SPACE CHARGE EFFECTS FOR THE ERL PROTOTYPE AT DARESBURY LABORATORY

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Abstract

Daresbury Laboratory is currently building an Energy Recovery Linac Prototype (ERLP) that will operate at a beam energy of 35 MeV. In this paper we examine the space charge effects on the beam dynamics in the ERLP transfer line. This is done in two ways. The first is based on an analytic formula derived via the envelope equations and a Kapchinsky-Vladimirsky (KV) distribution. This formula gives an appropriate estimate of the space charge effects in the case that no quadrupoles or dipoles are present in the transfer line. The second estimate is given by the multi-particle tracking code ASTRA for the whole transfer line both with and without quadrupoles. The methods are compared and are found to be in good agreement. Typical examples of beam transfer lines are given together with specific calculations for the ERLP.

INTRODUCTION

Daresbury Laboratory has been given funding to build an Energy Recovery Linac Prototype (ERLP) [1] which will operate at 35 MeV and drive an FEL. In order to drive an FEL, it is necessary to have a high brightness electron beam with a small transverse emittance. One of the biggest contributing factors to emittance degradation is space charge, particularly at low energies. Because of the low beam energy (< 10 MeV) in the ERLP transfer line, the estimation of space charge is important as it may hinder the successful operation of the accelerator. This report gives various estimates of space charge effects in a drift space and transfer line, and a comparison between them. The first involves an analytic estimate of space charge through a drift [2] and is described in some detail as it does not appear frequently in the literature. The second was obtained through modelling the transfer line using the particle tracking code ASTRA [3]. Comparisons between the analytic and ASTRA calculations show good agreement.

ANALYTICAL FORMULA

For a round beam with radius a, the equation of transverse motion of a particle in the beam has the form

$$\gamma m \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = \frac{2eI}{\gamma^2 \nu a^2} r \qquad (1)$$

when the paraxial approximation ($|\mathrm{d}r/\mathrm{d}t| \ll \nu$) is used and the beam is laminar. Equation (1) is also true for particles at the boundary of the beam ($r = a$). Therefore, letting $z = t\nu$, equation (1) becomes

$$\frac{\mathrm{d}^2 a}{\mathrm{d}z^2} = \frac{2I}{(\beta\gamma)^3 I_0} \frac{1}{a}$$

where $I_0 \simeq 17$ kA (for electrons) is the Alfvén current.

For a uniform elliptical beam, a Kapchinsky-Vladimirsky (KV) distribution [4] may be chosen and the transverse equations of motion in the presence of space charge and external focusing are then

$$x'' = -K_x(z)x + \frac{4I}{(\beta\gamma)^3 I_0} \frac{x}{a(a+b)} \qquad (2)$$

$$y'' = -K_y(z)y + \frac{4I}{(\beta\gamma)^3 I_0} \frac{y}{b(a+b)} \qquad (3)$$

where $a^2 = \epsilon_x \beta_x$, $b^2 = \epsilon_y \beta_y$ (ϵ_x and ϵ_y are the effective emittances in both planes) and $K_{x,y}$ are the focusing rigidities. Note that, for a KV distribution, the rms emittance is related to the effective emittance via $\epsilon_x^{\mathrm{rms}} = \epsilon_x/4$.

The envelope equations for the KV distribution can be seen to be

$$a'' + K_x(z)a - \frac{4I}{(\beta\gamma)^3 I_0} \frac{1}{(a+b)} - \frac{\epsilon_x^2}{a^3} = 0 \qquad (4)$$

$$b'' + K_y(z)b - \frac{4I}{(\beta\gamma)^3 I_0} \frac{1}{(a+b)} - \frac{\epsilon_y^2}{b^3} = 0 \qquad (5)$$

and are known as the Kapchinsky-Vladimirsky equations [4]. From equations (4) and (5) it can be seen that, for a round beam ($b = a, \epsilon_x = \epsilon_y = \epsilon$), the laminar approximation is only valid as long as

$$\frac{\epsilon}{\beta_x} \ll \frac{2I}{(\beta\gamma)^3 I_0}$$

in other words, space charge dominates the transverse dynamics.

Consider a beam passing through a drift of length L, locally space charge is equivalent to a quadrupole field (defocusing in both x and y planes) of varying strength. The horizontal focusing strength is given by

$$\frac{1}{F_x} = -\frac{4I(s)}{(\beta\gamma)^3 I_0} \frac{L}{a(a+b)} \qquad (6)$$

where F_x is the focal length and s is the coordinate along the bunch and with a similar expression for the vertical plane. The sigma matrix transformation $J = TJ_0T^{-1}$ with T the usual thin lens quadrupole, gives

$$\begin{aligned} J &= \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_x} & 1 \end{pmatrix} \begin{pmatrix} \alpha_{x_0} & \beta_{x_0} \\ -\gamma_{x_0} & -\alpha_{x_0} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{F_x} & 1 \end{pmatrix} \\ &= \begin{pmatrix} \alpha_{x_0} + \frac{\beta_{x_0}}{F_x} & \beta_{x_0} \\ -\gamma_{x_0} - \frac{2\alpha_{x_0}}{F_x} - \frac{\beta_{x_0}}{F_x^2} & -\alpha_{x_0} - \frac{\beta_{x_0}}{F_x} \end{pmatrix} \end{aligned}$$

Averaging over the bunch length gives the rms projected emittance as

$$\begin{aligned} \epsilon_x^2 &= \epsilon_{x0}^2 (< \beta_x >< \gamma_x > - < \alpha_x >^2) \\ &= \epsilon_{x0}^2 \left[1 + \beta_{x0}^2 \left\langle \frac{1}{F_x^2} \right\rangle - \beta_{x0}^2 \left\langle \frac{1}{F_x} \right\rangle^2 \right] \end{aligned} \quad (7)$$

where ϵ_{x_0} is the initial emittance, and the final emittance is

$$\epsilon_x = \epsilon_{x0} \sqrt{1 + \beta_{x0}^2 \left[\left\langle \frac{1}{F_x^2} \right\rangle - \left\langle \frac{1}{F_x} \right\rangle^2 \right]}. \quad (8)$$

For a Gaussian bunch the current is given by $I(s) = I_{\max} \exp\left(-\frac{s^2}{2\sigma^2}\right)$, where s is the coordinate along the bunch, and

$$\left\langle \frac{1}{F_x^2} \right\rangle - \left\langle \frac{1}{F_x} \right\rangle^2 = \left(\frac{1}{F_x} \right)_{\max}^2 \left(\frac{1}{\sqrt{3}} - \frac{1}{2} \right) \quad (9)$$

hence substituting (6) into (8), it can be seen that, for a round beam and a bunch with longitudinal Gaussian profile, the final emittance depends only on the initial emittance and not on the transverse beam size.

DRIFT SPACE

The particle tracking code ASTRA was used to track 10000 particles through a drift at the ERLP injection energy of 8.85 MeV. The evolution of the (rms) spot size and normalized emittance are plotted in Figure 1 for various initial (rms) beam sizes ($1 - 16$ mm), a bunch length of 4 ps, a bunch charge of 80 pC and an initial normalized emittance of 3 μm. Shown, for comparison, is the estimate made using the analytical formula (8). Although the ASTRA results depend on the initial spot size, the analytical formula appears to consistently give an upper bound.

Figures 2 and 3 show the emittance growth in a drift for various initial bunch lengths (2 ps, 3 ps, 4 ps and 5 ps) and an initial normalized emittance of 3 μm, and, for various initial normalized emittances (1 μm, 3 μm and 5 μm) and a bunch length of 4 ps. It can be seen that the rate of emittance growth increases drastically for bunch lengths below 3 ps and that the emittance increase depends heavily on the initial emittance.

ERLP TRANSFER LINE

The low energy part (350 keV) of the ERLP from the gun to the booster was modelled with ASTRA [5]. Two different models of the transfer line (booster to main linac), with different lengths, were created and matched with MAD8. To have a better idea of the space charge effect, both transfer lines were tracked with ASTRA. In both the short and long models, dipoles were replaced by focusing elements with very similar β functions as ASTRA does not yet model dipoles.

Tracking of the two transfer lines was performed using an initial normalized emittance of 3 μm and a

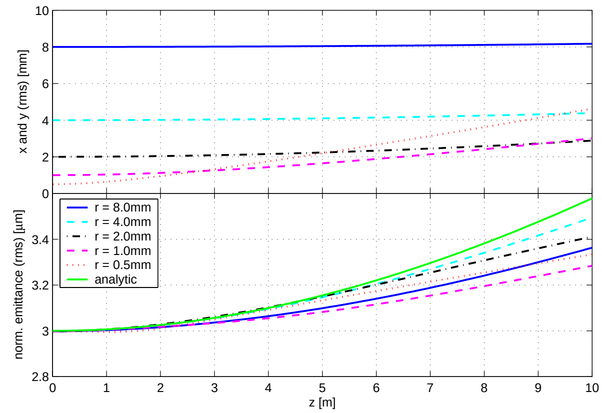

Figure 1: rms beam size and normalized emittance increase for $\epsilon_0 = 3$ μm, bunch length of 4 ps and charge of 80 pC.

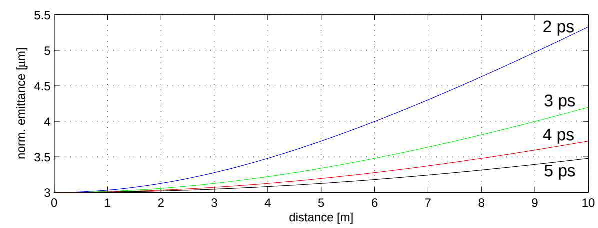

Figure 2: Emittance growth for 2 ps, 3 ps, 4 ps and 5 ps bunch lengths, 3 μm initial normalized emittance and 80 pC bunch charge.

uniform/Gaussian distribution transversally/longitudinally. The results are shown in Figures 4, 5 and 6. Figure 4 shows the original β functions that were obtained with MAD8 together with the tracked equivalents, both with and without space charge. Figures 5 and 6 show the transverse emittance growth and beam size for both models. As expected the emittance growth can be attributed fully to space charge forces. It is possible to compare the ASTRA results with those obtained using equation (8) for a drift. The analytic treatment again gives a very good measure for the emittance increase for a Gaussian bunch. For the first (short) model (Figure 6), the emittance growth is more prominent

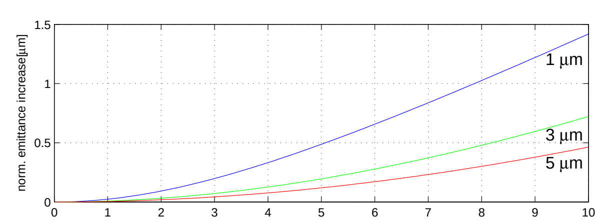

Figure 3: Emittance growth for 1 μm, 3 μm and 5 μm initial normalized emittances, 4 ps bunch length and 80 pC bunch charge.

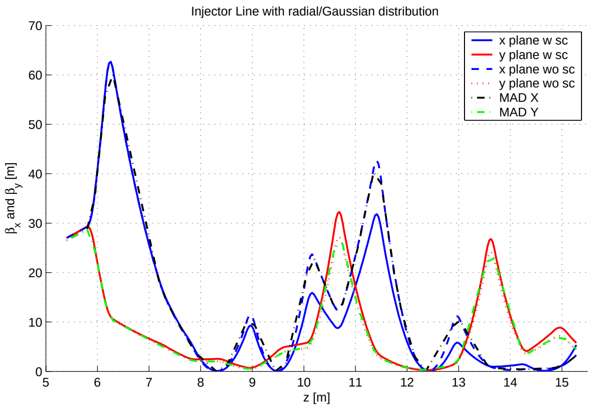

Figure 4: β function behaviour in the first (short) transfer line with and without space charge and original MAD matching.

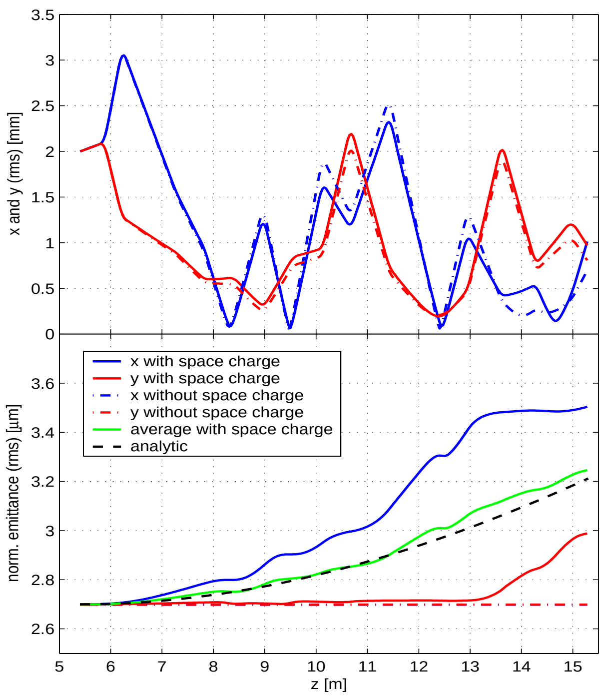

Figure 5: Transverse beam sizes and normalized emittance with and without space charge together with the analytic estimate for the first (short) transfer line.

in the x plane, nevertheless, the analytic formula agrees well with the average emittance growth ($\epsilon = (\epsilon_x + \epsilon_y)/2$) predicted by ASTRA.

CONCLUSIONS

Good agreement between the analytic formula and tracking with the code ASTRA has been found for the prediction of emittance growth for various examples of transfer lines. It appears that this formula gives a very good 'first

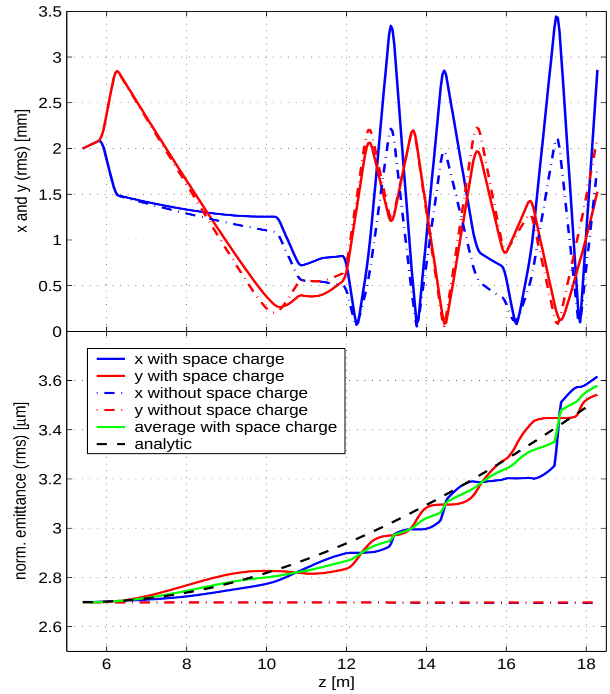

Figure 6: Transverse beam sizes and normalized emittance with and without space charge together with the analytic estimate for the second (long) transfer line.

guess' of emittance growth due to space charge in the case of a transfer line consisting of drifts and quadrupoles.

The results also show almost twice the percentage (18.5 % and 33.5 %) growth in the average emittance for the two models of the ERLP transfer line. As a consequence, the shorter version of the transfer line was chosen for the ERLP. However, a more complete study with dipoles properly modelled remains to be done.

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