

# MEASUREMENT OF BEAM-BEAM TUNE-SHIFT PARAMETER AT KEKB

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## Abstract

We have measured the coherent beam-beam tune shift at KEKB, an asymmetric double-ring electron/positron collider. The relationship between the tune shift and the beam-beam parameter is nonlinear, because the tunes are very close to a half integer. The estimated horizontal emittance is much larger than the calculated value, which is evidence of a dynamic effect due to the beam-beam interaction. The luminosity extrapolated from that of a single bunch, based on its vertical tune shift, is inconsistent with the luminosity measured for all bunches.

## 1 INTRODUCTION

When electron and positron beams collide in a storage ring, the space charge force from one beam gives the other beam a transverse kick and changes the betatron tune. The tune shift parameters give us absolute values of the luminosity and the horizontal emittance. In single-ring colliders, where the two beams have a common tune, the perturbed tune and the non-perturbed tune are simply related. In a double-ring collider, however, the beam-beam tune shift is more complicated because of the difference in the tunes of the two rings [1].

KEKB[2] is an asymmetric electron-positron collider with two rings: a 3.5 GeV Low Energy Ring (LER) for positrons and an 8.0 GeV High Energy Ring (HER) for electrons. More than 1000 bunches are stored in each ring to collide them at one interaction point (IP) with a horizontal crossing angle of 22 mrad. The main machine parameters are listed in Table 1. The betatron tunes are set just above a half integer and are separated by 0.01 or less in the horizontal plane and by 0.02 in the vertical one between the two rings.

## 2 BEAM-BEAM TUNE-SHIFT PARAMETER

The beam-beam interaction produces a new set of two betatron tunes by mixing two unperturbed tunes. Assuming the rigid Gaussian model, the resultant tunes are represented by [3]

$$\cos \mu = \frac{\cos \mu_0^+ + \cos \mu_0^-}{2} - \pi \Xi_q^+ \sin \mu_0^+ - \pi \Xi_q^- \sin \mu_0^- \pm \frac{1}{2} \sqrt{D}, \quad (1)$$

where  $D = 16\pi^2 \Xi_q^+ \Xi_q^- \sin \mu_0^+ \sin \mu_0^- + [\cos \mu_0^+ - \cos \mu_0^- - 2\pi \Xi_q^+ \sin \mu_0^+ + 2\pi \Xi_q^- \sin \mu_0^-]^2$ . Here,  $\mu_0^\pm$  is the betatron phase advance per turn without collision, where the subscript  $q$  stands for either  $x$  or  $y$  direction and the superscript  $\pm$

denotes positron and electron bunches. The unperturbed tune is given by  $\nu_0^\pm = \mu_0^\pm / 2\pi$ .

Table 1: Machine parameters used in the experiment.

Parameters	LER	HER	
Horizontal Emittance	18	24	nm
Betatron Tune, $\nu_x / \nu_y$	45.51 / 43.57(.56)	44.52(.51) / 41.59(.58)	
Beta's at IP, $\beta_x^* / \beta_y^*$	59/0.65	63/0.7	cm
Rms Bunch Length	6.5 ~ 8.0	6.5 ~ 7.0	mm
Synchrotron Tune	0.022	0.020	
Particles per Bunch	3.7 ~ 6.2	3.1 ~ 4.2	$\times 10^{10}$
Bunch Spacing	2.4		m
RF Frequency	508.886		MHz
Harmonic Number	5120		

The coherent beam-beam parameter  $\Xi_q^\pm$  is given by

$$\Xi_q^\pm = \frac{N_b^+ r_e}{\gamma^\pm} \frac{\beta_q^\pm}{2\pi \Sigma_q (\Sigma_x + \Sigma_y)}. \quad (2)$$

Here,  $N_b$  is the number of particles in a bunch,  $r_e$  the classical electron radius,  $\gamma$  the relativistic factor,  $\Sigma_q = \sqrt{(\sigma_q^+)^2 + (\sigma_q^-)^2}$  is the effective beam size and  $\beta_q$  is the betatron function at the IP. The coherent beam-beam parameter is just half of the incoherent beam-beam parameter,  $\xi_q$ ; when both of the sizes are equal, it is  $\xi_q \approx \Xi_q^+ + \Xi_q^-$ . We call the two new modes the higher tune ( $H$ -) mode and the lower tune ( $L$ -) mode; both tunes are affected by the beam-beam interaction, unlike the  $\Sigma$ - and the  $\pi$ - modes in a single-ring collider. The tunes are calculated as a function of the coherent beam-beam parameter, as shown in Fig.1, when  $\Xi_q^+ = \Xi_q^-$  is assumed. Both of the tunes are shifted to higher values as the beam-beam parameter increases. By eliminating  $D$  in Eq. (1), we get

$$\cos \mu_H + \cos \mu_L - (\cos \mu_0^+ + \cos \mu_0^-) = 2\pi \{ \Xi_q^+ \sin \mu_0^+ + \Xi_q^- \sin \mu_0^- \}, \quad (3)$$

where  $\mu_H$  and  $\mu_L$  are the perturbed betatron phases.

The total beam-beam tune shift, defined by  $\Delta \nu_{b-b} = \nu_H + \nu_L - \nu_0^+ - \nu_0^-$ , can be obtained as a function of the sum of the coherent beam-beam parameters,  $\Xi_q^+ + \Xi_q^-$ , as shown in Fig. 2. The relationship depends on the unperturbed tunes. Note that the Yokoya factor [4] is not included in Eq.(3). Under the condition that the value of  $\Xi_q^+ + \Xi_q^-$  is kept constant, even if the ratio  $\Xi_q^+ / \Xi_q^-$  changes from 0.2 to 5.0, the total beam-beam tune shift

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varies less than  $\pm 7\%$ . This means that the sum of the coherent beam-beam parameters,  $\Xi_q^+ + \Xi_q^-$ , is approximately determined by the total beam-beam tune shift. Each beam-beam parameter can be estimated from the shift of an individual tune.

Important parameters are derived from the beam-beam parameter. The luminosity is expressed by [5]

$$L = \frac{f_0 B_N}{r_e} \cdot \frac{N^+ N^- \gamma_+ \gamma_-}{N^+ \gamma_+ + N^- \gamma_-} \cdot \left[ \frac{\Xi_x^+ + \Xi_x^-}{\beta_x^+ + \beta_x^-} + \frac{\Xi_y^+ + \Xi_y^-}{\beta_y^+ + \beta_y^-} \right] \times R. \quad (4)$$

Here,  $f_0$  is the revolution frequency,  $B_N$  is the number of bunches and  $R$  is the reduction factor caused by an offset in collision, the finite angle crossing and the ‘‘hourglass’’ effect. Assuming that  $\beta_{x,y}^+ = \beta_{x,y}^- = \bar{\beta}_{x,y}^*$ ,  $\beta_x \gg \beta_y$  and no offset error, the luminosity is proportional to the vertical beam-beam parameter and is represented by

$$L = \frac{f_0 B_N \gamma_-}{r_e} \cdot \frac{N^-}{1+T} \cdot \left[ \frac{\Xi_y^+ + \Xi_y^-}{\bar{\beta}_y^*} \right] \cdot R. \quad (5)$$

Here, the parameter  $T$  defined by the energy transparency factor  $T = N^- \gamma^- / N^+ \gamma^+$  is larger than 1.0 in KEKB. The horizontal emittance is estimated from the horizontal tune shift. From Eq. (2) with a flat beam condition, we have

$$\Xi_x^+ + \Xi_x^- = \frac{r_e}{(\varepsilon_x^+ + \varepsilon_x^-)} \cdot \frac{N^+}{2\pi\gamma^-} (1+T). \quad (6)$$

Here,  $\beta_x^+ = \beta_x^-$  is assumed.

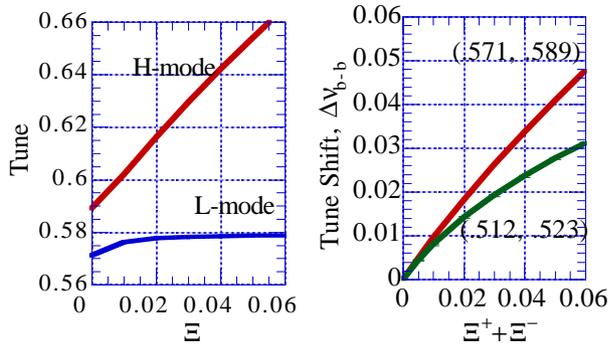


Figure 1(left): Calculated tunes as a function of the coherent beam-beam parameter, assuming  $\Xi_q^+ = \Xi_q^-$ , where the unperturbed fractional tunes are  $\nu_x = .571$  and  $\nu_y = .589$ .

Figure 2(right): Relation between the total beam-beam tune shift and the sum of the coherent beam-beam parameters.

### 3 MEASUREMENT

The tune is measured by a swept frequency method and can be read from the resonant frequency in the spectrum. The tune measurement system is coupled with a transverse feedback loop, where the loop is always closed. The transverse feedback enhances the damping of the betatron oscillation to cope with instabilities, which results in lowering the  $Q$ -value in the tune spectrum. The

feedback is not compatible with a precise tune measurement. Thus, a gate inserted on the feedback line cuts off the feedback power to a single bunch for tune measurement using a fast switch. Another gate for the tune measurement selects only the signal of the selected bunch to be measured. The bunch selection is carried out in units of the rf-bucket.

Both rings are operated with a single bunch-train followed by an empty gap. Bunches are spaced by 8 ns or 4 rf-buckets while keeping their intensity within  $\pm 5\%$ . Additional bunches called *pilot bunches* are placed after the last bunch of a train, at different locations in each ring so that they don't collide.

The tune measurement was performed during regular operation. First, the tunes of the pilot bunches were measured. The bunch appears to be unstable in the vertical spectrum of the LER. The vertical instability is related to the electron cloud that causes a modulation of the tune along a bunch train [6]. It was experimentally verified that the bunch-by-bunch tune shift is negligible, when bunches with equal intensity are equally spaced at the tail part of a train. The tune shift along the train in the HER was less than 0.001, so the tune shift due to trapped ions in the HER is found to be negligible.

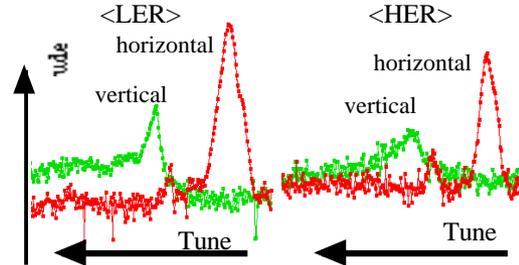


Figure 3: Typical tune spectra of colliding bunches measured at a bunch current of 0.85mA in the LER (left) and 0.63mA in the HER (right). The vertical scale is amplitude in dB and the horizontal scale is frequency or tune with a full scale of 0.09 in the LER and 0.075 in the HER. Note that the tune increases from right to left and that fractional tunes of the pilot bunches are  $\nu_{x0}^+ = 0.5124$ ,  $\nu_{y0}^+ = 0.5712$ ,  $\nu_{x0}^- = 0.5221$  and  $\nu_{y0}^- = 0.5903$ .

Next, the measurement was shifted to the nearest colliding bunches. Typical spectra of colliding bunches are shown in Fig. 3. Large amplitude horizontal oscillations are observed in both rings with equal tune. The observed tune is just in the middle of the tunes of the two pilot bunches. Since the amplitude in the LER is larger than that in the HER, the horizontal oscillation may originate in the LER. The beam-beam interaction can transfer the instability from the LER to the HER, since both unperturbed tunes approach by 0.01. Two peaks at the horizontal spectrum in both rings are separated by 0.024, the lower tune with large amplitude corresponds to the *L*-mode and the second peak with small amplitude corresponds to the *H*-mode. In addition, the vertical spectrum is broader than the horizontal one, especially in

the HER. Though only one peak is seen in each vertical spectrum with a different tune, a small discontinuity corresponding to a tune of the other mode is noticed in each spectrum. When the tune of the HER was changed, for example, we confirmed that the tunes of both rings changed.

The total beam-beam tune shift as a function of the LER beam current is shown in Fig. 4, when the pilot bunch tunes are  $\nu_{x0}^+ = 0.513$ ,  $\nu_{y0}^+ = 0.565$ ,  $\nu_{x0}^- = 0.512$  and  $\nu_{y0}^- = 0.583$ . The tune shifts in both directions tend to increase with the beam current. The sum of the coherent beam-beam parameter is obtained from the total tune-shift data using the curves shown in Fig. 2. The horizontal coherent beam-beam parameter  $\Xi_x^+ + \Xi_x^-$  is estimated to be 0.05 to 0.065 without the Yokoya factor. Since the horizontal tunes of the pilot bunches are nearly equal in this case, we have  $\Xi_x^+ + \Xi_x^- = 0.04 \sim 0.05$ , considering the Yokoya factor [4] to be 1.31. On the other hand, the vertical beam-beam parameter is  $\Xi_y^+ + \Xi_y^- = 0.02$  to 0.045. Since the vertical tunes are separated by 0.02, the Yokoya factor would be small [1]. The measured tune shifts are smaller in the *H*-mode and larger in the *L*-mode than those calculated using  $\Xi_q^+ = \Xi_q^-$ , which suggests  $\Xi_q^+ > \Xi_q^-$  in both directions.

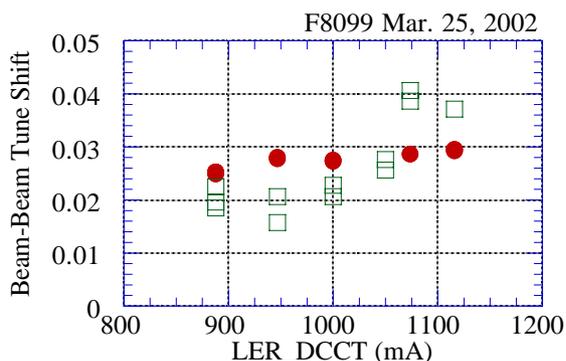


Figure 4: Total beam-beam tune shift as a function of LER beam current. Dots are horizontal tune shifts and squares are vertical tune shifts.

#### 4 DISCUSSION

We can estimate the sum of the horizontal emittances from the horizontal beam-beam parameter. An estimated emittance using the Yokoya factor is  $74 \pm 4$  nmrad, which is about 1.8 times larger than the calculated emittance of 42 nmrad. This disparity may be caused by the dynamic effect, since the horizontal tune is very close to a half integer. According to linear dynamic effects, the perturbed emittance is 1.7 times larger than the original value, when the incoherent beam-beam parameter is  $\xi_x = 0.05$ . The estimated emittance agrees with the calculation.

The luminosity can be estimated from the vertical beam-beam parameter using Eq. (5). Figure 5 shows the luminosity obtained from the vertical tune shift together

with the Belle luminosity monitor. It is noted that the estimated luminosity uses an assumption that all bunches have an equal luminosity. There is a disparity in the current dependence between the luminosity monitor and the tune shift. One candidate cause of this disparity is bunch-by-bunch dependence. The measured bunches were the last bunch of a train and were affected by the electron cloud and other wake fields. However, the bunch-by-bunch luminosity monitor [7] did not indicate such dependence. Another candidate is a misreading of peaks in the spectrum. Sidebands and two-horn peaks were observed in the vertical spectrum. During the collision, the orbit at the IP and the tunes are always manipulated to maximize the luminosity. These manipulations slightly change the colliding conditions, which makes the tune shift measurement difficult.

In summary, the coherent beam-beam tune shift was measured at KEKB. The sum of the horizontal coherent beam-beam parameter is from 0.04 to 0.05 including the Yokoya factor. The estimated horizontal emittance is 1.8 times larger than the designed value. The dynamic effect contributes to the emittance growth. The luminosity extrapolated from the vertical tune shift is inconsistent with that of the luminosity monitor. Further investigation is needed to understand the disparity.

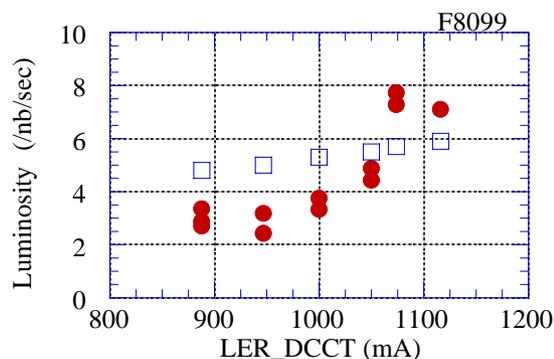


Figure 5: Dots indicate the luminosity estimated from the tune shift and squares are the luminosity monitor as a function of the LER beam current. The parameters, the reduction factor  $R = 1.0$  and  $\bar{\beta}_y^* = 6.75$  mm, are used for the estimation.

#### 5 REFERENCES

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