# THE OBSERVATION OF THE NEGATIVE MASS INSTABILITY IN THE PROTON SYNCHROTRON TRAPP 

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#### Abstract

In this paper we present the results of the experiments for observation of the negative mass instability. This experiments was attended in the proton synchrotron TRAPP designed for medical applications. The numerical simulations and analytical estimations are in good agreement with the experimental data.


## INTRODUCTION

The longitudinal bunching of the beam was observed during the improving of the proton synchrotron TRAPP [1]. The investigation showed that a so-called negative mass instability produces this phenomenon. A random increase of the beam density cause the arising of longitudinal forces which repel neighbor particles. These neighbor particles move to the area of enlarged density in case if $d \omega / d E<0$, that is the particles revolution frequency $\omega$ decreases when the particles energy $E$ increases. In consequence of this process the fluctuations density are grow with time leading to fast bunching of the beam. The proton synchrotron TRAPP (See Table 1) was designed for cancer therapy and tomography survey.

Table 1. Parameters of the synchrotron TRAPP.


## THEORY

The analytical estimations in linear approximation shows that the bunching has a place if a next conditions are accomplished [2]:

$$
\begin{equation*}
\frac{\partial \omega}{\partial E}<0 ; \frac{d E}{E}<\sqrt{\frac{2 \cdot r_{0} \cdot \Lambda \cdot N}{\pi \cdot R \cdot K \cdot \gamma^{3}}}=\left(\frac{d E}{E}\right)_{C R I T} \tag{1}
\end{equation*}
$$

where: dE is the energy spread in the beam cross-section, E is the total energy of particles, $\mathrm{r}_{0}$ is the classical proton radius, K is the slip factor, N is the total number of
particles, $\Lambda \approx 2$ is the factor defined by the beam and vacuum chamber geometry. The instability criteria (1) don't depend on the azimuth length of the beam density perturbation.

In a longwave approximation the perturbation growth time is determined by the next expression [2] :

$$
\begin{equation*}
\tau=2 \cdot\left(q \cdot K \cdot \omega \cdot \sqrt{\left(\frac{d E}{E}\right)_{C R I T}^{2}-\left(\frac{d E}{E}\right)^{2}}\right)^{-1} \tag{2}
\end{equation*}
$$

where q is the harmonic of the density perturbation.
In consequence of the beam grouping the bunches are formed and some bunches have a stable in time figuration. The longitudinal distribution of the beam density consisted of q equal bunches can be expressed by the next form: $\xi=\xi_{0} \cdot(1-\cos (q \cdot \theta)) \quad$ where $\xi_{0}=N \cdot e /(2 \cdot \pi \cdot R), \theta$ is the azimuth angle. Under influence of the beam self-fields the particles in these bunches move inside the separatrix analogous the particle motion in accelerated RF field. The maximum variation of the energy at one turn is:

$$
\begin{equation*}
d E_{\text {об }}=2 \cdot \pi \cdot q \cdot e \cdot \xi_{0} \cdot \Lambda \cdot \sin (q \cdot \theta) / \gamma^{2} \tag{3}
\end{equation*}
$$

The period of quasi-synchrotron oscillations can be found after the simple calculations and equal:

$$
\begin{equation*}
T=2 \cdot \pi \cdot \tau \tag{4}
\end{equation*}
$$

The maximal energy spread in the bunch is:

$$
\begin{equation*}
\left(\frac{d E}{E}\right)_{M A X}=\left(\frac{d E}{E}\right)_{C R I T} \tag{5}
\end{equation*}
$$

## COMPUTER SIMULATION

The 3-dimenshional programme based on the PP-PM method [4] was created for computer simulation of the collective phenomena in the TRAPP. The beam is described as the set of uniform charged spherical macroparticles interacting between themselves. The number of the macroparticles acquired the value $10^{5}$.

The particle motion in the external fields of accelerator structure was simulated by matrix method. The perturbation of density in the sinusoidal form (80-th harmonic) with amplitude $3 \%$ was used for the comparison with the analytical estimations. The increment value (Fig.1.) is 3.8 turns. From equation (2) for same parameters the increment value is $\tau=4.3$ turns.


Fig.1. The instability growth for sinusoidal form perturbation (80-th harmonic) in the coast beam.

Let's consider the dynamic of the short monochromatic bunch (Fig.2.). In consequence of influence of longitudinal forces in the fronts the particle distribution transforms in time. At first the beam is shortened and accordingly the density in the centre of bunch and the energy spread increases.


Fig.2. The dynamic of the short monochromatic bunch.
After this the bunch is split at two parts, particles from head and tail of the bunch change over and bunches go in different directions. The difference between bunch's average energy is $2 \cdot 10^{-4}$. At further motion bunches have the stable form. The maximal energy spread in the bunch is $\mathrm{dE} / \mathrm{E}_{\mathrm{cin}}=4 \cdot 10^{-4}$. The equation (5) predict the value of maximum energy spread about $3.3 \cdot 10^{-4}$. The coordinate variation of the test particle is shown in Fig.3. The particle oscillate about the bunch centre with period about 300 turns. The period of quasi-synchrotron oscillations
can be found from equation (4), $\mathrm{T}_{2}=252$ turns that is in good agreement with results of the simulation.


Fig.3. The coordinate variation of the test particle.

## EXPERIMENTAL DATA

For beam observations the signal from the cylindrical electrostatic electrode with effective length 1 cm was used. The typical scenario of the instability growth is shown in Fig.4. As is seen from Fig.4. the first signs of the longitudinal modulation of density appear after 6 turns. After 10 turns the modulation increase by $10 \%$, after 100 turns $-80 \%$. The initial bunching of the beam is close to equable and accords with 80 -th harmonic of the revolution frequency. However after 2000 turns only about 10 bunches remain. The value of instability increment can be found from the cadre's progression $\tau \approx 4$ turns. For the average bunch intensity $2.5 \cdot 10^{9}$ from equation (2) the increment value for 80 -th harmonic is 2.4 turns. Adducing estimations and numerical simulations can't completely explain the experiment results because take into account the particle losses during the bunching.


Fig.4. The longitudinal profile of beam after injection: a) 6 turns, b) 10 turns, c) 100 turns d) 2000 turns.

The decrease of the number of bunches can be explained either in consequence of the bunch's fusion or because of the irregularity of particle losses from different bunches. From the observations of beam dynamic (Fig.5.) the conclusion that the bunches fusion is the main cause of the decreasing bunches number can be made. It is possible to assume that the slow fusion process is determined by other collective phenomena.


Fig.5. The longitudinal profile of beam in different time after injection in one cycle of injection.

Two experiments are the important confirmation that the negative mass instability produces the longitudinal beam bunching.

In first experiment the growth of instability was damped by energy spread in the injected beam. Therefore the bunch expanded into the all ring perimeter. The time dependencies of beam intensity for different value of the energy spread are shown in Fig.6.


Fig.6. The time dependencies of beam intensity for different value of the energy spread.

Other experiment bases in the modification of synchrotron optics with aim to change the sign of $d \omega / d E$.

The longitudinal profile of beam after 2000 turns for positive and negative signs of the slip factor are shown in Fig. 7.

| 2000 turns K $>0$ | 2000 turns $\mathrm{K}<0$ |
| :---: | :---: |
|  | B25 |
| 0 mos 1,2 | $0 \mathrm{mcs} 1,2$ |

Fig.7. The longitudinal profile of beam for different momentum compaction factors.

The negative mass instability influence on the particle losses was studied. From the results of experiments can be concluded that the negative mass instability increase the particle losses. Adducing data belong to so-called betatron regime: the constant magnetic field and without accelerated voltage. Let's consider the possible mechanisms of the losses increase. The most apparent cases are the next:

1. It is the increase of radial beam size. Simulation shows that the proton energy variation in the beam with the number of particles $5 \cdot 10^{9}$ acquires $\pm 5 \cdot 10^{-3}$. This value agrees with the variation of the ideal orbit radius about 8 mm , that determines the particle losses in incipient turns after injection.
2. The periodic variations of transverse space-charge forces because of the particles quasi-synchrotron motion increase the particle losses through the excitation of synchro-betatron resonance's.
3. Accordingly the results of the magnetic measurements and studying of the magnet structure the nonlinearities of magnet field have a place that produce the decrease of dynamic aperture. Quasi-synchrotron oscillations on account of the negative mass instability increase this effect.

## CONCLUSION

In paper the good agreement between theory, computer simulation and experiment results in the initial stage of the instability growth is shown. The experiments also shows that the negative mass instability increase the particle losses.

## REFERENCES

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