VELOCITY OF SIGNAL DELAY CHANGES IN FIBRE OPTIC CABLES

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Abstract

Most timing systems used for particle accelerators send their time or reference signals via optical single mode fibres embedded in cables. An important question for the design of such systems is how fast the delay changes in the fibre optic cable take place, subject to the variation of the ambient air temperature. If this information is known, an appropriate method for delay compensation can be chosen, to enable a phase stabilised transmission of the timing signals. This is of interest particularly with regard to RF synchronisation applications.

To characterise the velocity of the delay change, the delay behaviour after a sudden temperature change will be described.

When trying to determine the step response, two problems occur. On the one hand, the material parameter of the coating, necessary for the calculation, is typically unknown. On the other hand, the measurement of the step response under realistic conditions is very laborious.

Thus in this presentation it will be shown how the step response and, accordingly, the velocity of the delay change in a fibre optic cable can be calculated by means of theoretical considerations, utilizing the typical geometry of fibre optic cables.

INTRODUCTION

The following analysis was performed as part of the development of the timing system for the cavity synchronisation of the Facility for Antiproton and Ion Research (FAIR) [1-3]. Loose tube cables frequently used in telecommunications are considered. The coating of this cable type can be described as a tube in which the standard single mode fibres (SMF) are loosely inserted (Fig. 1). The fibres and the coating are thus mechanically decoupled. This and the fact that the fibres are about 2% longer than the cable coating means that the tension on the cable to a certain extent does not act on the fibres and thus does not cause any delay change (see Eq. (3)). Accordingly, only temperature fluctuations are responsible for a delay change.

In the following, a way of calculating analytically not only the absolute delay change but also the speed at which such change takes place is presented for the first time.

ABSOLUTE DELAY CHANGE

Signals need the group delay τ to pass through a fibre of length L and group index N_g [4]

$$\tau = \frac{N_g \cdot L}{c}, \tag{1}$$

wherein c stands for the speed of light in a vacuum. With a transmission length of L = 1 km and a group index of $N_g \approx 1.5$, delays of approximately 5 µs, for example, occur. According to [4], a change in the group delay can have two causes. First of all, fluctuations in the temperature T

$$\frac{d\tau}{L \cdot dT} = \frac{1}{c}\left(N_g \frac{dL}{L \cdot dT} + \frac{dN_g}{dT} \right) \tag{2}$$

and secondly, mechanical tension σ acting on the length of the fibre

$$\frac{d\tau}{L \cdot d\sigma} = \frac{1}{c}\left(N_g \frac{dL}{L \cdot d\sigma} + \frac{dN_g}{d\sigma} \right). \tag{3}$$

Both bring about both a change in the length of the fibre and a change in the group index. For a non-jacketed fibre this results, according to Eq. (2) with N_g = 1.4682 at a wavelength of 1550 nm [5], the length expansion coefficient $dL/(L \cdot dT) = 5.6 \cdot 10^{-7}$ K^{-1} and the temperature coefficient of the group index of $dN_g/dT = 1.2 \cdot 10^{-5}$ K^{-1} [6], in a value of

$$\left(\frac{d\tau}{L \cdot dT} \right)_{fibre} = 40 \frac{ps}{km \cdot K}. \tag{4}$$

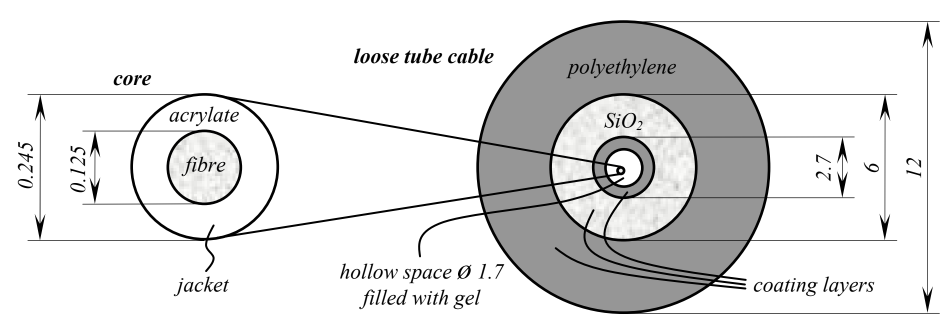

Figure 1: Cross-section of a loose tube fibre optic cable.

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In loose tube cables, however, the fibres are frequently provided with thin jackets (Fig. 1, left), i.e. hollow space of the cable contains tight buffeted cables. According to [4], in tight buffeted cables tensions occur between the fibre and the jacket because of their different temperature expansion coefficients. The tension acting on the fibre in turn, according to Eq. (3) brings about a change in the delay. For the resulting impact on a jacketed fibre,

$$\left(\frac{d\tau}{L\cdot dT}\right)_{jacketed} = \frac{A_j}{A_f}E_j k_j\left(\frac{d\tau}{L\cdot d\sigma}\right)_{fibre} + \left(\frac{d\tau}{L\cdot dT}\right)_{fibre} \quad (5)$$

is obtained, with the cross-sectional areas A_j and A_f of the jacket as well as the fibre, the modulus of elasticity E_j and the linear expansion coefficient k_j of the jacket*. Here, the product $E_j k_j$ represents the change in tension per Kelvin acting on the fibre in a proportion of A_j/A_f. The extent to which delay depends on the temperature is thus greatly influenced by the properties of the fibre jacket. With so-called phase-stable optical fibres [7], the jacket exhibits a negative expansion co-efficient. As a result, the first addend in Eq. (5) becomes negative and partly compensates the second addend. As a result, the function of temperature falls below the value in Eq. (4) exhibited by the non-jacketed fibres.

By way of specific example, delay as a function of temperature will be calculated with Eq. (5) for the conventional loose tube cable shown in Fig. 1. Assuming the following material parameters of the jacket of $E_j = 3.3\cdot 10^{-5}$ N/mm^2 and $k_j = 7.7\cdot 10^{-5}$ K^{-1} (Tab. 1), a value of

$$\left(\frac{d\tau}{L\cdot dT}\right)_{core} = \left(\frac{d\tau}{L\cdot dT}\right)_{cable} = 76\frac{ps}{km\cdot K} \quad (6)$$

results.

If it is now possible to succeed in determining the temperature curve in the fibre after a sudden change in ambient air temperature, the delay change in the cable over time and thus the step response can be determined in conjunction with Eq. (5).

SPEED OF DELAY CHANGE

An exact calculation is not possible since, firstly, the cable manufacturers did not deliver any data on the relevant material parameters and, secondly, the values found in the literature in some cases differ widely (Tab. 1). The only approach that can be taken here, therefore, is to estimate the speed range of the change. In the course of establishing this, it will however be revealed that this estimation is conservative, making it possible to say which value with certainty will not be exceeded.

Two key variables throttle the speed at which the delay changes after a rise in the temperature of the ambient air: the thermal resistance and the thermal capacity of the cable. Heat transfer resistance from the ambient air to the cable can be calculated by [8, p. 315]

$$R_{a\to c} = \frac{1}{\alpha 2\pi r_c L} \quad (7)$$

Here, α is the heat transfer coefficient, r_c stands for the radius and L for the length of the cable. The thermal resistance of the various layers from which the cable is made is calculated according to [8, p. 318]

$$R_{l,n} = \frac{1}{\kappa_n 2\pi L}\ln\left(\frac{r_{e,n}}{r_{i,n}}\right), \quad (8)$$

with thermal conductivity κ, the external radius r_e and the inner radius r_i of the respective layer n. It is assumed that the ambient air moves slightly† at a speed of 2 m/s and thus amounts to $\alpha = 13.6$ W/(m^{2}K) [8, p. 637]. Moreover, $r_c = 6$ mm and $\kappa \geq 0.33$ W/(m·K) for all coating layers. Under these conditions,

$$R_{a\to c} >> R_{l,n} \quad (9)$$

and also significantly greater than the transfer resistances between the layers. By way of approximation, therefore, all thermal resistances except $R_{a\to c}$ are treated as negligible, as a result of which the speed at which the complete cable is heated is underestimated. The heating period now depends only on $R_{a\to c}$ and the thermal capacity of the entire cable, the latter representing the sum of the thermal capacities of all individual layers

$$C_c = \sum_{n=1}^{N} c_{sh,n}\rho_n V_n , \quad (10)$$

wherein c_{sh} is specific thermal capacity, ρ density and V the volume of the respective layer n. The product $c_{sh}\cdot\rho$ differs only slightly for the materials typically used and is the lowest in silica glass (see Tab. 1)

$$\left(c_{sh}\rho\right)_{SiO_2} = 1.66\cdot 10^6\frac{Ws}{m^3K}. \quad (11)$$

For Eq. (10), the following can thus be written

$$C_c \geq \left(c_{sh}\rho\right)_{SiO_2} V \quad (12)$$

by which the variable C_c was attributed to two known variables. The heating process for the cable now behaves similar to the charging process of a capacitor connected in series to an ohmic resistance. For the time curve of the temperature change in the cable ΔT_c after a sudden rise in the ambient temperature by ΔT_a ,

$$\Delta T_c\left(t\geq 0\right) = \Delta T_a\left(1-e^{-\frac{t}{R_{a\to c}C_c}}\right), \quad (13)$$

* In this approximation, [4] assumes that the modulus of elasticity of the fibre is substantially greater than that of the jacket and the expansion coefficient of the fibre substantially smaller than that of the jacket. This is the case, e.g., with acrylate and polyethylene. Note that the parameters of the fibre are similar to those of fused silica (Tab. 1).

† Strong movements of the air in cable ducts and buildings in which the fibre optic cables are laid in the FAIR are unlikely.

Table 1: Parameter of materials used for fibre optic cables

	k [K^{-1}]	E [N/mm^2]	κ [$W/(m{\cdot}K)$]	c_{sh} [$J/(kg{\cdot}K)$]	ρ [kg/m^3]	$c_{sh}\rho$ [$kg/(m^3{\cdot}K)$]
SMF [6]	$5.6{\cdot}10^{-7}$					
Fused Silica SiO₂ [9]	$5.5{\cdot}10^{-7}$	71700	1.38	754	2200	$1.66{\cdot}10^{6}$
Acrylate [10]	$7{\cdot}10^{-5}$ to $7.7{\cdot}10^{-5}$	2400 to 3300	0.17 to 0.19	1400 to 1500	1190	$1.67{\cdot}10^{6}$ to $1.79{\cdot}10^{6}$
Polyethylene [10]	$1{\cdot}10^{-4}$ to $3.0{\cdot}10^{-4}$	100 to 300	0.33	1900 to 2300	920	$1.75{\cdot}10^{6}$ to $2.12{\cdot}10^{6}$
Aramid fibre [10]	$-2.0{\cdot}10^{-6}$	59000 to 124000	0.04	1400	1440	$2.02{\cdot}10^{6}$
Steel [8]	$1.6{\cdot}10^{-5}$	190000	15.00	500	7800	$3.90{\cdot}10^{6}$

results with the time constant of the cable $R_{a\to c}C_c$, for which the following can be written using Eq. (7) and (12)

$$R_{a\to c}C_c \geq \frac{\pi r_c^2 L}{\alpha 2\pi r_c L}\left(c_{sh}\rho\right)_{SiO_2} = \frac{r_c}{\alpha 2}\left(c_{sh}\rho\right)_{SiO_2}. \quad (14)$$

The step response of the delay change is now obtained by multiplying Eq. (13) by the absolute delay change $d\tau/(L{\cdot}dT)$ of the core[‡]

$$\left(\frac{d\tau}{L}\right)_{cable} = \Delta T_a\left(1-e^{-\frac{t}{R_{a\to c}C_c}}\right)\left(\frac{d\tau}{L\cdot dT}\right)_{core}. \quad (15)$$

To determine the speed of the delay change, Eq. (15) is differentiate with respect to the time t

$$\left(\frac{d\tau}{L\cdot dt}\right)_{cable} = \frac{\Delta T_a}{R_{a\to c}C_c}e^{-\frac{t}{R_{a\to c}C_c}}\left(\frac{d\tau}{L\cdot dT}\right)_{core}. \quad (16)$$

In this expression we use Eq. (14) as well as $t = 0$ and obtain the maximum delay change speed

$$\left(\frac{d\tau}{L\cdot dt}\right)_{\substack{\max\\cable}} \leq \frac{2\alpha}{\left(c_{sh}\rho\right)_{SiO_2}}\frac{\Delta T_a}{r_c}\left(\frac{d\tau}{L\cdot dT}\right)_{core}. \quad (17)$$

According to this conservative estimate it can be said that, as the diameter of the cable becomes larger, the speed of the change in the temperature and thus also the delay change in the fibre decreases linearly. In the specific transmission cable in Fig. 1, the following results for a sudden rise in ambient temperature from ΔT_a = 1 K and the value in Eq. (6)

$$\left(\frac{d\tau}{L\cdot dt}\right)_{\substack{\max\\cable}} \leq 0.208\frac{ps}{km\cdot K}. \quad (18)$$

If the fibre were to have only the directly contacting acrylate jacket, the delay, assuming the same sudden rise in temperature, would change at ≤ 10.2 ps/(km·s) or 49 times faster. By choosing a thick-coated loose tube cable, the speed of the delay change after a sudden rise in temperature can be significantly throttled.

‡ $\Delta T_c = dT$

SUMMARY

A formula was established by means of which it is possible to calculate the speed of the delay change in a fibre optic cable. The thermal conductivity, specific thermal capacity and density of the cable coating material do not have to be known for this. Using the analytical formula, it is possible to calculate on the basis of the known cable geometry a value which, based on the approximations, with a high degree of probability will not be exceeded. Of key importance is that the change speed, assume an identical core, is approximately inversely proportional to the cable diameter.

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