NUMERICAL SIMULATIONS OF DYNAMIC LORENTZ DETUNING OF SC CAVITIES

G. Devanz, M. Luong, A. Mosnier, DSM/DAPNIA/SACM, CEA Saclay, 91191 Gif-sur-Yvette

Abstract

Most of high power accelerator projects rely on bulk niobium superconducting cavities technology. When pulsed operation is required, cavities are submitted to time varying radiation pressure proportional to the square of accelerating field. This excitation couples to the mechanical system constituted of the cavity and auxiliary components, and may excite mechanical modes at resonance. Subsequent deformation of the cavity induces a time-varying detuning. When high accelerating gradients or low beta cavities are of concern, this detuning could, if not controlled, seriously impair RF operation because of extra power required, and add constraints on the RF stabilization system to keep the beam quality high. We have studied the dynamical behavior of SC cavities under pulsed operation using a modal approach and coupling mechanical simulation to RF calculations. Stiffening, mechanical modes damping and finite stiffness of tuning system and helium vessel are included in the numerical model. Modal Lorentz detuning coefficients are extracted from these calculations on which basis a simple second order system algebraic model can be set up.

1 INTRODUCTION

Superconducting (SC) cavities technology has proven to be the best choice for high current CW machine. The number of designs of high peak power pulsed linacs relying on SC technology is growing, including HEP machine like TESLA [1] and high power proton linacs for neutron spallation sources like ESS [2]. Progress of SC cavities performance lead designers to rely on high accelerating gradients. Radiation pressure

\[ P_{rad} = \frac{1}{4} (\mu_0 H^2 - \epsilon_0 E^2) \]

depends quadratically on accelerating field \( E_{acc} \). Its effects therefore becomes a prominent topic when high gradients are contemplated. The consequence of \( P_{rad} \) is a mechanical deformation of the cavity and a subsequent frequency shift. This shift can usually be reduced to a non-zero minimum by means of a stiffening system. For a CW machine, the cavities can simply be detuned to compensate the static frequency shift. The Lorentz detuning coefficient \( K \) accounts for the detuning \( \Delta f \) through the relation \( \Delta f = -|K| E_{acc}^2 \). For pulsed operation the time varying detuning must be studied in order to design the RF control system [3], and evaluate the effects on beam dynamics [4]. Depending on the time structure of the RF, mechanical eigenmodes of the cavity can be excited by time varying \( P_{rad} \). All examples shown in this paper correspond to the medium \( \beta \) 5-cells cavity of the 704.4 MHz SC linac proposed for ESS [2].

2 STATIC DETUNING MINIMISATION

Lorentz force detuning could be reduced by stiffening the cavity, for example designing it with a thicker wall, at the expense of tunability. As it was first proposed for TESLA cavities, a better solution is to weld rings between cells [5]. The principle is to dispose a fixed point in the cavity wall in order to balance the electric and magnetic part of the detuning. Figure 1 represents the cavity equipped with rings.

The optimisation of ring position was done for static detuning only. The cavity is modeled using the FEM mechanical code CASTEM. Since static \( P_{rad} \) tends to shrink the cavity, the boundary conditions at beam tubes should account for the finite stiffness \( K_{ext} \) of the He tank and tuning system. Elements with a fixed stiffness included in the beam tube model simulate an external stiffness of 100 kN/mm. The radiation pressure distribution \( P_{rad}(z, r) \) is obtained from Superfish calculations. The mechanical simulation consists in applying this pressure on cavity RF surface. The mesh of original cavity shape and the displacement field are used to produce two Superfish input files. This procedure ensures that both geometry descriptions are based on the same nodes and guarantees the accuracy of the frequency difference between the two shapes.

Figure 1: medium \( \beta \) cavity geometry

Figure 2: Ring radius optimisation
The optimisation of ring radius for the medium $\beta$-5-cells is illustrated in figure 2. For this cavity, made of 3.8 mm thick niobium, the optimum ring radius is 70 mm for a Lorentz detuning coefficient $K=1.7 \text{ Hz/(MV/m)}^2$. The cavity stiffness $\kappa_{\text{cav}}$ is kept to a low value of 1.7 kN/mm with the optimal rings, which should help operating the cavity with a low tuning force. The effect of finite $\kappa_{\text{ext}}$ results in a two fold amplification of $K$, but no significant difference for the optimum radius location. The value of $K$ without rings is given as a reference.

3 DYNAMIC BEHAVIOR MODEL

3.1 Second order model

A second order model can be set up to include the possible resonant behavior of cavity detuning $\Delta f$. Corresponding to each mechanical eigenmode of the cavity with angular frequency $\omega_m$ and quality factor $Q_m$, we use the equation

$$\frac{d^2 \Delta f_m(t)}{dt^2} + \frac{\omega_m}{Q_m} \frac{d \Delta f_m(t)}{dt} + \omega_m^2 \Delta f_m(t) = -k_m \omega_m^2 E_{\text{acc}}^2(t)$$

(2)

to describe the contribution $\Delta f_m$ of $m^{th}$ mode to the cavity detuning. In this equation, $k_m$ is the dynamic Lorentz coefficient of $m^{th}$ mode. The total cavity detuning is

$$\Delta f(t) = \sum_{m=1}^{N} \Delta f_m(t),$$

(3)

where $N$ is the number of mechanical modes included in the model. When all coefficients of equation 2 are known, the time dependant detuning can be computed for an arbitrary RF pulse by solving the N independant 2nd order equations numerically.

3.2 Principle of harmonic analysis

A convenient method to determine $k_m$ is to modulate the radiation pressure at angular frequency $\omega_m$ in order to excite the $m^{th}$ mode only. Substituting $E_{\text{acc}}^2(t)$ by $E_0^2 \sin \omega_m t$ in equation 2 and using standard methods of linear systems analysis, one can derive the steady state of $\Delta f_m(t)$:

$$\Delta f_m(t)|_{\text{harmonic}} = E_0^2 k_m Q_m \sin(\omega_m t - \pi/2)$$

(4)

4 NUMERICAL COMPUTATION OF $K_M$

4.1 Mechanical modes

CASTEM 2D numerical simulations have been carried out to determine the axi-symmetrical mechanical modes of the cavity with optimised rings. Frequency distribution is shown in figure 3. A band structure can be observed: modes under 1 kHz correspond to axial displacements of groups of cells, last modes to cell modes. Figure 4 represents typical modes: the first one is the lowest frequency mode. The second is a result of the coupling of individual cell modes. The last one is a combination of higher order cell modes. The last two modes have been chosen since their corresponding $k_m$ are among the highest.

![Figure 3: Eigenfrequencies of medium $\beta$ cavity](image)

![Figure 4: Three typical mechanical modes. Red contour is the deformed cavity shape (amplified)](image)
monically in turn, and a set of deformed shapes in steady state regime are sampled inside a mode period. The RF frequency of each of these shapes is computed with Superfish. The value of $k_m$ is derived from the fit of frequency versus time data to equation 4. The first 55 coefficients have been computed and are shown in figure 5. For mechanical modes with frequencies above 8 kHz, coefficients are decreasing to values of the order of 2% of strongest $k_m$.

![Image](https://example.com/image1.png)

Figure 5: First 55 $k_m$ of medium β cavity

### 4.3 Step response reconstruction

The cavity response to a unit step of radiation pressure is modeled by setting $E^2_{acc} (t) = E^2_0 \Theta(t)$ where $\Theta(t)$ is the Heaviside function and $E_0 = 1$ MV/m in equation 2. The steady state is then found to be $\lim_{t \to \infty} \Delta f_m(t) = k_m$ for $m^{th}$ mode. The total detuning is thus simply $\Delta f = \sum_{m=1}^{N} k_m$ when steady state has been reached. Using the 55 $k_m$ values computed for the medium β cavity, $\sum_{m=1}^{N} k_m = 1.65$ Hz/(MV/m)$^2$, which is to be compared to the $|K| = 1.7$ Hz/(MV/m)$^2$ coefficient given by static detuning calculation. This indicates that a sufficient number of modes have been taken into account in order to reproduce the static behaviour successfully.

### 5 EXTERNAL STIFFNESS

The influence of $\kappa_{ext}$ on static $K$ can be approximated the following way: the loaded cavity exerts a force on the tuner represented by stiffness $\kappa_{ext}$. The variation of cavity length $\Delta l_{ext}$ is thus proportional to $P_{rad}/\kappa_{ext}$. If $\kappa_{ext} \gg \kappa_{cav}$, extra detuning is $\Delta f_{ext} = \Delta l_{ext} df/dl$. In this condition, the static coefficient $K$ can be expressed as the sum of $\kappa_{ext}$ dependent and independent detunings:

$$|K(\kappa_{ext})| = |K_\infty| + \frac{F_z}{\kappa_{ext}} \frac{df}{dl}$$

(5)

where $K_\infty$ is static $K$ computed for an infinitely stiff tuner, and $F_z$ the axial component of the force at beam tube end due to radiation pressure for $E_{acc} = 1$ MV/m.

In dynamic analysis, modes should behave in distinct ways. Modes whose displacement field is low at beam tube, such as high frequency cell modes, should respond to radiation pressure independently of $\kappa_{ext}$. In contrast, cavity modes of the first band involve large displacements at cavity ends: taking lower values of $\kappa_{ext}$ should greatly enhance the $k_m$ coefficients of this particular modes. This is illustrated on figure 6 where the first 40 computed $k_m$s for $\kappa_{ext}$ equal to 20 kN/mm and 100 kN/mm can be compared: $k_m$ of the first two passbands increase dramatically for the lower value of $\kappa_{ext}$. This clearly favors a stiffer tuning system for pulsed operation.

![Image](https://example.com/image2.png)

Figure 6: Influence of $\kappa_{ext}$ on the first 40 $k_m$

### 6 CONCLUSION

We evaluated the dynamic Lorentz coefficients using a combination of mechanical and RF computations. The method of modal analysis with N=65 modes was found to be adequate for mechanical calculations. The $k_m$ coefficients can be included in a second order model of the time dependent detuning of the cavity. This model allows one to determine the frequency shift induced by an arbitrary RF pulse. The $k_m$ coefficients of the optimised medium β cavity were computed for a stiff and a looser tuning system. Choosing a stiffer tuner improves dramatically the dynamic detuning behaviour of the cavity.

### 7 REFERENCES

[3] M. Luong et al., Minimizing RF Power Requirement and Improving Amplitude/Phase Control for High Gradient Superconducting Cavities, these proceedings
[4] A. Mosnier, Control of SCRF Cavities in High Power Proton Linacs, these proceedings

2222