SMITH–PURCELL RADIATION IN VIEW OF PARTICLE BEAM DIAGNOSTICS

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Abstract

During the last years Smith–Purcell radiation which is generated when a charged particle beam passes close to the surface of a diffraction grating became topic of interest as non–invasive tool for particle beam diagnostics. In some publications the use of Smith–Purcell radiation for longitudinal as well as for transversal monitoring was already considered. The proposed methods are based on the idea that the relevant beam parameters can be extracted by a comparison of the measured spectral intensity distribution with the theoretical one. In the case of Smith–Purcell radiation this is a non-trivial task because the spectral intensity distribution is strongly influenced by the form of the grating profile, and the theoretical description of the radiation factors describing this dependency require extensive numerical calculations. As consequence a careful choice of the grating is essential or methods are preferable by which the wavelength dependent influence can be minimized. In the case of longitudinal beam diagnostics it should be possible by a proper choice of the form of the grating profile. In the case of transversal beam monitoring it is demonstrated that direct imaging of the beam profile is possible with Smith–Purcell radiation taking advantage of the specific emission characteristics at ultra relativistic beam energies.

INTRODUCTION

The development of the next generation high quality electron beams necessary for future high luminosity linear colliders and short wavelengths free electron lasers presents an enormous challenge for both the diagnostic measurement of beam parameters and the accurate positioning and control of these beams. The existing monitors are based on a number of different physical principles, and one technique is to use the radiation that can be produced by the beam itself. In this context nowadays the beam diagnostics based on optical transition radiation is widely used [1, 2, 3, 4]. However, the transition radiation technique entails the disadvantage of an interaction of the beam with the target leading to either the destruction of the high quality beam parameters due to small angle scattering of the electrons in the target foil or at high beam currents even of the screen. Hence the development of non–invasive, low cost, and compact beam monitors is demanded. Monitors based on synchrotron radiation, while non–invasive, are disadvantageous because they cannot be used in linear beam geometries. Another approach rather similar to the transition radiation technique is to exploit the radiation characteristics of diffraction radiation which is emitted when the electrons pass close to an obstacle [5, 6, 7, 8, 9]. In recent publications beam diagnostics based on resonant diffraction radiation is proposed which originates from electrons moving through an ideally conducting tilted target which is made by strips separated by vacuum gaps [10, 11].

A rather similar and also non–destructive approach is to use Smith–Purcell (SP) radiation as a compact and inexpensive beam profile monitor. The radiation is generated when the electron beam passes a periodic structure like a diffraction grating at a fixed distance close to the surface. The radiation mechanism was predicted by Frank in 1942 [12] and observed in the visible spectral range for the first time by Smith and Purcell [13] using a 250–300 keV electron beam.

Soon after the discovery of the SP effect also potential applications became topic of interest. The possibility to use coherent SP radiation as bunch length diagnostic was proposed in [14, 15] and design studies of bunch length monitors were reported e.g. in reference [16]. The feasibility to use SP radiation for longitudinal bunch shape measurements was recently demonstrated in Frascati [17]. SP radiation as high resolution position sensor is discussed in view of possible applications for ultra relativistic beam energies up to 500 GeV in Ref. [18]. Experimental studies performed at the Mainz Microtron MAMI demonstrated that SP radiation from ultra relativistic energies can be used both as transversal beam size as well as beam position monitor. The investigations presented here were performed in context with the experiment reported in Ref. [19]. SP radiation generated with the MAMI low emittance 855 MeV electron beam [20] was investigated in the visible spectral range. By measuring the spatial intensity distribution emitted perpendicular to the grating surface beam position and beam size in the horizontal plane could be determined.

SMITH–PURCELL RADIATION PROPERTIES

According to the theory of di Francia [21], the emission mechanism of SP radiation can be interpreted in analogy to the diffraction of light as the diffraction of the field of the electrons (virtual photons) which pass the grating at a distance d away from its surface by the grating grooves. One characteristic signature of SP radiation is that it must fulfill the dispersion relation [13]

$$\lambda = \frac{D}{|n|}(1/\beta - \cos\theta\sin\Phi)\,. \quad (1)$$

In this equation λ is the wavelength of the emitted radiation, D the grating period, n the diffraction order, β = v/c

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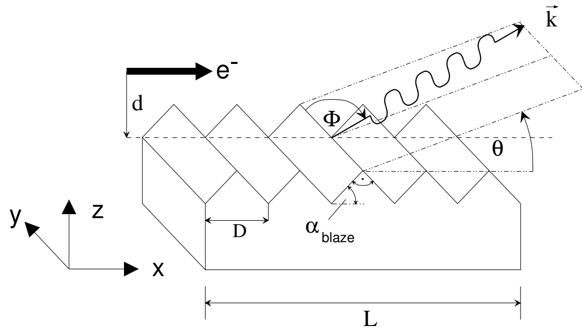

Figure 1: Definition of the geometry. The electron moves with constant reduced velocity β = v/c at a distance d parallel to the grating surface in x direction. The grooves, oriented in the y direction, repeat periodically with the grating period D. The blaze angle α_{blaze} characterizes the échelette gratings which were used in the experiment [19]. The direction of the photon wave vector $\vec{k}$ is described in the emission plane resulting from the z = 0 plane by a rotation about the y axis by the angle θ. In the emission plane the $\vec{k}$ vector makes an angle Φ with the y axis. The length of the grating is denoted by L.

the reduced electron velocity, and θ, Φ the emission angles as introduced in Fig. 1.

The angular distribution of the emitted power radiated into the nth order is [21]

$$\frac{\mathrm{d}P_n}{\mathrm{d}\Omega} = \frac{eIn^2L}{2D^2\varepsilon_0}\frac{\sin^2\theta\sin^2\Phi}{(1/\beta-\cos\theta\sin\Phi)^3}|R_n|^2 \times\exp\left(-\frac{d}{h_{int}}\sqrt{1+(\beta\gamma\cos\Phi)^2}\right), \quad (2)$$

with e the elementary charge, I the beam current, L the grating length, ε_0 the permittivity of free space, and d the distance of the beam above the grating. The radiation factors $|R_n|^2$ which are analogous to the reflection coefficients of optical gratings are a measure for the grating efficiency. Their properties will be discussed in the following subsection.

According to Eq. (2) the intensity decreases exponentially with increasing distance d between electron beam and grating surface. The interaction length

$$h_{int} = \frac{\lambda\beta\gamma}{4\pi}\ , \quad (3)$$

where $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor, describes the characteristic finite range of the virtual photons emitted and re-absorbed by the electrons.

To achieve a good coupling between the electrons and the radiation field via the grating the mean spot size of the electron beam and its distance from the grating should be in the order of the interaction length h_{int}. However, in the experiment of Smith and Purcell [13] the beam spot size of 0.15 mm was much larger than the interaction length which was of order of $h_{int} \simeq 10^{-8}$ m. As a consequence only a small fraction of the beam could contribute to the radiation emission. At higher beam energies, for instance 855 MeV which is the current maximum energy of the Mainz Microtron MAMI, the interaction length in the optical spectral region amounts to $h_{int} \simeq 70\ \mu$m. Taking, in addition, advantage of the low vertical emittance $\varepsilon_z = 1\ \pi\ \mathrm{nm\,rad}$ of the MAMI electron beam, a beam spot size as small as a few μm can be achieved, i.e. all electrons contribute to the emitted radiation in nearly the same way.

Furthermore, at ultra relativistic electron energies, according to Eq. (2) the radiation is emitted in a very narrow angular region around $\Phi = 90°$, i.e. in the plane containing the grating normal and the electron beam. For ultra relativistic beam energies the characteristic opening angle (FWHM) can be written as

$$\Delta\Phi = 2\sqrt{\left(\frac{\ln 2}{4\pi}\frac{\lambda}{d}\right)^2 + \frac{1}{\beta\gamma}\frac{\ln 2}{2\pi}\frac{\lambda}{d}}\ , \quad (4)$$

i.e. the angular width can be controlled by the beam energy, the wavelength of observation, and the distance between beam and grating surface. Taking additionally into account the distance dependence of the intensity, for an effective coupling the distance d should be in the order of the interaction length Eq. (3). With $d = h_{int}$, Eq. (4) can be rewritten as $\Delta\Phi = 2.733/\gamma$, i.e. the opening angle of SP radiation is slightly larger than the characteristic opening angle $\alpha_c \sim 2/\gamma$ of transition or synchrotron radiation used for particle beam diagnostics. If the restriction imposed on the effective coupling $d \leq h_{int}$ is given up, even $\Delta\Phi$ smaller than α_c can be achieved at the expense of a reduced radiation intensity. For typical experimental parameters γ = 1673, λ = 360 nm, and d = 100 μm [19], an angular width of $\Delta\Phi$ = 1.0 mrad results compared to α_c = 1.2 mrad. The feature of the strongly collimated emission can be used either to discriminate SP radiation against background components as described in Ref. [19] or for particle beam diagnostic purposes as shown in the present article.

Radiation Factors

In a series of experiments Bachheimer demonstrated that the intensity of SP radiation is strongly influenced by the shape of the grating profile [22]. This dependency is expressed by the radiation factor $|R_n|^2$ which is additionally a function of beam energy and observation geometry.

The radiation factors which are analogous to the reflection coefficients of optical gratings are calculated according to the theory of Van den Berg [23, 24] which is formulated in terms of a boundary value problem for partial differential equations and yields predictions only after extensive numerical calculations. Experiments performed in the low [25] as well as in the high energy region [19] show a satisfactory agreement with the underlying theory.

An alternative approach is based on the interpretation of SP radiation caused by induced surface currents which arise when the beam electrons traverse the grating surface

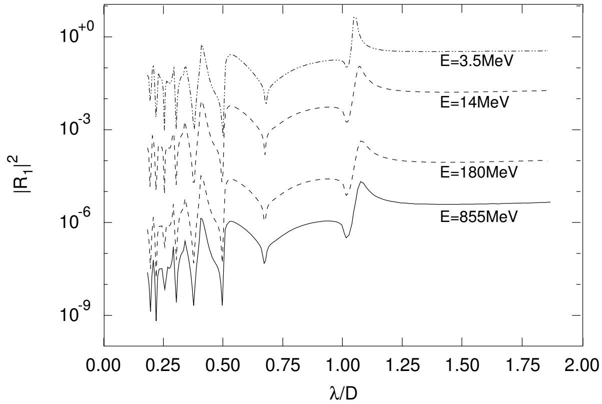

Figure 2: Calculated functional dependence of the radiation factor $|R_1|^2$ on the normalized wavelength λ/D according to Ref. [23] for a grating with α_{blaze} = 41.12° as used in Ref. [19]. The quoted beam energies correspond to the different accelerator sections of the Mainz Microtron MAMI.

[19, 26]. Unfortunately, at ultra relativistic beam energies this kind of model failed to predict the measured intensities by orders of magnitude [19, 27]. Therefore, the subsequent considerations are based on the Van den Berg theory.

In Fig. 2 calculated radiation factors are plotted as function of the ratio λ/D with the beam energy as parameter for experimental conditions accessible at MAMI. The striking feature is that (i) the $|R_n|^2$ exhibit pronounced resonance structures as function of the observation angle (which according to Eq. (1) corresponds to λ/D) and that (ii) the radiation factors are extremely small and scale inversely proportional to γ^2.

The first aspect known from optical grating theories [28] and usually named "Wood–Rayleigh" anomaly is associated with the passing off of a spectral order. The corresponding "Rayleigh" wavelength at which the anomaly occurs is given by $\lambda_R = \frac{D}{|n|}(1 + 1/\beta)$. As a consequence for beam diagnostic measurements it is to consider that if the beam parameters have to be extracted from a comparison of the measured spectral intensity distribution with the theoretical one, the influence of the anomalies can blur the signature on the relevant parameters as shown in the following section.

The second aspect, i.e. the decrease of the radiation factors with increasing beam energy, was found empirically and a well–founded explanation for the γ^{-2} dependency is still due. Nevertheless as a consequence, for beam diagnostic measurements at ultra relativistic energies the radiated intensity is extremely small. As an example, for the experiment at MAMI the number of SP photons amounted typically $\sim 10^{-9}$/electron at a distance d = 100 μm between electron beam and grating surface. In comparison to that the intensity of optical transition radiation which originated from beam electrons striking the grating when the beam center was 10 μm away from the surface amounted typically $\sim 10^{-5}$ photons per electron [29]. Therefore, for beam diagnostics with SP radiation at ultra relativistic beam energies extremely clear experimental conditions and a good background suppression are required.

LONGITUDINAL BEAM DISTRIBUTION

Similar to frequency domain techniques based on transition or synchrotron radiation the determination of the longitudinal charge distribution resp. the bunch length via SP radiation is based on the study of the coherent radiation process. Neglecting the influence of transverse beam dimensions the radiated power emitted by a bunch of N_e particles can be written as

$$\left(\frac{\mathrm{d}P_n}{\mathrm{d}\Omega}\right)_{coh} = \left(\frac{\mathrm{d}P_n}{\mathrm{d}\Omega}\right)_{inc} \times \Big[1 + (N_e - 1)\ f(\sigma_x, \lambda)\Big] \quad (5)$$

with $(\mathrm{d}P_n/\mathrm{d}\Omega)_{inc}$ the incoherent power emitted by the bunch as given by Eq. (2) and $f(\sigma_x, \lambda)$ the longitudinal bunch form factor which describes the time coherence in the emission process. The form factor is the square of the Fourier transform of the normalized longitudinal charge distribution function $S(x)$ [30, 31], i.e.

$$f(\sigma_x, \lambda) = \left| \int_{-\infty}^{+\infty} \mathrm{d}x\, S(x)\, e^{i2\pi x \cos\theta/\lambda} \right|^2 . \quad (6)$$

For wavelengths in the order of the bunch length σ_x the form factor approaches unity and the radiated intensity is increased by the number of particles in the bunch. Since this number is typically very large (at MAMI up to $\sim 10^6$) the coherent signal is much more intense than the incoherent radiation when the form factor equals zero.

Basis of bunch length diagnostics with coherent radiation is the investigation of the emitted power Eq. (5) as function of the wavelength. From the spectral decomposition of the measured intensity it is possible to determine the bunch form factor Eq. (6) from which $S(x)$ has to be reconstructed. In contrast to transition or synchrotron radiation where usually autocorrelation measurements are performed in order to analyze the spectral content of a polychromatic spectrum, SP radiation is dispersive according to Eq. (1). Therefore, a direct measurement of the spectral intensity distribution has to be done. Due to the dispersion relation the variation of the wavelength corresponds to a variation of the observation angle θ (the detector placed under Φ = 90°). Therefore, a detector which is rotatable in θ direction is required by which the emitted intensity can be measured as function of λ.

In Fig. 3 spectral intensity distributions are plotted as function of θ. The calculations were performed under the assumption of a Gaussian distribution for $S(x)$ with bunch lengths of σ_x = 300 μm (which corresponds to the bunch length at MAMI) and of 50 μm and a constant radiation factor $|R_1|^2$. A variation of the bunch length results in a shift of the maximum of the intensity distribution, i.e. the angular position of the maximum is a measure for the bunch length. Additionally from the shape of the emitted power as function of the wavelength it is possible to determine the bunch form factor. From that the longitudinal

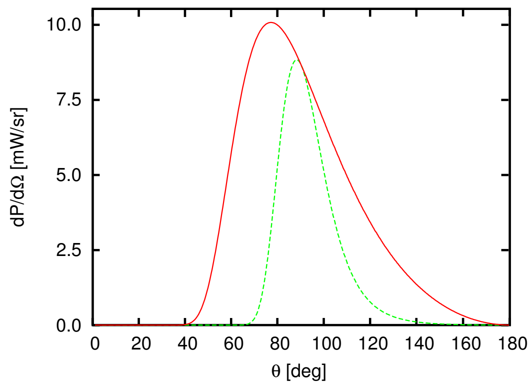

Figure 3: Emitted power in first order as function of angle of observation for a Gaussian longitudinal charge distribution with σ_x = 50 μm (solid line) and 300 μm (dashed line). The beam energy was 3.5 MeV which corresponds to the output energy of the MAMI injector linac and the beam current 100 μA with 10^6 electrons per bunch. Grating: D = 500 μm, L = 5 cm, $|R_1|^2$ = 0.3. Distance between grating surface and beam: 0.5 mm.

beam profile can be extracted by fitting the measured form factor supposing the functional dependency of $S(x)$. Methods based on this principle was proposed in Ref. [14, 15], and in a recent publication the determination of the longitudinal bunch shape was reported with the 1.8 MeV electron beam at the ENEA FEL Facility at Frascati [17] with the result of a triangular beam profile and a bunch length of about 14 ps.

As pointed out before the radiation factors strongly depend on θ. Therefore, in Fig. 4 the same calculated spectral intensity distributions are plotted than in Fig. 3 taking into account the angular dependency of $|R_1|^2$. In this case the maximum of the spectral power is determined by the maximum of the radiation factor. A noticeable influence of the different bunch lengths is only to recognize in the tails where the intensity is already strongly reduced.

As consequence the influence of the radiation factors of

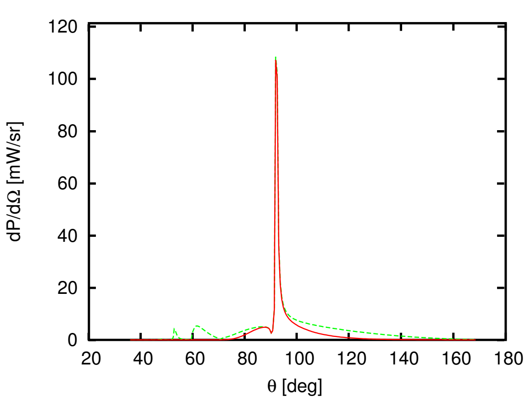

Figure 4: Emitted power in first order as function of angle of observation taking into account the angular dependence of $|R_1|^2$ for the échelette grating of Fig. 2.

the grating can blur the signature on the bunch leading to a misinterpretation of bunch shape and length. Therefore, for beam diagnostic measurements a careful choice of the grating structure should be done. From a comparison of radiation factors of different grating structures it is to conclude that especially gratings with sinusoidal profile seem to be suitable because the influence of the anomalies is strongly reduced [24] while gratings with échelette profiles as used in Ref. [19] show rather pronounced anomalies and thus should be avoided.

Additionally it is to keep in mind that for SP based beam diagnostics $S(x)$ is the induced bunch charge distribution on the grating surface rather than the direct beam profile, which results in an additional broadening, c.f. Ref. [14]. However, this effect can be calculated and therefore taken into account for high resolution beam diagnostics.

TRANSVERSE BEAM DISTRIBUTION

The use of SP radiation as high–resolution position sensor for ultra relativistic electron beams was proposed already in Ref. [18]. The basic idea of the method is that a horizontal displacement of the beam above the grating is connected with a change in the observation angles θ, Φ under which a detector mounted at a fixed position will see the emitted radiation. Therefore, such a displacement will result in a change of the measured intensity.

At the one hand in order to transform the intensity change in a horizontal displacement it is to consider that the radiation factors depend on both observation angles and consequently the calibration requires numerical effort. At the other hand due to the strongly collimated emission of SP radiation the dynamical range of such a monitor would be extremely small, whereby it would make possible to achieve a sensitivity on horizontal displacements in the order of 1 μm. Anyhow methods are preferable similar to beam monitoring via synchrotron radiation which do not rely on absolute intensities.

At this it is possible to exploit the fact that according to Eq. (4) SP radiation is emitted in a very narrow angular region around Φ = 90° and to perform a direct imaging similar to beam diagnostics based on synchrotron radiation [32]. In contrast to the latter imaging with SP radiation even would have the advantage that a resolution broadening due to the focus depth can be avoided because the source point for the emitted radiation is the grating surface.

In connection with an experiment at the Mainz Microtron MAMI [19, 27] this fact was used to measure the horizontal beam profile with optical SP radiation at wavelengths of 360 nm and 546 nm. The radiation was generated with the low emittance 855 MeV electron beam focused down to a vertical spot size of 2 μm (1 σ) which passed over optical diffraction gratings of echelle profiles with blaze angles of 17.27° and 41.12° and with a grating period of 0.833 μm. In Fig. 5 a sketch of the experimental setup is shown (left). Because no spatial resolving detector was at disposal a 400 μm wide slit aperture mounted

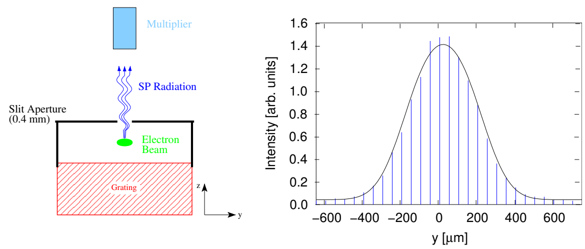

Figure 5: Left: Experimental arrangement for the horizontal SP beam monitor. Right: Measured horizontal intensity profile at observation angle θ = 69.8° corresponding to λ = 546 nm, $|n|$ = 1. Dashed line: best fit supposing a normal distributed beam profile and a rectangular distribution function for the aperture resolution.

9 mm above the grating and designed originally for background suppression was used to achieve sensitivity on the horizontal (y) coordinate. In the experiment the slit aperture together with the grating was moved in y direction in order to scan the horizontal SP intensity profile which was recorded by a photo multiplier placed under a fixed angle corresponding to a diffraction order of SP radiation. In Fig. 5 (right) the multiplier intensity is plotted as function of the position of the slit aperture together with the best fit which takes into account the horizontal beam profile (normal distribution) and the resolution function of the aperture (rectangular distribution). The result for the horizontal beam width of $\sim$ 270 μm (FWHM) was in accordance with the result from an independent measurement of $\sim$ 250 μm performed with a wire scanner.

By this the proof of principle was done that SP radiation can be applied for beam diagnostic purposes. Nevertheless the problem of the extremely low radiation intensity (typically in the order of 10^{-9} photons per electron) must be overcome before to think about applications of SP monitors in the standard operation mode of an accelerator. Again the question is how to increase the extremely low radiation factors. As pointed out in Ref. [33] based on calculations according to the surface current model large radiation factors can be expected for strip gratings. However, calculations based on the theory of Van den Berg result in values comparable to the ones for echelle–type gratings [34].

CONCLUSIONS

The present article summarizes the main characteristics of SP radiation in view of applications for particle beam diagnostics. It is demonstrated that this radiation mechanism has interesting features which can be used either for bunch length measurements or as transverse beam monitor. Nevertheless the use of SP radiation involves difficulties which are mainly connected with the grating structure. Therefore, the grating properties have to be considered carefully before to apply for standard diagnostic purposes and in addition extensive experimental investigations are required to support the underlying theories.

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