ON THE LIMITATIONS OF LONGITUDINAL PHASE SPACE MEASUREMENTS USING A TRANSVERSE DEFLECTING STRUCTURE

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Abstract

High-brightness electron bunches with low energy spread, small emittance and high peak currents are the basis for the operation of high-gain Free-Electron Lasers (FELs). As only part of the longitudinally compressed bunches contributes to the lasing process, time-resolved measurements of the bunch parameters are essential for the optimisation and operation of the FEL. Transverse deflecting structures (TDS) have been proven to be powerful tools for time-resolved measurements. Operated in combination with a magnetic energy spectrometer, the measurements of the longitudinal phase space can be accomplished. Especially in case of ultra-short electron bunches with high peak currents for which a time resolution on the order of 10 fs would be desireable, both the TDS and magnetic energy spectrometer have intrinsic limitations on the attainable resolution. In this paper, we discuss the fundamental limitations on both the time and energy resolution, and the relation between them.

INTRODUCTION

Recently developed Free-Electron Lasers for the generation of photons in the extreme ultraviolet and soft X-ray regime are based on an exponential gain of the radiation power in a single pass through a long undulator magnet system. These high-gain FELs put stringent demands on the electron bunch parameters. In order to initiate the lasing process and to reach power saturation in reasonable undulator lengths, a high charge density which is related to the peak current, a low energy spread, and a small emittance is mandatory.

Projected measurements of the electron bunch parameters are not sufficient to understand and to control the lasing process. For this reason, sliced electron bunch measurements are essential, which can be accomplished by Transverse Deflecting Structures (TDS). In combination with a magnetic energy spectrometer, the longitudinal phase space can be investigated.

In order to get the most information concerning the lasing process, the measurements may be carried out in front of the undulators. This is very challenging since the electron bunches are, in particular in front of the undulators, very short with bunch lengths in the femtosecond range and peak currents on the order of kiloampere.

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GENERIC BEAMLINE LAYOUT

An experimental layout for sliced beam parameter measurements is presented schematically in Fig. 1. The entire beamline is equipped with quadrupole magnets to ensure the required optics for standard machine operation as well as for dedicated sliced beam parameter measurements. For operation without affecting the entire bunch train, e.g. bunch profile measurements, a fast kicker can be used to pick out individual bunches for off-axis screen operation. The magnetic energy spectrometer consists of at least one dipole magnet, followed by a drift section for building up dispersion.

Further requirements are imaging screens, e.g. OTR-screens or scintillators, and optical camera systems providing high spatial resolutions.

ACCELERATOR OPTICS

In order to obtain desired time[1] and energy resolutions, the accelerator optics has to be designed and adapted according to the following considerations.

In linear beam dynamics, the general transverse motion of charged particles can be described as combination of betatron motion and dispersion trajectory. The deflection plane of the energy spectrometer is assumed to be in the horizontal, e.g. the x-plane. The horizontal particle motion is then given by

$$x(s) = x_\beta(s) + D_x(s) \cdot \delta \,, \tag{1}$$

with the horizontal betatron motion $x_\beta(s)$, horizontal dispersion function $D_x(s)$, and relative momentum deviation δ. After passing the magnetic energy spectrometer, the horizontal rms beam size at screen location s_1 can be expressed by $\sigma_x = \sqrt{\sigma_{x_\beta}^2 + D_x^2 \cdot \sigma_\delta^2}$. In order to achieve energy resolutions on the order of σ_δ, the beam size due to dispersion and energy spread has to be larger than the natural rms beam size $\sigma_{x_\beta} = \sqrt{\epsilon_x \cdot \beta_x}$. This condition yields

$$D_x(s_1) \cdot \sigma_\delta > \sqrt{\epsilon_x \cdot \beta_x(s_1)} \,, \tag{2}$$

with the horizontal geometric emittance ϵ_x and beta function β_x. The reachable energy resolution is then given by

$$\sigma_\delta > \sqrt{\epsilon_x} \cdot \frac{\sqrt{\beta_x(s_1)}}{D_x(s_1)} \,. \tag{3}$$

[1] The time resolution expresses the resolution of the longitudinal coordinate ζ within the bunch. The relation is given by $t = \zeta/c$.

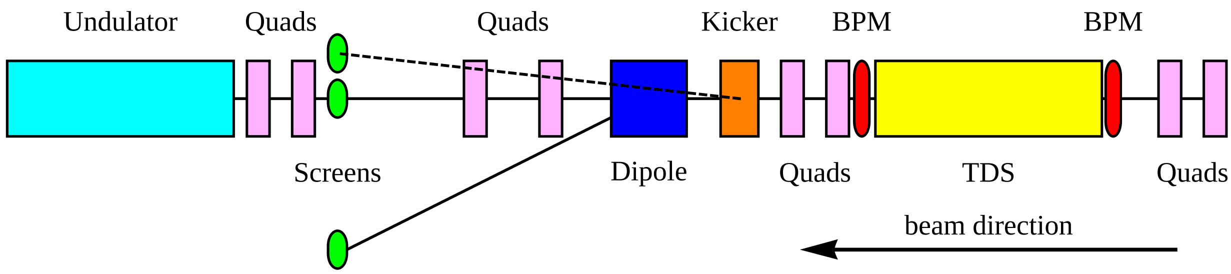

Figure 1: Beamline layout for dedicated sliced beam parameter measurements and investigations of the longitudinal phase space. The different components are labelled and assigned to colours. The electron beam direction is indicated by the arrow (from right to left).

For using a TDS in combination with a magnetic energy spectrometer, the deflection induced by the TDS has to be in the vertical. In case of a LOLA-type TDS [3], the particle motion is given by [1, 2]

$$y(s) = y_\beta(s) + S_y(s) \cdot \left(\zeta + \frac{\sin(\Psi)}{k \cdot \cos(\Psi)} \right) , \quad (4)$$

with the streak or shear function

$$S_y(s) = \sqrt{\beta(s_0)\beta(s)} \cdot \sin(\Delta\Phi_y) \cdot \frac{eV_0 k}{pc} \cdot \cos(\Psi) , \quad (5)$$

and the internal longitudinal bunch coordinate ζ. The parameters V_0 and k denote the deflecting voltage and the wavenumber. The beam energy is given by $E \approx pc$, and $\Delta\Phi_y$ is the vertical phase advance from location s_0 to s. The expression in (4) is valid for the approximation that the TDS is considered as to be a drift section with an instantaneous deflection at the centre of the structure. In order to avoid centroid deflections and achieve best resolution, the TDS is assumed to be operated near zero-crossing, i.e. the RF phase is set to $\Psi \approx 0$. Thereby, Eq. (4) is reduced to

$$y(s) = y_\beta(s) + S_y(s) \cdot \zeta , \quad (6)$$

which is of similar form as Eq. (1). In order to achieve a longitudinal resolution on the order of σ_ζ, the beam size at screen location s_1 due to streaking with the TDS has to be larger than the natural beam size due to betatron motion. This yields the condition

$$S_y(s_1) \cdot \sigma_\zeta > \sqrt{\epsilon_y \cdot \beta_y(s_1)} . \quad (7)$$

By using Eq. (5) and $\Psi \approx 0$, the attainable longitudinal resolution is then given by

$$\sigma_\zeta > \frac{\sigma_{y_\beta}(s_1)}{S(s_1)} = \frac{\sqrt{\epsilon_y} \cdot pc}{\sqrt{\beta(s_0)} \cdot \sin(\Delta\Phi_y) \cdot eV_0 k} . \quad (8)$$

TRANSVERSE DEFLECTING STRUCTURE

The considerations in the previous section represent the attainable resolutions for sliced beam parameter measurements in terms of linear beam dynamics. More detailed studies of the particle dynamics within the TDS reveal additional resolution limitations.

The physics of RF deflectors originates from the Panofsky-Wenzel theorem [4], which makes a general statement of the transverse momentum gained by fast particles moving through RF fields. The theorem states

$$\Delta\vec{p}_\perp = \left(\frac{e}{\omega}\right) \int_0^L (-i)\nabla_\perp E_z dz , \quad (9)$$

and it follows that transverse deflection is only possible if a transverse gradient of the longitudinal electric field $\nabla_\perp E_z$ is present. From $\nabla_\perp E_z \neq 0$ follows the existence a longitudinal accelerating field E_z. In case of uniform deflection over the whole aperture, i.e. abberation-free, it follows that E_z depends linearly on the transverse coordinates. Assuming that the field E_z can be factorised by $E_{z,\parallel}(z,t) \cdot E_{z,\perp}(x,y)$, the relative momentum gain within a TDS of length L can be expressed by

$$\delta = \frac{\Delta p}{p} = \left(-\frac{e}{pc} \int_0^L E_{z,\parallel} dz \right) \cdot E_{z,\perp} . \quad (10)$$

For abberation-free deflection, this leads to additional energy spread. In case of the LOLA-type TDS for vertical deflection, the longitudinal electric field is given by [3, 2]

$$E_z(x,y,z,t) = E_0\, k\, y \cos(\Psi(z,t)) . \quad (11)$$

For zero-crossing operation ($\Psi \approx 0$), the evaluation of Eq. (10) yields

$$\delta = \frac{eV_0 k}{pc} \cdot y , \quad (12)$$

with the equivalent deflecting voltage $V_0 = E_0 L$. Using Eq. (6), the additional energy spread induced by the TDS splits into two parts:

$$\delta = \frac{eV_0 k}{pc} \cdot y_\beta + \frac{eV_0 k}{pc} \cdot S \cdot \zeta . \quad (13)$$

The first part results in an rms energy spread given by

$$\sigma_\delta = \frac{eV_0 k}{pc} \cdot \sigma_y , \quad (14)$$

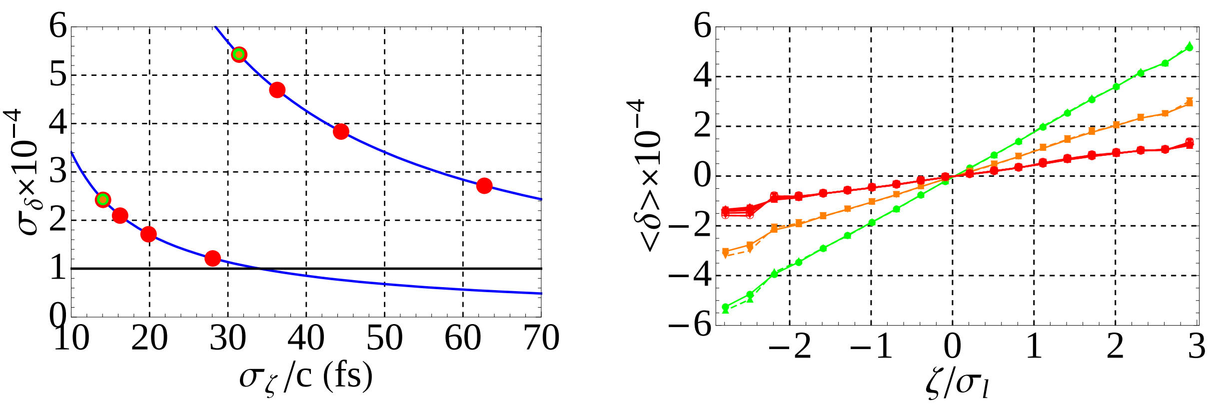

Figure 2: Left: Induced energy spread as a function of the longitudinal resolution. Dots indicate the results of simulations with the code `elegant`; blue lines represent the expression in Eq. (16). Right: Mean energy spread along the bunch for the cases in the left plot with the same colour coding. V_0: 20 MV (red), 30 MV (orange), and 40 MV (green).

whereas the second part corresponds to a linear energy correlation along the bunch. By means of the shear function in Eq. (5) and the vertical rms beam size in the TDS, the induced energy spread can be expressed by

$$\sigma_\delta = \frac{S(s_1)}{\sqrt{\beta_y(s_1)\beta_y(s_0)}\sin(\Delta\Phi)} \cdot \sqrt{\epsilon_y \cdot \beta_y(s_0)}\,. \qquad (15)$$

With the longitudinal resolution introduced in Eq. (7), it follows

$$\sigma_\delta \cdot \sigma_\zeta > \frac{\epsilon_y}{\sin(\Delta\Phi)}\,. \qquad (16)$$

Evaluating the second part of (13) with Eq. (5) and the assumption that the beta function β_y is almost constant in the TDS, yields

$$< \delta(\zeta) > \; \sim \; L \cdot \left(\frac{eV_0 k}{pc}\right)^2 \cdot \zeta\,, \qquad (17)$$

which gives the mean energy spread along the bunch. It depends quadratically on the deflecting voltage V_0 and is not affected by the beta function β_y. In order to check the relations in (16) and (17), simulations using the code `elegant` [5] were performed. The simulation and Gaussian bunch parameters are listed in Table 1. The quoted beta functions represent the mean values within the TDS. According to the longitudinal resolutions σ_ζ/c given in Table 1 and the expression (16), the TDS induces energy spreads which are shown as dots in the left plot of Fig. 2. The lines are plotted according to (16) for a phase advance of $\Delta\Phi = \pi/2$ and two different emittance values. For each simulation point, also the mean energy spread is shown in the right plot. Three combinations of β_y and V_0 give the same resolutions, but due the dependency in Eq. (17), different slopes of the mean energy spread along the bunch are generated.

CONCLUSIONS

We have presented the required optics conditions in order to achieve a desired time and energy resolution for sliced beam parameter measurements. In case of the LOLA-type TDS, we have also shown that there exists a basic relation between the attainable energy spread and time resolution for longitudinal phase space measurements.

Table 1: Simulation and Initial Gaussian Bunch Parameters: beam energy $E = 1$ GeV, bunch length $\sigma_l = 50\ \mu$m, initial energy spread $\delta = 1 \cdot 10^{-4}$, normalised emittance $\epsilon_{N,y} = \epsilon_y \beta\gamma$, TDS wavenumber $k \approx 60\ \text{m}^{-1}$ and length $L = 3.826$ m.

β_y	V_0	σ_ζ/c $\epsilon_{N,y} = 2\ \mu$m	σ_ζ/c $\epsilon_{N,y} = 10\ \mu$m
10 m	20 MV	29 fs	63 fs
20 m	20 MV	20 fs	45 fs
30 m	20 MV	17 fs	37 fs
40 m	20 MV	15 fs	32 fs
17.78 m	30 MV	15 fs	32 fs
10 m	40 MV	15 fs	32 fs

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