

# DATA ACQUISITION AND ERROR ANALYSIS FOR PEPPERPOT EMITTANCE MEASUREMENTS

S. Jolly, Imperial College, London, UK

J. Pozimski, STFC/RAL, Chilton, Didcot, Oxon, UK/Imperial College, London, UK

J. Pfister, IAP, Frankfurt am Main, Germany

O. Kester, NSCL, East Lansing, Michigan, USA

D. Faircloth, C. Gabor, A. Letchford, S. Lawrie, STFC/RAL, Chilton, Didcot, Oxon, UK

## Abstract

The pepperpot provides a unique and fast method of measuring emittance, providing four dimensional correlated beam measurements for both transverse planes. In order to make such a correlated measurement, the pepperpot must sample the beam at specific intervals. Such discontinuous data, and the unique characteristics of the pepperpot assembly, requires special attention be paid to both the data acquisition and the error analysis techniques. A first-principles derivation of the error contribution to the rms emittance is presented, leading to a general formula for emittance error calculation. Two distinct pepperpot systems, currently in use at GSI in Germany and RAL in the UK, are described. The data acquisition process for each system is detailed, covering the reconstruction of the beam profile and the transverse emittances. Error analysis for both systems is presented, using a number of methods to estimate the emittance and associated errors.

## INTRODUCTION

The use of pepperpots in measuring transverse emittance is widespread. The pepperpot is unique in providing an instantaneous measurement of the 4 dimensional emittance of a beam in a single shot. To do so the pepperpot sacrifices position resolution by measuring the beam only at discrete intervals through an intercepting screen. With suitably fast analysing software, this provides the opportunity of measuring and visualising the emittance of the beam in real time. The disadvantage of using a pepperpot is that they are highly destructive to the beam, primarily due to the intercepting screen, and the discontinuous nature of the position measurement that results from segmenting the beam.

To fully categorise emittance measurement error, a first principles analysis of the propagation of errors through the calculation of *rms* emittance has been carried out. This results in a general formula for the calculation of errors from any method of emittance measurement. This error analysis procedure is demonstrated for two contrasting pepperpot designs.

## PEPPERPOT SYSTEMS

Error analysis has been carried out for two pepperpot systems: from the HITRAP project at GSI [1] and the Front End Test Stand (FETS) at RAL [2].

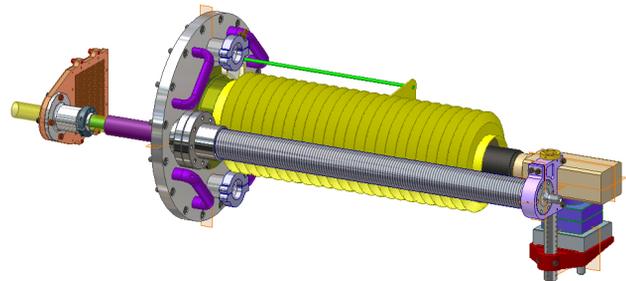


Figure 1: 3-D model of the FETS pepperpot assembly [3].

A CAD model of the FETS pepperpot assembly is shown in Fig. 1: full description of the FETS pepperpot device is given in [3]. The intercepting screen is a  $100\ \mu\text{m}$  thick tungsten foil with a square array of  $41 \times 41$  holes, each  $50 \pm 5\ \mu\text{m}$  in diameter, on a  $3 \pm 0.01\ \text{mm}$  pitch, giving a total imaging area of  $120 \times 120\ \text{mm}^2$ . The beam is imaged with a quartz scintillator, 10 mm from the tungsten screen, and a  $2048 \times 2048$  pixel PCO 2000 high speed camera: the camera-to-screen distance of 1100 mm gives a resolution of  $65\ \mu\text{m}$  per pixel and an angular resolution of 6.5 mrad. Data is recorded from the camera direct to a multi-image TIFF file and analysed with Matlab. Calibration is carried out using a series of calibration marks on the rear copper plate facing the camera: 4 lines, forming a  $125\ \text{mm} \times 125\ \text{mm}$  square around the intercepting screen, provide the necessary calibration information on the size, location and rotation of the pepperpot holes.

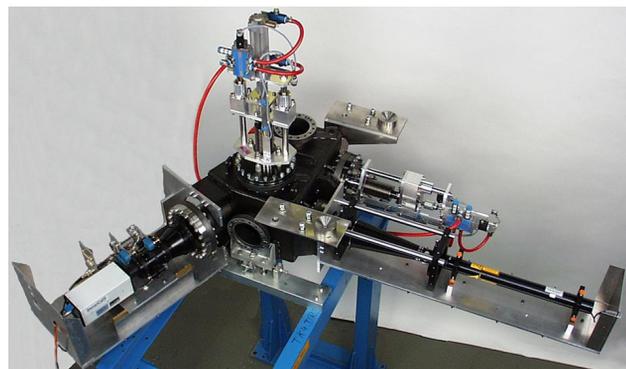


Figure 2: The HITRAP pepperpot setup (*cf.* [5]).

The setup of the GSI pepperpot system for the HITRAP

experiment is outlined in [4], with recent modifications detailed in [5]: the complete system is shown in Fig. 2. The pepperpot can be equipped with various intercepting screens: for recent measurements a 100  $\mu\text{m}$  thick tungsten foil was used, with an array of  $19 \times 19$  holes, each  $100 \pm 10 \mu\text{m}$  in diameter, on a  $1.6 \pm 0.02 \text{ mm}$  pitch, to image an area of  $29 \times 29 \text{ mm}^2$ . An  $\text{Al}_2\text{O}_3$  scintillator is mounted 150 mm from the screen: pepperpot images are recorded with a cooled fast shutter CCD camera with a  $1280 \times 1024$  pixel resolution, giving an angular resolution of 0.3 mrad. A laser is used to calibrate the system: an image is recorded of the resulting light spots on the imaging screen, these calibration spots are projected to horizontal and vertical axes and the maxima are defined as calibration positions for each individual row and column.

## EMITTANCE ERROR ANALYSIS

In order to calculate an error on the emittance, it is necessary to derive an emittance error formula by propagating the errors on each measured quantity through the formula for emittance. The *rms* emittance is used as it is mathematically well defined, allowing such a first-principles approach to be used. Such an approach is valid only if the errors on each variable are also well defined: this is addressed in the next section. In the  $x$ -plane, the definition of  $\varepsilon_{rms}$  is:

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \quad (1)$$

$$= \sqrt{\frac{\left( \sum_{i=1}^N \rho_i x_i^2 \right) \left( \sum_{j=1}^N \rho_j x_j'^2 \right) - \left( \sum_{i=1}^N \rho_i x_i x_i' \right)^2}{\left( \sum_{i=1}^N \rho_i \right)^2}} \quad (2)$$

To calculate an error on the emittance, a variance,  $\sigma_{\varepsilon_{rms}}^2$ , and hence a standard deviation,  $\sigma_{\varepsilon_{rms}}$ , must be derived. The variance on one term of the  $\sum_{i=1}^N \rho_i x_i^2$  series is given by:

$$\sigma_{\rho x^2}^2 = 4\rho^2 x^2 \sigma_x^2 + x^4 \sigma_\rho^2 \quad (3)$$

Since, for addition, variances also add, the variance for the complete series,  $\sigma_{\sum \rho x^2}^2$  is:

$$\sigma_{\sum \rho x^2}^2 = \sum_{i=1}^N (4\rho_i^2 x_i^2 \sigma_{x_i}^2 + x_i^4 \sigma_{\rho_i}^2) \quad (4)$$

...since every position measurement,  $x_i$ , has its own error,  $\sigma_{x_i}$ , and each intensity measurement,  $\rho_i$ , has its own error  $\sigma_{\rho_i}$ . The same calculation holds true for  $\sum_{j=1}^N \rho_j x_j'^2$  and its associated variance,  $\sigma_{\sum \rho x'^2}$ :

$$\sigma_{\rho x'^2}^2 = 4\rho^2 x'^2 \sigma_{x'}^2 + x'^4 \sigma_\rho^2 \quad (5)$$

$$\sigma_{\sum \rho x'^2}^2 = \sum_{j=1}^N (4\rho_j^2 x_j'^2 \sigma_{x_j'}^2 + x_j'^4 \sigma_{\rho_j}^2) \quad (6)$$

The variance for the product of these two terms,  $\sigma_{\sum \rho x^2 \sum \rho x'^2}$ , is given by:

$$\begin{aligned} \sigma_{\sum \rho x^2 \sum \rho x'^2}^2 &= \left( \sum_{i=1}^N 4\rho_i^2 x_i^2 \sigma_{x_i}^2 + x_i^4 \sigma_{\rho_i}^2 \right) \left( \sum_{j=1}^N \rho_j x_j'^2 \right)^2 \\ &+ \left( \sum_{i=1}^N 4\rho_i^2 x_i'^2 \sigma_{x_i'}^2 + x_i'^4 \sigma_{\rho_i}^2 \right) \left( \sum_{j=1}^N \rho_j x_j^2 \right)^2 \end{aligned} \quad (7)$$

Two more variances are needed: for the  $xx'$  and  $\rho^2$  terms in Eqn. 2. Following the same procedure:

$$\sigma_{\rho xx'}^2 = x^2 x'^2 \sigma_\rho^2 + \rho^2 x'^2 \sigma_x^2 + \rho^2 x^2 \sigma_{x'}^2 \quad (8)$$

$$\sigma_{\sum \rho xx'}^2 = \sum_{i=1}^N \left( x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2 \right) \quad (9)$$

$$\begin{aligned} \sigma_{(\sum \rho xx')^2}^2 &= 4 \left( \sum_{i=1}^N x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2 \right) \\ &\times \left( \sum_{j=1}^N \rho_j x_j x_j' \right)^2 \end{aligned} \quad (10)$$

And:

$$\sigma_{\sum \rho}^2 = \sum_{i=1}^N \sigma_{\rho_i}^2 \quad (11)$$

$$\sigma_{(\sum \rho)^2}^2 = 4 \left( \sum_{i=1}^N \sigma_{\rho_i}^2 \right) \left( \sum_{j=1}^N \rho_j \right)^2 \quad (12)$$

As such, the variance for the numerator in Eqn. 2 is:

$$\begin{aligned} \sigma_{\sum \rho x^2 \sum \rho x'^2 - (\sum \rho xx')^2}^2 &= \left( \sum_{i=1}^N 4\rho_i^2 x_i^2 \sigma_{x_i}^2 + x_i^4 \sigma_{\rho_i}^2 \right) \left( \sum_{j=1}^N \rho_j x_j'^2 \right)^2 \\ &+ \left( \sum_{i=1}^N 4\rho_i^2 x_i'^2 \sigma_{x_i'}^2 + x_i'^4 \sigma_{\rho_i}^2 \right) \left( \sum_{j=1}^N \rho_j x_j^2 \right)^2 + \\ &4 \left( \sum_{i=1}^N x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2 \right) \left( \sum_{j=1}^N \rho_j x_j x_j' \right)^2 \\ &- 8 \left( \sum_{i=1}^N \rho_i^2 x_i^4 x_i'^4 \sigma_{\rho_i}^2 + \rho_i^4 x_i^2 x_i'^4 \sigma_{x_i}^2 + \rho_i^4 x_i^4 x_i'^2 \sigma_{x_i'}^2 \right) \end{aligned} \quad (13)$$

The subtraction comes about through the cancellation of terms in Eqn. 2. Propagating the errors through the division and square root gives a variance on  $\varepsilon_{rms}$ ,  $\sigma_{\varepsilon_{rms}}^2$ :

$$\begin{aligned} \sigma_{\varepsilon}^2 = & \frac{\left( \sum_{i=1}^N \rho_i^2 x_i^2 \sigma_{x_i}^2 + \frac{x_i^4 \sigma_{\rho_i}^2}{4} \right) \left( \sum_{j=1}^N \rho_j x_j'^2 \right)^2}{\varepsilon^2 \left( \sum_{k=1}^N \rho_k \right)^4} \\ & + \frac{\left( \sum_{i=1}^N \rho_i^2 x_i'^2 \sigma_{x_i'}^2 + \frac{x_i'^4 \sigma_{\rho_i'}^2}{4} \right) \left( \sum_{j=1}^N \rho_j x_j^2 \right)^2}{\varepsilon^2 \left( \sum_{k=1}^N \rho_k \right)^4} \\ & + \frac{\left( \sum_{i=1}^N x_i^2 x_i'^2 \sigma_{\rho_i}^2 + \rho_i^2 x_i'^2 \sigma_{x_i}^2 + \rho_i^2 x_i^2 \sigma_{x_i'}^2 \right) \left( \sum_{j=1}^N \rho_j x_j x_j' \right)^2}{\varepsilon^2 \left( \sum_{k=1}^N \rho_k \right)^4} \\ & - \frac{2 \left( \sum_{i=1}^N \rho_i^2 x_i^4 x_i'^4 \sigma_{\rho_i}^2 + \rho_i^4 x_i^2 x_i'^4 \sigma_{x_i}^2 + \rho_i^4 x_i^4 x_i'^2 \sigma_{x_i'}^2 \right)}{\varepsilon^2 \left( \sum_{k=1}^N \rho_k \right)^4} \\ & + \frac{\varepsilon^2 \left( \sum_{i=1}^N \sigma_{\rho_i}^2 \right)}{\left( \sum_{k=1}^N \rho_k \right)^2} \end{aligned} \quad (14)$$

The error on the *rms* emittance,  $\sigma_{\varepsilon_{rms}}$ , is the square root of this value.

## PEPPERPOT EMITTANCE ERRORS

The next stage is to identify the errors on each measured quantity and use these to calculate  $\sigma_{\varepsilon_{rms}}$ . For pepperpot measurements, the dominant errors are: the spacing of the holes for  $\sigma_x$ ; the camera resolution and pepperpot-to-scintillator distance – defining the angular resolution – for  $\sigma_{x'}$ ; and the inherent beam variation and signal noise in the measurement apparatus for  $\sigma_{\rho}$ . Certain errors, such as the shape and diameter of the holes, contribute to both  $\sigma_x$  and  $\sigma_{\rho}$ : however, the dominant contribution to  $\sigma_x$  is clearly the hole spacing, and careful analysis of the hole size would allow this error to be removed. It is assumed that the hole size is smaller than the relative pixel size of the camera, allowing simple angles to be calculated through ray tracing, and that the camera orientation defines the beam orientation. As such, calibration errors contribute only to errors on  $\sigma_{x'}$ .

The emittance values derived from measurements for the two systems, along with the corresponding error estimate for each contributing error, is shown in Table 1. Where errors are negligible, or error analysis has not been carried out, no figure is shown. For the FETS system, an additional angle error of  $\sim 1$  mrad per mm is included due to the inaccuracy of the calibration. Two sources of intensity error are considered: beam noise, corresponding to the stochastic pulse-to-pulse variation in the beam, and the noise floor,

Table 1: Percentage Emittance Error Contributions for Pepperpot Measurements

	FETS		HITRAP	
	Value	$\sigma$ (%)	Value	$\sigma$ (%)
Beam radius (mm)	45	–	17	–
$\varepsilon_x$ ( $\pi$ mm mrad)	0.61	–	0.24	–
Hole spacing (mm)	3	1.8	1.6	2.2
Angle res. (mrad)	6.5	1.6	0.3	0.2
Beam noise (%)	10	1.3	10	0.3
Noise floor (%)	2	$\sim 0$	10	1.2
$\sigma_{\varepsilon}$ ( $\pi$ mm mrad)	0.029	4.8	0.010	3.9

a pessimistic figure representing the constant level of background noise (quoted as a percentage of the maximum signal). For the FETS system, each source of error contributes approximately equally to the final error figure of  $\pm 4.8\%$ : For the GSI system, the dominant error is clearly the hole spacing, with a contribution from the background noise: the angle resolution is considerably better than the FETS system and this is reflected in the error values. An interesting effect is that the beam noise contributes significantly more for the FETS system but is dominated by the noise floor in the HITRAP system: this is a result of the smaller beam and lower light intensity producing a less intense pepperpot image for the HITRAP pepperpot. This also contributes to the larger position error.

## CONCLUSIONS

The formula for calculating *rms* emittance errors has been applied successfully to 2 different pepperpot setups, with promising results. Further work is required to categorise errors not included in this analysis, since these affect the accuracy of the emittance measurement while not contributing to the error estimate. This has particular importance when dealing with cut selection, something dealt with in considerable detail by the SCUBEE algorithm (see [6] and Refs therein). As such, this method constitutes a minimum estimate of the emittance error.

## ACKNOWLEDGEMENTS

The authors gratefully acknowledge the contributions of Roger Barlow, Louis Lyons and Martin Jolly in the analysis and derivation of emittance errors.

## REFERENCES

- [1] J Pfister *et al.*, EPAC'08, THPP037, p. 3449.
- [2] A Letchford *et al.*, EPAC'08, THPP029, p. 3437.
- [3] S Jolly *et al.*, DIPAC'07, WEO2A01, p. 218.
- [4] T Hoffmann *et al.*, BIW 2000, AIP Conf. Proc. 546, p. 432.
- [5] J Pfister *et al.*, LINAC'08, TUP074, p. 555.
- [6] M Stockli *et al.*, Rev. Sci. Instrum. **75** (2004) 1646.