ADVANCED DIAGNOSTICS OF LATTICE PARAMETERS IN HADRON COLLIDERS

J.- P. Koutchouk, CERN, Geneva, Switzerland

Abstract

With a beam stored energy exceeding by several orders of magnitude the quench level of the magnets and non-negligible non-linear field components, the control of the beam dynamics and losses in LHC must be very precise [1]. This is a strong incentive to strengthen as much as possible the potential of beam diagnostics. This paper reviews some of the developments in various laboratories that appear to have a large potential. They either allow for a much better access to classical beam parameters or for the measurement of quantities formerly not accessible. Examples are a fast measurement of the betatron tunes, the use of PLL for reliable tune tracking and feedback, new methods or ideas to measure the chromaticity with the potential of feedback systems and similarly for the betatron coupling, the measurement of high-order non-linear fields and resonances and the potential of AC dipole excitation. This list is bound to be incomplete as the field is fortunately very dynamic.

FAST TUNE MEASUREMENTS

The most common method for tune measurement is to Fourier transform the transverse oscillation of a beam after a transverse kick. If observed over N turns, the tune resolution is 1/2N. The tunes shall usually be controlled to some 0.001, requiring a measurement over 500 turns. This method has limitations. The decoherence due to non-linearities from various sources and the beating effect due to a non-vanishing chromaticity are often significant in the first 500 turns and degrade the accuracy. This is particularly the case when the purpose of the tune measurement is to fix a pathological situation which most often enhances the decoherence.

Methods have been available for some time to either improve the accuracy of the tune measurement for a given observation time or reduce the latter significantly to capture faster phenomena.

Interpolation in the frequency domain

The measurement of the linear parameters like the tunes only requires small amplitude beam oscillations. A simple Fourier Transform does not take advantage of the knowledge that this oscillation is a simple sine wave. It is however possible to re-interpret the spectrum in the light of this hypothesis. This leads to an analytic interpolation between the FFT lines on either side of the true tune, improving significantly the precision [2]:

$$Q_{true} = \frac{1}{N}\left(k + \frac{|\phi_{k+1}|}{|\phi_k| + |\phi_{k+1}|}\right), \quad \text{where } \phi_k \text{ and } \phi_{k+1}$$

are the harmonics of largest amplitude and k/N the FFT approximation of the tune. The tune accuracy improves like $1/N^2$ in the absence of noise.

Cross-correlation in the time domain

Instead of Fourier transforming the data set and interpolating, it is possible to cross-correlate the measured time series with a sinusoidal model using a guessed tune. For continuous signals, the tune difference between the theoretical and measured tunes would simply be given by the abscissa of the peak of the cross-correlation function. For sampled signals, the tune shift is derived from two or more values of the cross-correlation function [3]. The analytical evaluation shows that this algorithm is robust against noise, with an error proportional to $1/N\sqrt{N}$. This method was experimented in LEP.

Windowing in time domain

The transverse beam oscillation signal may be viewed as the product of an infinite sinusoidal signal with a rectangle window of length *N*. The resulting spectrum is the convolution of a Dirac function with the Fourier transform of the rectangle window i.e. merely the window spectrum shifted in frequency so that its main peak is centred on the tune. The signal power concentrated on the Dirac impulse of the Fourier transform of the infinite sine wave is spread over the main peak and the side lobes of the transform of the rectangle.

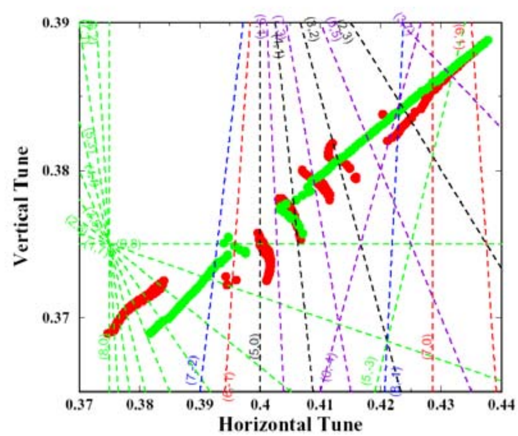

Figure 1: Experimental frequency map of the ESRF [5]

To study the non-linearities in the frequency domain, it is necessary to use the shortest possible window *N* (e.g. N<1000) and yet to measure the tune to high accuracy (e.g. 10^{-5}). Plotting the tunes for a range of oscillation amplitudes gives a representation of dynamical systems in the frequency space (Fig. 1). The method, developed for celestial mechanics [4] relies on *i)* windowing with a Hanning window of order 2, *ii)* interpolating in the frequency domain *iii)* followed by an iterative subtraction of the identified frequency components. The Hanning window significantly reduces the side lobes. In the absence of noise the tune accuracy improves like $1/N^3$. This spectacular improvement is somewhat reduced in practice for realistic noisy signals. The simulations [5] show that windowing still improves somewhat the accuracy as compared to a simple FFT. A fast and accurate tune measurement can be used to measure the frequency map (or tune footprint) of an accelerator or investigate the tune diffusion in time. The prominent resonances in ESRF [6] are put in evidence on Figure 1.

CONTINUOUS TUNE MEASUREMENT

The PLL method for tune tracking is certainly not new in its concept but remains a challenge for hadron colliders: The transverse beam excitation shall be small enough to prevent any significant emittance blow-up over long periods of use. The sensitivity of the pickup to transverse signals shall thus be large in presence of a strong parasitic longitudinal signal when the beam is not centered. The lock should not be lost when the beam transfer function

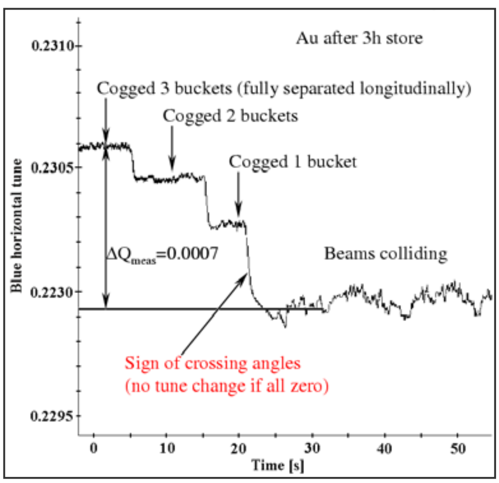

Figure2: PLL-based beam-beam tune shift measurement at RHIC [7]

suffers significant perturbations e.g. due to a change of the betatron frequency spread or betatron coupling. This is indeed the time where measurements are most useful.

Very interesting results have been obtained e.g. at RHIC [6]. The tune monitoring during the ramp provides a fast response with a moderate resolution (~0.001). Smaller loop gains yield a tune resolution of a few 10^{-6}. This is most useful for studies of the tune variations versus slowly (seconds) varying parameters. This principle was used e.g. for studies on the triplet non-linearity (this paper) and the beam-beam tune shift versus beam separation [7] (figure 2: the continuous variation of the beam-beam tune shift versus the crossing azimuth is a signature of a parasitic crossing angle). The PLL system has been found sensitive to variations of the BTF (jump of harmonics), to coupling… An international collaboration on PLL-based tune-meters has been launched [9].

ON-LINE CHROMATICITY MEASUREMENTS

The most natural way of measuring the chromaticity is by measuring the change of tune following a static momentum offset. The method has been upgraded by slowly modulating the RF frequency and tracking the tunes. However, to prevent longitudinal blow-up, the RF frequency modulation is generally very slow (versus the synchrotron period) in order not to interfere with the RF feedback loops. This does not allow tracking the chromaticity at the rate required for the LHC. Indeed, at the beginning of the acceleration ramp, a sudden reorganization of the persistent currents in the super-conducting material causes the chromaticity $Q' = \partial Q / \partial \delta$ to change at a rate of 1/s to 3/s. Faster chromaticity measurements (about 10 Hz for LHC) require faster methods for changing the beam momentum.

Measurement of the Head-Tail Phase Shift

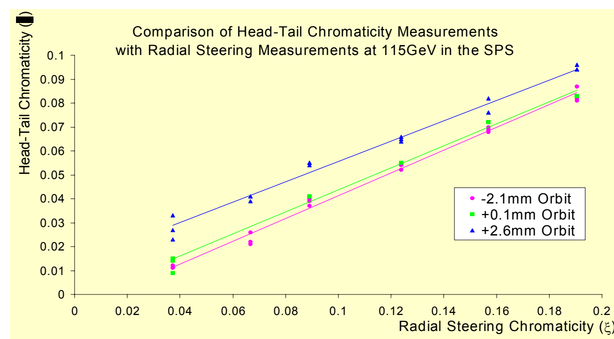

Figure 3: Head-tail versus classical Q' measurement [10]

This elegant method [10] [11] relies on the synchrotron oscillation to change the particle momentum. It is thus much faster than the above-mentioned methods.

Over half a synchrotron period, the particles forming initially the head will become the tail and vice-versa. In this half oscillation, the average momentum deviation of the particles moving from head to tail will be positive (above transition) while it will be negative for the particles moving from tail to head. If the chromaticity does not vanish, the betatron phase accumulated in the two cases will be different. The difference $\Delta\psi$ is proportional to the linear chromaticity and time interval $\Delta\tau$ between head and tail: $\Delta\hat{\psi} = (2\omega_0/\eta) Q' \Delta\tau$.

In order to detect a coherent signal, the beam must be kicked transversely and the head and tail of the bunch measured separately on a turn-by-turn basis to detect the phase slip. A discrepancy on the value of the chromaticity by a factor of 2.2 with respect to the classical method was lately resolved [11]. Figure 3 demonstrate the consistency with a classical measurement and Figure 4 shows the variation of the chromaticity versus momentum in the SPS. It reveals the presence of a third-order chromaticity. In LHC, this term is expected from the b_5 component of the main field which need to be corrected to 20% for a well-behaved beam dynamics.

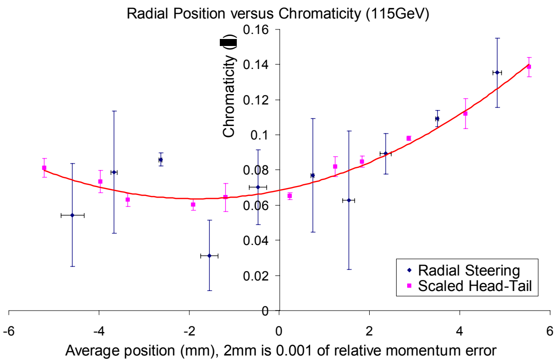

Figure 4: Non-linear chromaticity measured with head-tail oscillation [10]

Measurement via fast RF modulation

Another approach leading to a fast variation of the beam momentum is a fast modulation of the RF phase [12][13]. If this modulation is made outside of the beam frequencies, no dilution of the longitudinal emittance should occur. The modulation depth is limited by the corresponding reduction of the bucket area. This reduction shall not produce a leakage of particles outside the bucket nor create a synchrotron tune spread covering the excitation frequency. For the LHC, the resulting momentum modulation is of the order of 5 10^{-5} and two times more in an experiment carried out at the SPS, for a modulation frequency close to five times the synchrotron frequency.

While the RF phase is modulated, a high-precision tune meter shall follow the resulting high-frequency tune modulation. The method in [12] assumes a PLL-based tune-meter locked on the beam response. The signal its VCO is Fourier analysed. The amplitude of the harmonic

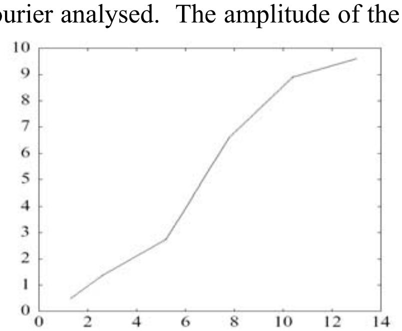

Figure 5: Amplitude of RF line versus set chromaticity [12]

at the RF modulation frequency is proportional to the modulus of the chromaticity. A calibration of the absolute chromaticity scale is possible by modulating the gradient of a quadrupole at the RF modulation frequency. If the amplitude and phase of the quadrupole modulation are adjusted to compensate the modulation due to the machine chromaticity, the signed chromaticity is measured. A first test with an incomplete setup done in the SPS shows a definite dependence of the amplitude of the transverse spectral line at the RF modulation frequency on machine chromaticity (figure 5). In [13], the beam signal is rather demodulated, filtered and Fourier analysed. The chromaticity sign is extracted from a comparison of an upper and lower sideband.

ON-LINE COUPLING MEASUREMENT

Betatron coupling is most often measured via the closest tune approach. Kick methods can be used as well: the coupling strength can be calculated from the beat period of the exchange of beam amplitude between the horizontal and vertical planes e.g. [14] or, if turn by turn observation is available, by the direct measurement of the resonance driving term [19]. In both cases, the beam must be kicked resulting in some emittance blow-up.

A method better adapted to a continuous measurement and opening the possibility of a feedback circuit is the coupling beam transfer function (BTF) [15]. The principle is to excite the beam over its eigen-frequencies at a very low level in one plane and observe the response in the other. The resulting BTF is proportional to the modulus of the coupling coefficient while its phase can in principle be recovered from the phase of the BTF outside of the beam eigen-frequencies. This method was extensively used in the ISR and other machines. A continuous use for hours would not produce any measurable beam blow-up.

While [15] relied on a simplified approach, a recent

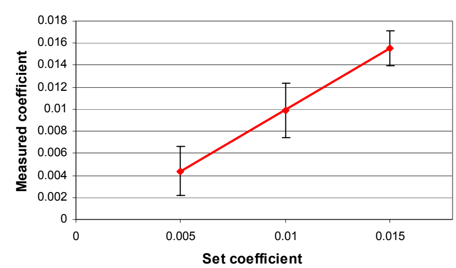

Figure 6: Coupling measurement via the BTF and AC dipole excitation [17]

exact first-order calculation of the single particle transfer function was carried out with the aim of predicting the observables in the general case [16]. This analysis shows that by measuring one or two points of the BTF, on either side of the beam spectrum and outside of it, it is possible to deduce the four parameters of linear coupling ($\vec{c}_-, \vec{c}_+$) and the local coupling parameters (in case of a local closed coupling bump). The method requires the use of two BPM's on either side of the beam excitor, spaced by a phase advance different from $k\pi$, without quadrupoles between them. Simulations show that an accuracy of 0.001 or better in |c| should be expected. The excitation outside the beam eigen-frequencies should prevent any emittance blow-up. The first test done at the SPS, to be published soon [17] gave already a measurement of the coupling to an accuracy of 0.001 albeit using a non-canonical set-up A ten times better accuracy is expected when the BTF phase can be measured. The coupling in the SPS is purely real and the sum resonance very weak. The same test is to be performed in the more difficult environment of a collider. The principle of this measurement seems well adapted to a feedback system.

LOCAL NON-LINEAR FIELDS

The dynamic aperture in superconducting hadron colliders is limited at injection energy by the non-linearity in the arc dipoles and at collision energy by the non-linearity of the low-beta quadrupoles. Multipolar correction circuits are provided. A beam-based correction requires the measurement of the multipoles up to the dodecapolar order. The feasibility of measuring high-order multipoles was demonstrated at RHIC[18]: a local closed orbit bump is created at the azimuth of the non-linearity to be measured. The beam displacement in the multipoles creates a feed-down to all lower orders. In practice, the feed-downs either to the orbit or tune order are measured versus the amplitude of the bump. The

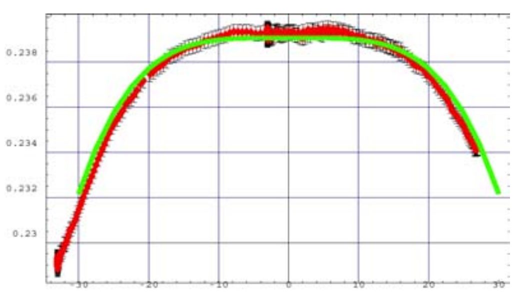

Figure 7: Signature of a dodecapole in RHIC

(tune shift versus bump amplitude [18]

degree of the variation of the tune/orbit with the bump amplitude is used to identify the multipole orders present and a fit provides their respective strengths. Using the PLL Q-meter at RHIC, the feed-down to the tune turned out to be the most accurate observable. The measured tune shift is given by:

$$\Delta Q \propto \int_{bump} \beta \begin{pmatrix} b_n \\ a_n \end{pmatrix} \begin{pmatrix} \Delta x_{bump} \\ \Delta y_{bump} \end{pmatrix}^{n-2} \frac{ds}{R^{n-2}} \quad (1)$$

where n is the multipolar order in the European convention (n=3 is the sextupole), R the reference radius.

In the low-β section, the β-function is modulated but the phase hardly advances. Expressing the orbit oscillation as a function of β in (1) shows that the measured tune shift in the plane of the bump is proportional to the `action kick' $\int \beta^{n/2} b_n ds$, i.e. directly proportional to the quantity to be corrected.

The resolution of the RHIC PLL was found to be 5 10^{-6}. This allowed a very precise measurement of multipoles up to the dodecapole order (figure 7). The statistical error is the % range. Redundant measurements can be obtained using horizontal and vertical bumps and tunes to exclude systematics.

RESONANCE DRIVING TERMS

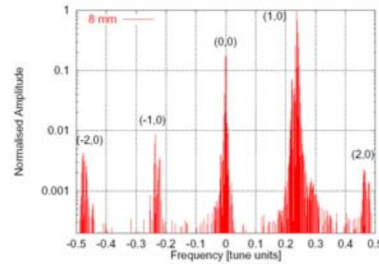

Figure 8: Transverse spectrum from [19]

In presence of non-linearities, the spectrum of the beam oscillation after a kick exhibits harmonics in addition to the main tune lines (figure 8). The method presented in [19] explains how to identify the harmonics with resonance conditions and how to relate their amplitudes with the resonance driving terms. It is summarized below.

To first-order in the non-linearities, the turn-by-turn transverse beam position takes a simple form in normalized complex coordinates:

$$\begin{aligned} \hat{x}(N) - i\,\hat{p}_x(N) &= \sqrt{2I_x}\, e^{i(2\pi Q_x N + \psi_0)} \\ &\quad - 2i \sum_{jklm>0} j\, f_{jklm} (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} \quad (2) \\ &\quad \times e^{i\left[(1-j+k)(2\pi Q_x N + \psi_{x0}) + (m-l)(2\pi Q_y N + \psi_{y0})\right]} \end{aligned}$$

The normalization is done with respect to the local β-function. The momentum is in practice replaced by the beam position measured at a second BPM spaced by 90 degrees. Q_x and Q_y are the betatron tunes and I_x, I_y the actions (normalized amplitudes). The complex coefficient f_{jklm} is directly related to the coefficients h_{jklm} of the expansion of the non-linear Hamiltonian into resonant terms which yields the driving terms.

$$f_{jklm} = h_{jklm} \Big/ \left[1 - e^{-i2\pi([j-k]Q_x + [l-m]Q_y)}\right] \quad (3)$$

The expansion of the Hamiltonian, in term of the action-angle variables I and ψ, is itself given by:

$$\begin{aligned} H &= \sum_{jklm>0} h_{jklm} (2I_x)^{\frac{j+k}{2}} (2I_y)^{\frac{l+m}{2}} \\ &\quad \times e^{-i\left[(j-k)(\psi_x + \psi_{x0}) + (l-m)(\psi_y + \psi_{y0})\right]} \end{aligned} \quad (4)$$

From a comparison between the formula (2) and (4), it comes out that the resonance conditions $(j-k)Q_x + (l-m)Q_y$ appears in the spectrum as the spectral line ($1-j+k,m-l$). For example, the horizontal third-order resonances (3,0,0,0), i.e. $3Q_x$ appears as line (-2,0). The resonance order is given by $j+k+l+m$.

The modulus of the resonance strength $|h_{jklm}|$ is computed from the amplitude of the spectral line after calibration of the kick. It shows the correct dependence on the amplitude (figure 9). In this formalism, the resonance strength is not Fourier expanded in azimuthal harmonics and thus retains its dependence on the machine azimuth. Its modulus remains constant in the absence of non-linearity and jumps when a non-linearity is passed. This was used to 'find' the SPS sextupoles [19]. The measurement accuracy is limited by the decoherence. The method showed a good accuracy at the third order.

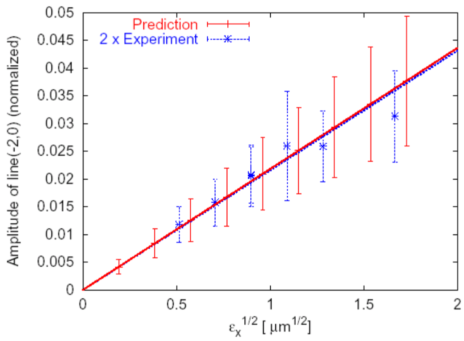

Figure 9: Normalized amplitude dependence of the 3Qx driving term [19]

AC DIPOLE EXCITATION

Most measurement methods, except those based on the Schottky signals, require the beam to oscillate coherently in at least one of its degrees of freedom. The kick methods are easiest and largely used. With the decoherence due to the non-linearities and space charge, the coherent oscillation is transformed into a blow-up of the incoherent amplitudes, i.e. of the emittance.

For the measurement of the **linear** parameters, tunes, coupling... the kick method may be advantageously replaced by a very low amplitude coherent excitation coupled with a proper detection technique such as resonant pick up's, PLL circuit, transfer function... yielding a high signal to noise ratio.

The measurement of the **non-linearities** requires a beam displacement large with respect to the beam size. A beam blow-up can in principle be avoided if the beam is excited outside of its eigen-frequencies. This principle has been formalized in [20] and implemented at RHIC. Various experiments have shown that, provided the excitation frequency is switched on in a progressive manner, Figure 10, (no overlap of the spectrum of the modulated oscillation with the beam spectrum), the emittance blow-up is indeed in practice negligible for oscillations of the order of the beam size.

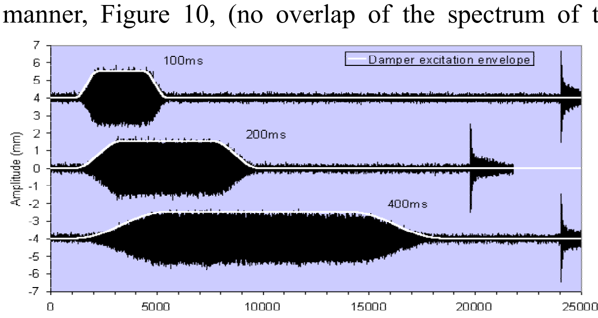

Figure 10: AC dipole excitation from[21]

Recent theoretical studies show the high potential of this excitation technique in the measurement of the linear optics parameters including linear coupling [16] and in the measurement of the non-linear parameters (detunings and resonance driving terms) [22][23]. Experiments carried out at the SPS in 2002 confirm the potential for the measurement of linear parameters. The measurement of non-linear parameters still appears as a challenge.

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