Data Acquisition and Error Analysis for Pepperpot Emittance Measurements

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DIPAC '09

Abstract

"The pepperpot provides a unique and fast method of measuring emittance, providing four dimensional correlated beam measurements for both transverse planes. In order to make such a correlated measurement, the pepperpot must sample the beam at specific intervals. Such discontinuous data, and the unique characteristics of the pepperpot assembly, requires special attention be paid to both the data acquisition and the error analysis techniques.

A first-principles derivation of the error contribution to the *rms* emittance is presented, leading to a general formula for emittance error calculation. Two distinct pepperpot systems, currently in use at GSI in Germany and RAL in the UK, are described. The data acquisition process for each system is detailed, covering the reconstruction of the beam profile and the transverse emittances. Error analysis for both systems is presented, using a number of methods to estimate the emittance and associated errors."

Error Analysis of Pepperpot Emittances

- Derivation of emittance error formula.
- Description of 2 contrasting pepperpot systems:
  - FETS pepperpot at RAL.
  - HITRAP pepperpot at GSI.
- Sources of error in each system.
- Results of error analysis.

Calculating Emittance Errors

- *rms* emittance is mathematically well defined:

$$\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

- Would like a mathematically sound method of calculating errors!
- Possible to propagate errors mathematically by summing variances:

$$f(x, y, z\ldots) \qquad \sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma_z^2 \ldots$$

Definition of *rms* Emittance

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

Position x_i, Angle x_i', Intensity ρ_i

$$= \sqrt{\left(\frac{\sum_{i=1}^{N} \rho_i x_i^2}{\sum_{i=1}^{N} \rho_i} \right) \left(\frac{\sum_{j=1}^{N} \rho_j x_j'^2}{\sum_{j=1}^{N} \rho_j} \right) - \left(\frac{\sum\limits_{i=1}^{N} \rho_i x_i x_i'}{\sum_{i=1}^{N} \rho_i} \right)^2}$$

$$= \sqrt{\frac{\left(\sum_{i=1}^{N} \rho_i x_i^2\right) \left(\sum_{j=1}^{N} \rho_j x_j'^2\right) - \left(\sum_{i=1}^{N} \rho_i x_i x_i'\right)^2}{\left(\sum_{i=1}^{N} \rho_i\right)^2}}$$

Calculation of Variances

We can derive variances for each term in the *rms* emittance equation. Start by looking at single terms of x^2 sum:

$$\sum_{i=1}^{N} \rho_i x_i^2$$

Variance on 1 term: $$\sigma^2_{\rho x^2} = 4\rho^2 x^2 \sigma_x^2 + x^4 \sigma_\rho^2$$

Variance on series: $$\sigma^2_{\sum \rho x^2} = \sum_{i=1}^{N} \left(4\rho_i^2 x_i^2 \sigma^2_{x_i} + x_i^4 \sigma^2_{\rho_i}\right)$$

Calculation of Variances (2)

Same applies to sum over x'^2:

$$\sum_{j=1}^{N} \rho_j x_j'^2$$

Variance on 1 term: $$\sigma^2_{\rho x'^2} = 4\rho^2 x'^2 \sigma^2_{x'} + x'^4 \sigma^2_{\rho}$$

Variance on series: $$\sigma^2_{\sum \rho x'^2} = \sum_{j=1}^{N} \left(4\rho_j^2 x_j'^2 \sigma^2_{x'_j} + x_j'^4 \sigma^2_{\rho_j} \right)$$

Calculation of Variances (3)

Variance of the product of these two terms:

$$\left(\sum_{i=1}^{N} \rho_i x_i^2\right)\left(\sum_{j=1}^{N} \rho_j x_j'^2\right)$$

$$\sigma^2_{\sum \rho x^2 \sum \rho x'^2} = \left(\sum_{i=1}^{N} 4\rho_i^2 x_i^2 \sigma_{x_i}^2 + x_i^4 \sigma_{\rho_i}^2\right)\left(\sum_{j=1}^{N} \rho_j x_j'^2\right)^2$$

$$+ \left(\sum_{i=1}^{N} 4\rho_i^2 x_i'^2 \sigma_{x_i'}^2 + x_i'^4 \sigma_{\rho_i}^2\right)\left(\sum_{j=1}^{N} \rho_j x_j^2\right)^2$$

Calculation of Variances (4)

2 more variances needed – for *xx'* term:

$$\left(\sum_{i=1}^{N}\rho_i x_i x_i'\right)^2$$

Variance on 1 term:

$$\sigma^2_{\rho x x'} = x^2 x'^2 \sigma^2_\rho + \rho^2 x'^2 \sigma^2_x + \rho^2 x^2 \sigma^2_{x'}$$

Variance on series:

$$\sigma^2_{\sum \rho x x'} = \sum_{i=1}^{N}\left(x_i^2 x_i'^2 \sigma^2_{\rho_i} + \rho_i^2 x_i'^2 \sigma^2_{x_i} + \rho_i^2 x_i^2 \sigma^2_{x_i'}\right)$$

Calculation of Variances (4)

2 more variances needed – for xx' term:

$$\left(\sum_{i=1}^{N}\rho_i x_i x_i'\right)^2$$

Variance on series squared:

$$\sigma^2_{(\sum \rho x x')^2} = 4\left(\sum_{i=1}^{N} x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2\right)$$

$$\times\left(\sum_{j=1}^{N}\rho_j x_j x_j'\right)^2$$

Calculation of Variances (4)

…and for the ρ^2 term:

$$\left(\sum_{i=1}^{N}\rho_i\right)^2$$

Variance on series:

$$\sigma^2_{\sum\rho} = \sum_{i=1}^{N}\sigma^2_{\rho_i}$$

Variance on series squared:

$$\sigma^2_{(\sum\rho)^2} = 4\left(\sum_{i=1}^{N}\sigma^2_{\rho_i}\right)\left(\sum_{j=1}^{N}\rho_j\right)^2$$

Calculation of Variances (5)

Now start combining: first the numerator…

$$\left(\sum_{i=1}^{N} \rho_i x_i^2\right)\left(\sum_{j=1}^{N} \rho_j x_j'^2\right) - \left(\sum_{i=1}^{N} \rho_i x_i x_i'\right)^2$$

Calculation of Variances (5)

$$\sigma^2_{\sum \rho x^2 \sum \rho x'^2 - (\sum \rho x x')^2} =$$

$$\left(\sum_{i=1}^{N} 4\rho_i^2 x_i^2 \sigma_{x_i}^2 + x_i^4 \sigma_{\rho_i}^2\right)\left(\sum_{j=1}^{N} \rho_j x_j'^2\right)^2$$

$$+\left(\sum_{i=1}^{N} 4\rho_i^2 x_i'^2 \sigma_{x_i'}^2 + x_i'^4 \sigma_{\rho_i}^2\right)\left(\sum_{j=1}^{N} \rho_j x_j^2\right)^2 +$$

$$4\left(\sum_{i=1}^{N} x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2\right)\left(\sum_{j=1}^{N} \rho_j x_j x_j'\right)^2$$

$$-8\left(\sum_{i=1}^{N} \rho_i^2 x_i^4 x_i'^4 \sigma_{\rho_i}^2 + \rho_i^4 x_i^2 x_i'^4 \sigma_{x_i}^2 + \rho_i^4 x_i^4 x_i'^2 \sigma_{x_i'}^2\right)$$

Calculation of Variances (6)

…now propagate errors through the division and square root:

$$\sqrt{\frac{\left(\sum_{i=1}^{N} \rho_i x_i^2\right)\left(\sum_{j=1}^{N} \rho_j x_j'^2\right) - \left(\sum_{i=1}^{N} \rho_i x_i x_i'\right)^2}{\left(\sum_{i=1}^{N} \rho_i\right)^2}}$$

Calculation of Variances (6)

$$\sigma_\varepsilon^2 = \frac{\left(\sum_{i=1}^N \rho_i^2 x_i^2 \sigma_{x_i}^2 + \frac{x_i^4 \sigma_{\rho_i}^2}{4}\right)\left(\sum_{j=1}^N \rho_j x_j'^2\right)^2}{\varepsilon^2 \left(\sum_{k=1}^N \rho_k\right)^4}$$

← Position error

$$+ \frac{\left(\sum_{i=1}^N \rho_i^2 x_i'^2 \sigma_{x_i'}^2 + \frac{x_i'^4 \sigma_{\rho_i}^2}{4}\right)\left(\sum_{j=1}^N \rho_j x_j^2\right)^2}{\varepsilon^2 \left(\sum_{k=1}^N \rho_k\right)^4} +$$

← Angle error

$$\frac{\left(\sum_{i=1}^N x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2\right)\left(\sum_{j=1}^N \rho_j x_j x_j'\right)^2}{\varepsilon^2 \left(\sum_{k=1}^N \rho_k\right)^4}$$

← Coupling term

$$- \frac{2\left(\sum_{i=1}^N \rho_i^2 x_i^4 x_i'^4 \sigma_{\rho_i}^2 + \rho_i^4 x_i^2 x_i'^4 \sigma_{x_i}^2 + \rho_i^4 x_i^4 x_i'^2 \sigma_{x_i'}^2\right)}{\varepsilon^2 \left(\sum_{k=1}^N \rho_k\right)^4}$$

← Cancelling terms

$$+ \frac{\varepsilon^2 \left(\sum_{i=1}^N \sigma_{\rho_i}^2\right)}{\left(\sum_{k=1}^N \rho_k\right)^2}$$

← Intensity error

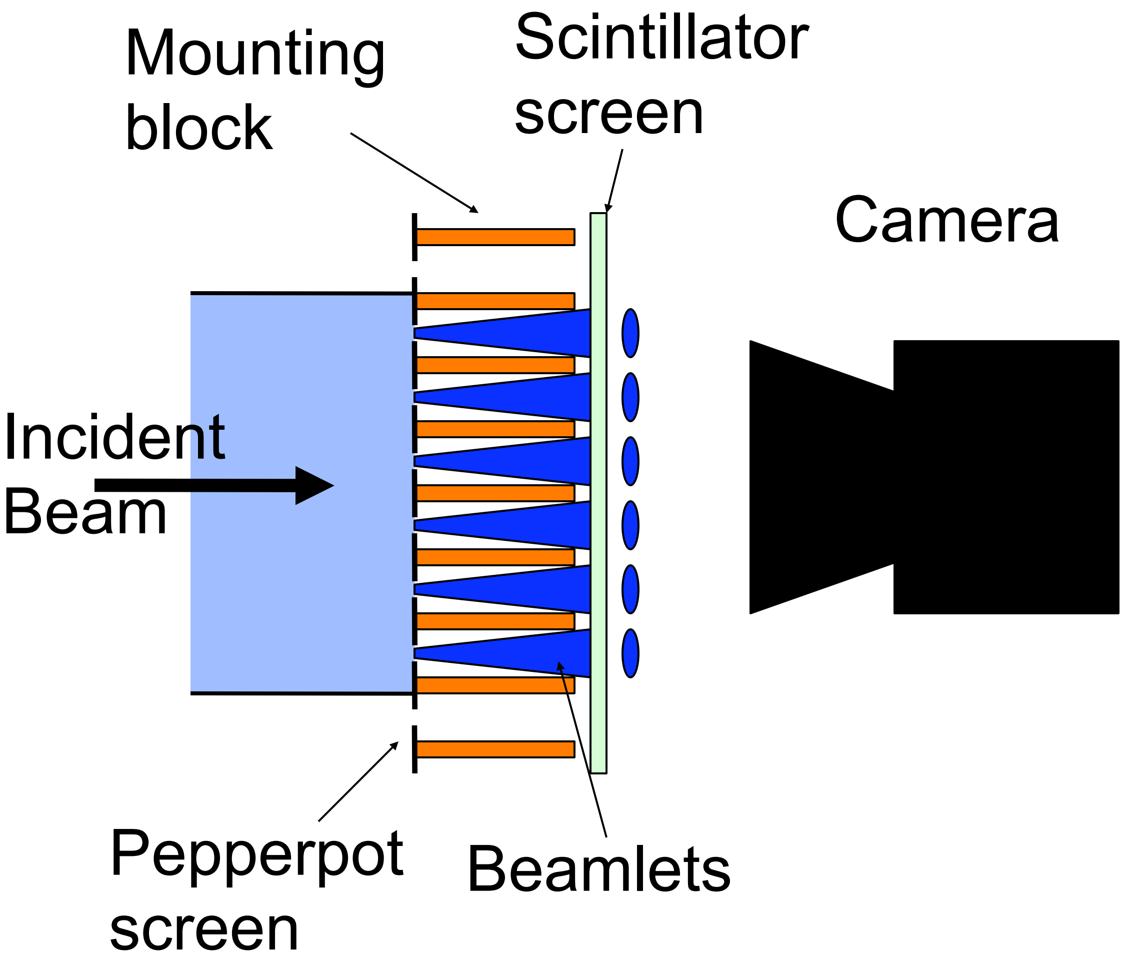
The variance on the emittance: error is square root of this value.

Application of Emittance Error

- We have now derived a formula for calculating emittance error: so what…?
- Apply it to pepperpot measurements:
  - Do the predicted results make sense?
  - How does each type of error contribute?
  - Can this help to improve our measurement?

Pepperpot Principle

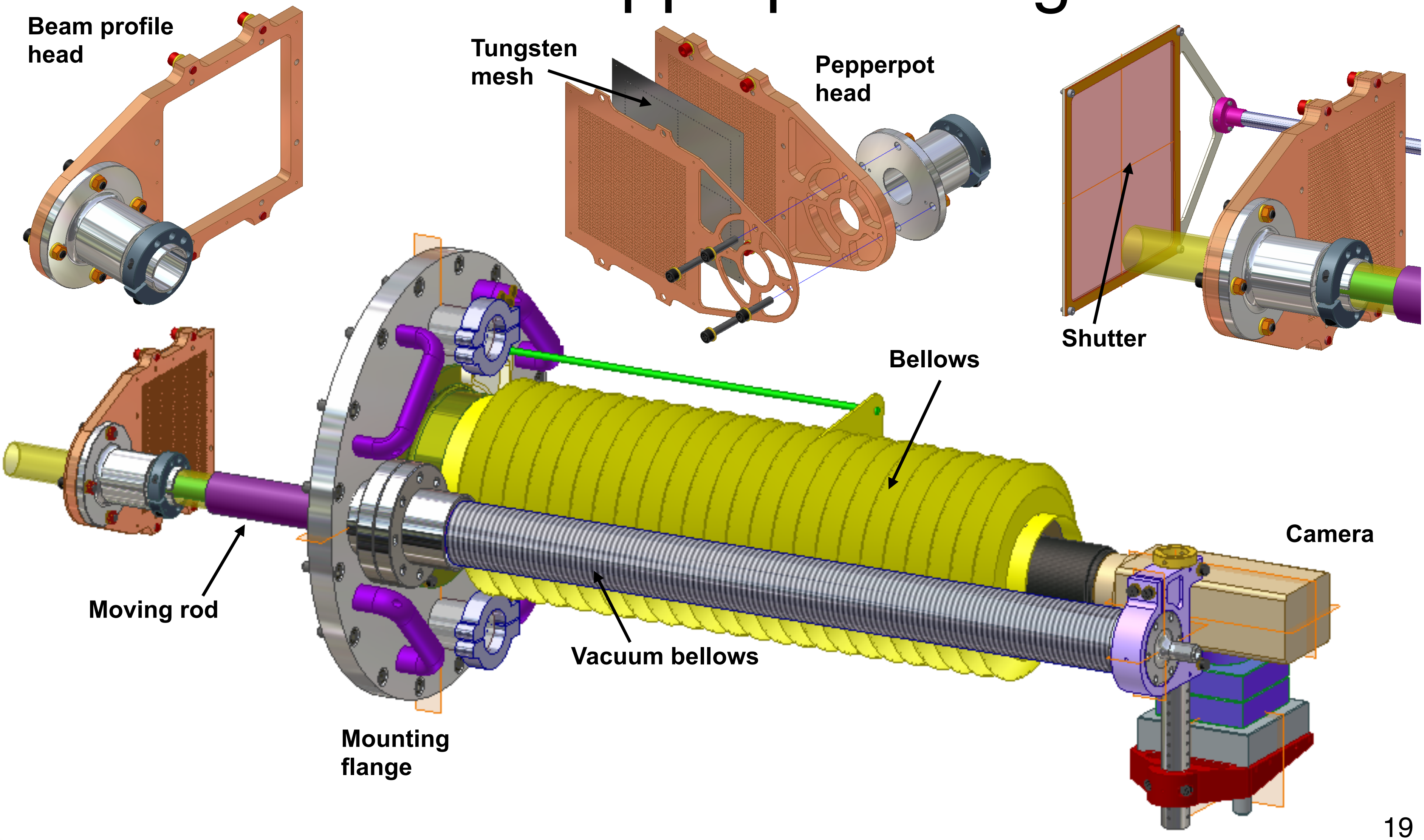
- Beam segmented by pepperpot screen.
- Beamlets drift before producing image on scintillator.
- Camera records image of light spots.
- Calculate emittance from spot distribution.



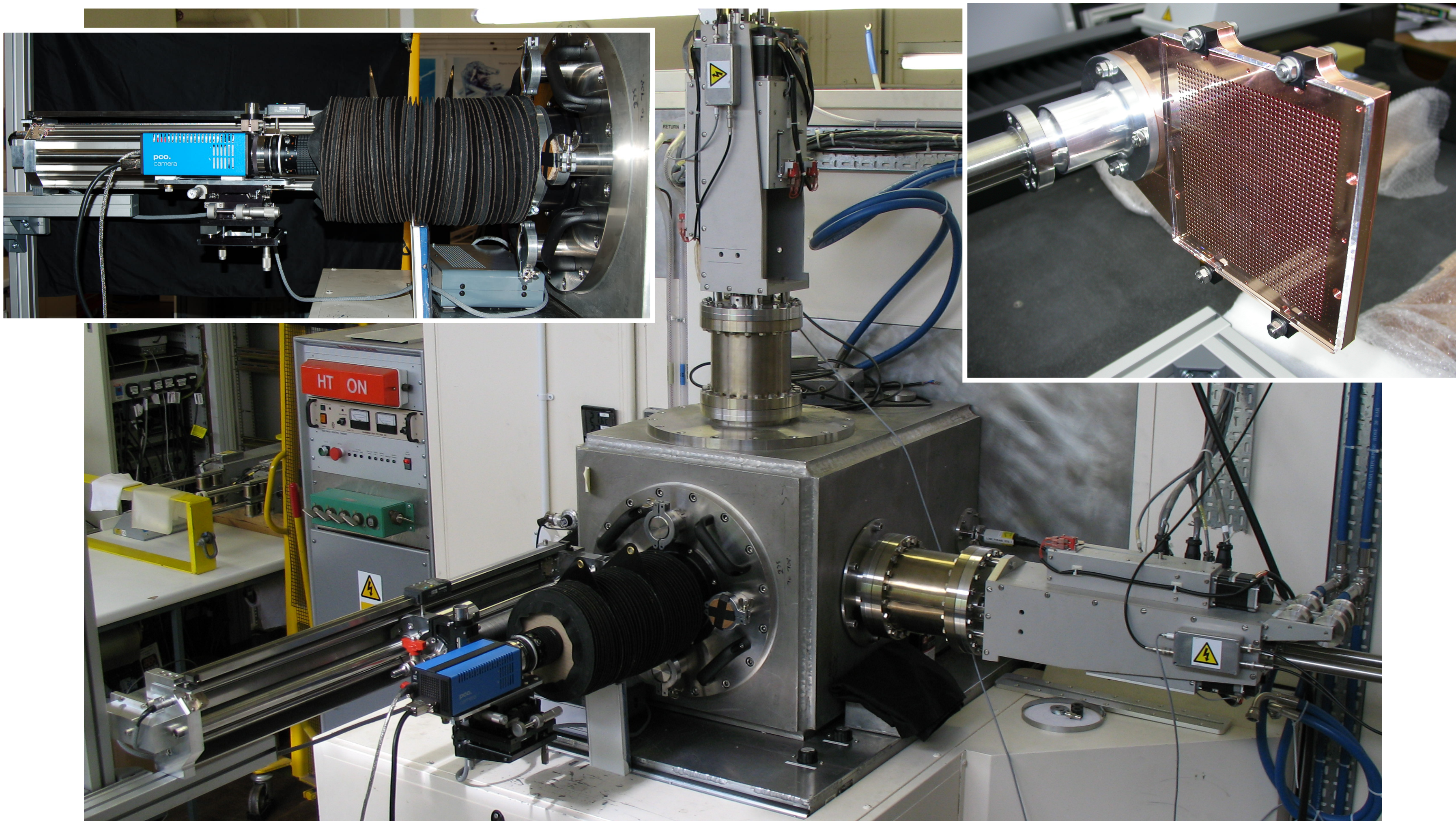
Pepperpot Systems

- Errors analysed for 2 different pepperpots:
  - Front End Test Stand system (FETS) at RAL (see Jolly *et al*, WEO2A01, DIPAC'07).
    - H^- ions, 60mA, 35keV, 30mm radius, emittance up to 1 π mm mrad.
    - Tungsten intercepting screen, 100 microns thick, 41x41 holes, 50 microns diameter on 3 mm pitch.
    - Quartz scintillator, 10 mm from tungsten screen; images captured with high speed PCO camera, 2048x2048 pixels.
    - Angular resolution of 6.5 mrad.
    - Calibration by eye from calibration markings around support structure.
  - HITRAP system at GSI (see Hoffmann *et al*, BIW 2000, AIP Conf. Proc. 546, p.432; Pfister *et al*, THPP037, EPAC'08; TUP074, LINAC'08).
    - Ni ions at 4 MeV/u, 17 mm radius, 0.2 π mm mrad emittance.
    - Tungsten foil, 100 microns thick, 19x19 holes, 100 microns diameter on 1.6 mm pitch.
    - Al_2O_3 scintillator, 150 mm from tungsten screen: images captured with cooled fast shutter CCD camera with 1280x1024 pixels.
    - Angular resolution of 0.3 mrad.
    - Laser calibrated: project X and Y, use maxima for calibration.

FETS Pepperpot Design

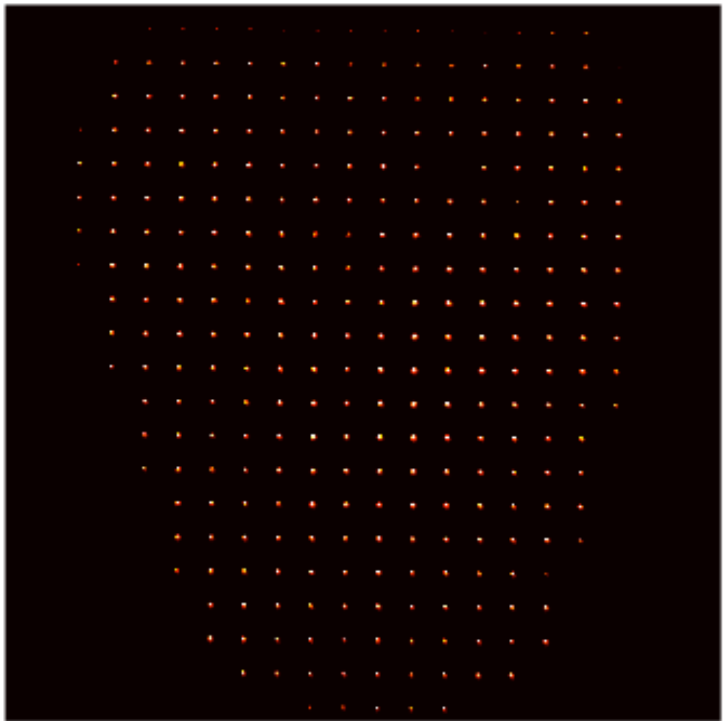


FETS Pepperpot Installation



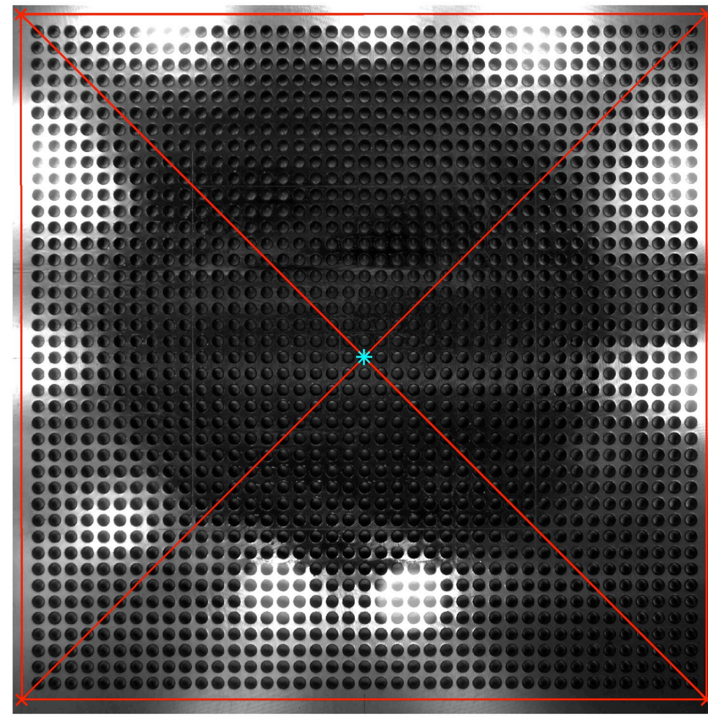
FETS Pepperpot Data Image

Raw data



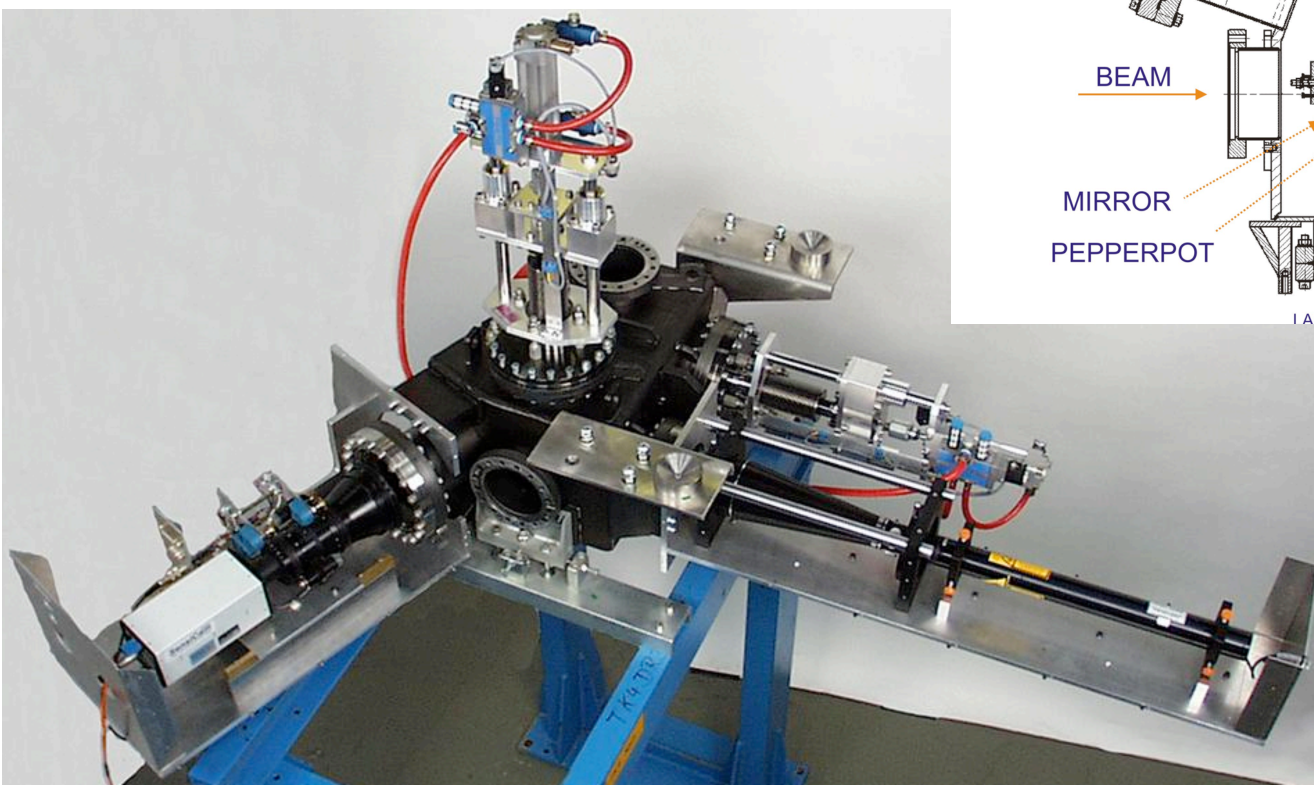
Colour enhanced raw data image, 60 x 60 mm^2.

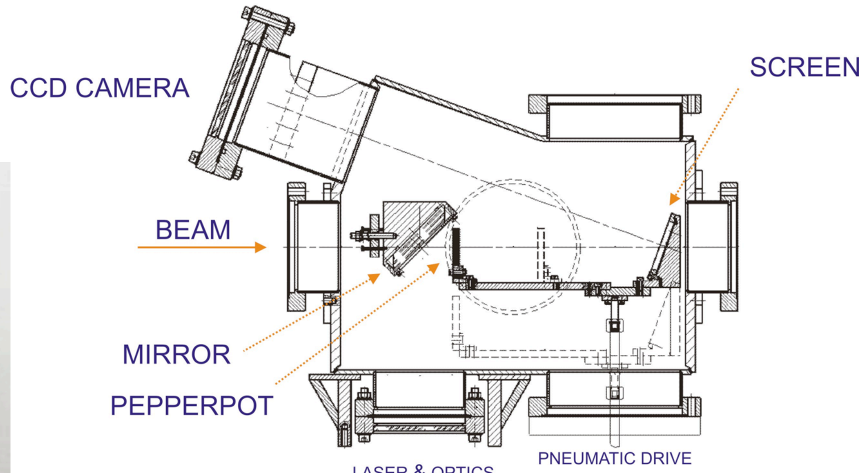
Calibration image



Calibration image: use corners of 125 x 125 mm square on copper plate to give image scaling, tilt and spot spacing.

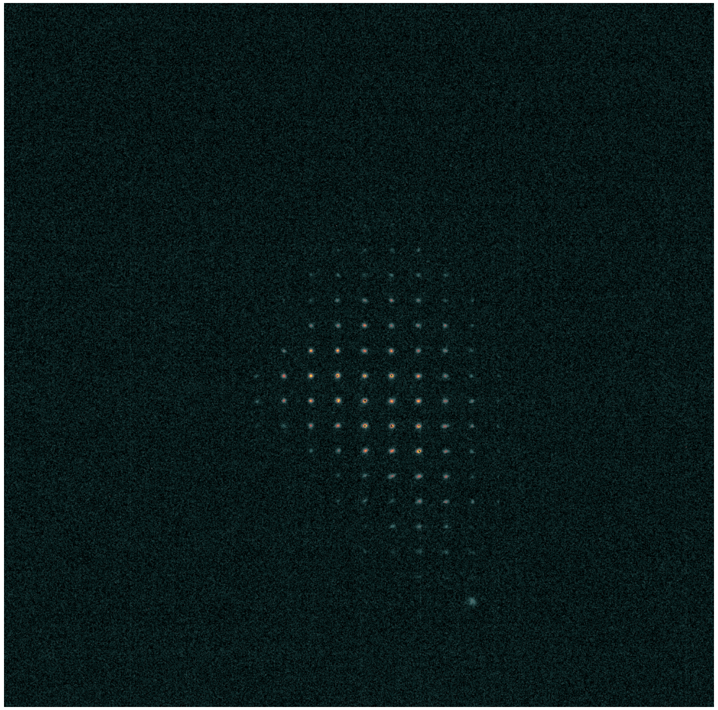
HITRAP Pepperpot (GSI)





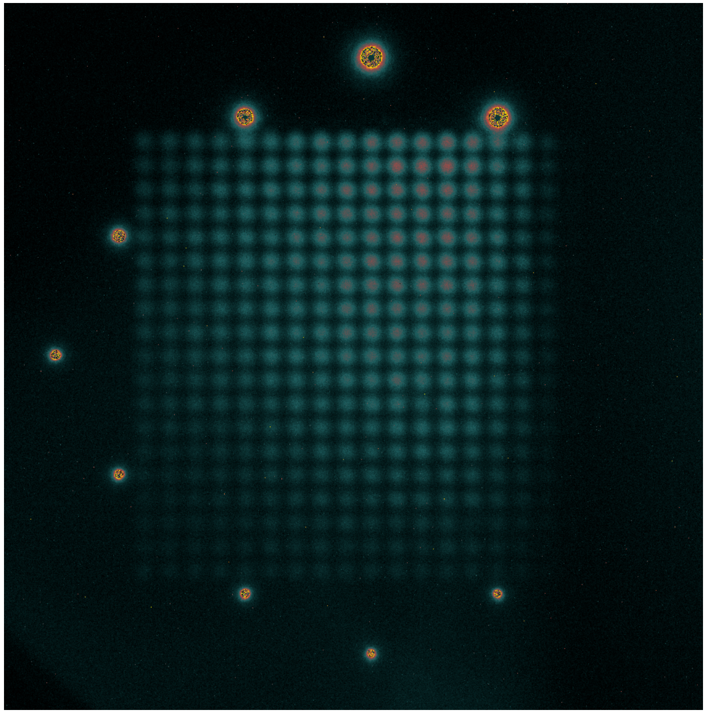
HITRAP Pepperpot Data Image

Raw data



Raw data image, 30 x 30 mm^2.

Calibration image



Calibration image: laser light spots.

Pepperpot Errors

- Using error formula as a guide, we can split our errors into 3 distinct groups:
  - Position (σ_x), eg.:
    - Mesh spacing.
    - Hole size and position.
  - Angle ($\sigma_{x'}$), eg.:
    - Camera resolution.
    - Calibration.
    - Optical aberrations.
  - Intensity (σ_ρ), eg.:
    - Beam noise (pulse-to-pulse variation).
    - Background noise (stray light etc.).
    - System noise (CCD readout etc.).

Pepperpot Errors: Assumptions

- Position measurement resolution is hole spacing, NOT size or accuracy of holes.
- Calibration errors ONLY affect angle measurement.
- Emittance plot offsets do not contribute (no error on means).
- Hole size is smaller than pixel size: can do ray tracing (don't need to consider hole size/shape as part of angle error).
- All the information is measured: mesh & screen are big enough to measure whole beam.

Pepperpot Error Sources

	FETS		HITRAP	
	Measurement	σ_i	Measurement	σ_i
Hole spacing	3 mm	$3/\sqrt{12}$	1.6 mm	$1.6/\sqrt{12}$
Angle resolution	6.5 mrad	$6.5/\sqrt{12}$	0.3 mrad	$0.3/\sqrt{12}$
Beam noise	10%	0.1ρ	10%	0.1ρ
Noise floor	10% of max.	0.1	2% of max.	0.02

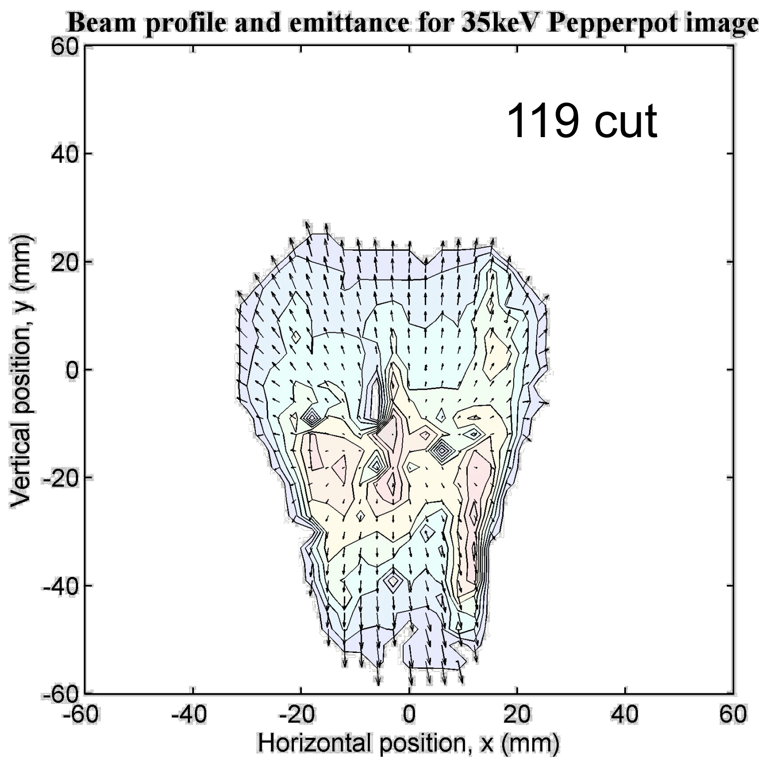
Pepperpot Error Contributions

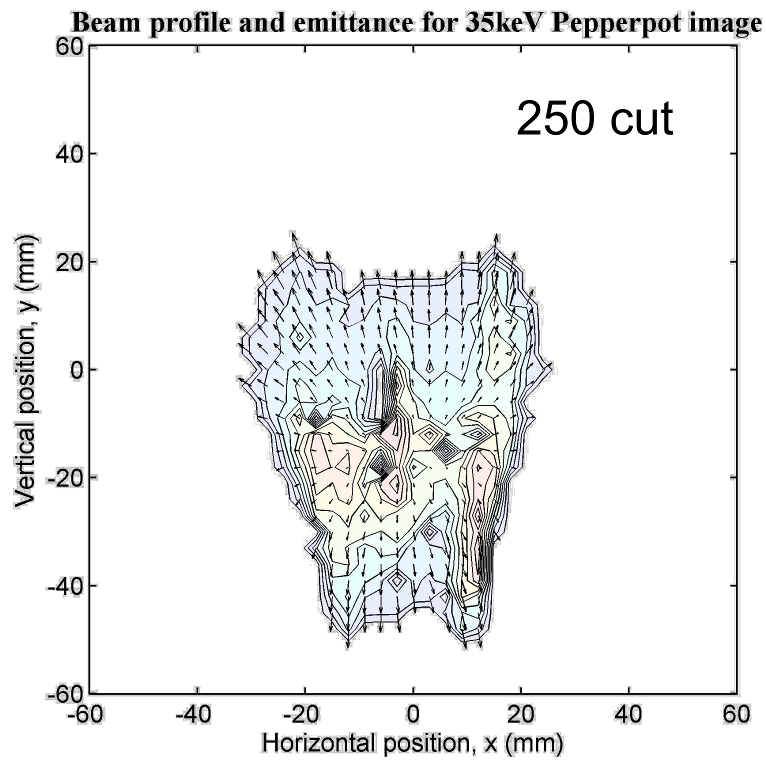
	FETS		HITRAP	
	Measurement	σ_ε (%)	Measurement	σ_ε (%)
Beam radius	45 mm	-	17 mm	-
ε_{rms}	0.61 π mm mrad	-	0.24 π mm mrad	-
Hole spacing	3 mm	1.8%	1.6 mm	2.2%
Angle resolution	6.5 mrad	1.6%	0.3 mrad	0.2%
Beam noise	10%	1.3%	10%	0.3%
Noise floor	2% of max.	~0	10% of max.	1.2
σ_ε	0.029 π mm mrad	4.8%	0.010 π mm mrad	3.9%

Preliminary Conclusions

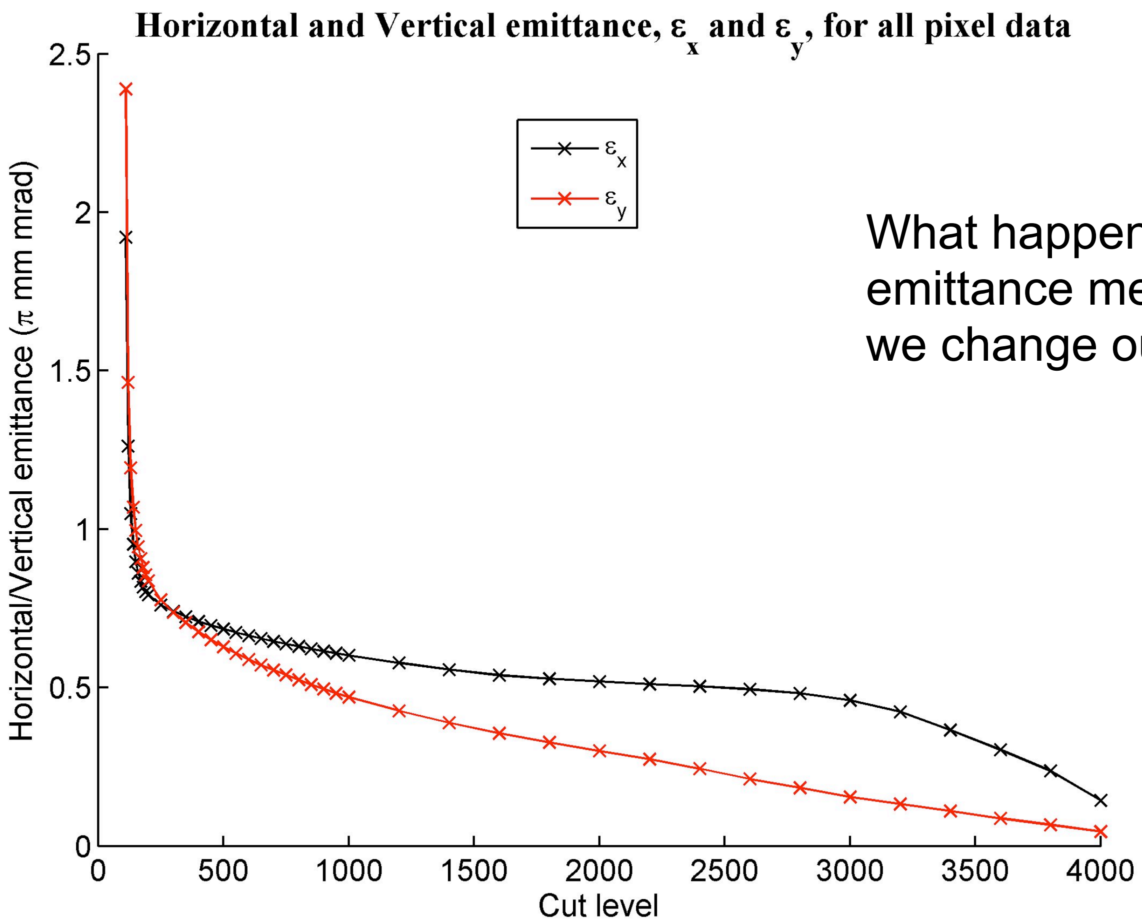
- Some promising results!
- Our first-principles formula gives sensible numbers for real experiment.
- Possible to see where one experiment wins out over the other, and where both can be improved.
- But…are we including everything…?

FETS Pepperpot: 119/250 Count Cut





The Unkindest Cut of All



What happens to the emittance measurement if we change our cut level?

Death by A Thousand Cuts

- Virtually every measurement includes some kind of “background rejection” cut.
- But for emittance errors to be absolutely valid, we must include ALL information.
- Is it possible to select consistent cut level to optimise emittance measurement but have meaningful errors?
- Definite overlap with SCUBEEx (Stockli *et al*, Rev. Sci. Instrum. **75** (2004) 1646).
- SCUBEEx (Self-Consistent Unbiased Emittance Exclusion algorithm) uses all emittance data and optimises ellipse to minimise noise contribution.

Conclusions

- Formula has been derived from first principles for calculating emittance errors: this is a general formula for ANY measurement of *rms* emittance.
- This formula has been applied successfully to 2 different pepperpot setups, with promising results.
- Analysis of the errors demonstrate the contributions of different parts of the system, such as the hole spacing and angular resolution, to the overall error estimate.
- Further work is required to categorise errors not included in this analysis, since these affect the accuracy of the emittance measurement while not contributing to the error estimate.
- Would be interesting to see how good errors are for many real measurements: also plan to use simulated pepperpot images with known emittance.
- Emittance error formula probably gives absolute limit in resolution: the best we can do given the experimental setup.
- More thought required on how to deal with cuts and selection/rejection of “good” data.
- Can we combine this with SCUBEEx…?

Acknowledgements

- Pepperpotters:
  - Jochen Pfister.
  - Jürgen Pozimski.
  - Christoph Gabor.
- Statisticians:
  - Roger Barlow.
  - Louis Lyons.
  - Martin Jolly.