmas with finite temperatures. The ion component presence is important to support soliton structure in the plasma. The ability of ions to support soliton becomes weaker in high temperature over $m_e c^2$. Fig. 3 shows that the role of the ions as a support of soliton is affected by increase of particles' thermal energies. The changes of $M(\varphi; \lambda, \alpha_m)$ and $M(E; \lambda, \alpha_m)$ are explained by the increase of ion's mobility with temperature increase. Relativistic nonlinear effects also help reducing differences among ion's mobility, electron's and positron's mobilities.

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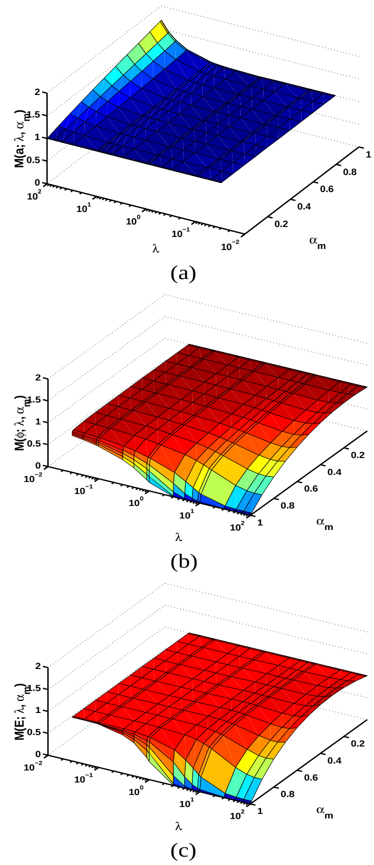

Figure 3: Figure 4. Plot of $M(\psi; \lambda, \alpha_m)$ with $\psi = a, \varphi,$ and E with respect to α_m. The direction of λ axis is inverted to show the changes explicitly in (b) and (c). (a) $M(a; \lambda, \alpha_m)$ (b) $M(\varphi; \lambda, \alpha_m)$ (c) $M(E; \lambda, \alpha_m)$

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