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Error magnetic field due to the median plane asymmetry and its applications

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On behalf of
Beam physics group in TRIUMF

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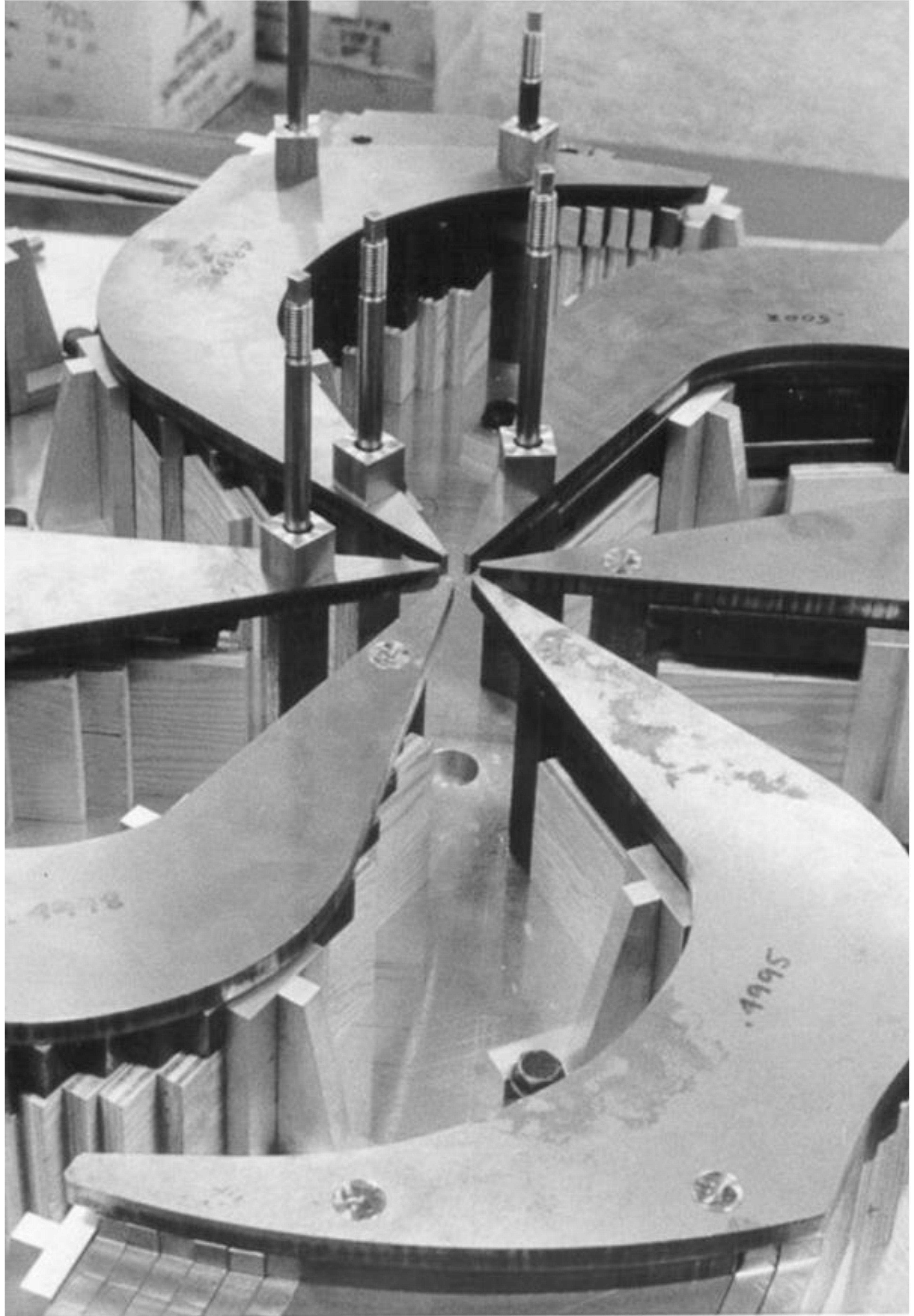


Discovery, accelerated

Outline

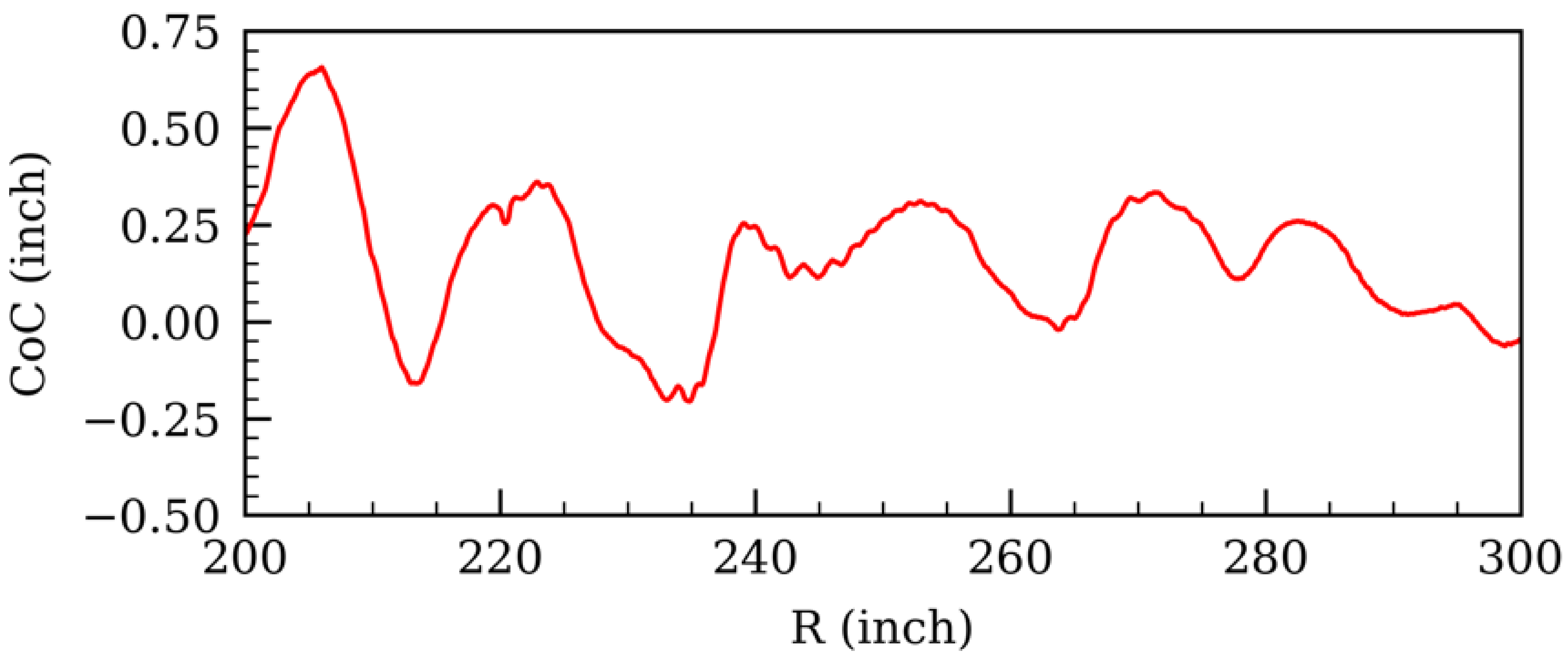
- Introduction
- Treatment of median plane field map
  - Expansion out of median plane
  - Redundant in the field survey data
- Applications:
  - Correcting the error in the field survey data
  - Measuring the vertical tune
  - Correcting the linear coupling resonance
- Conclusion

Introduction

- Median plane field map is important for cyclotron beam dynamics studies.

- The pole's geometric error and the unevenly magnetized soft iron give rise to non-zero asymmetrical fields in the geometric median plane.

- Effect of the asymmetrical field.



Expansion out of median plane: Gordon's approach

It is derived from the scalar potential Ψ that satisfies Laplace's equation

$$\Psi = \Psi_o + \Psi_e,$$

$$\Psi_o = zB - \frac{z^3}{3!}\nabla_2^2 B + \frac{z^5}{5!}\nabla_2^4 B - \ldots,$$

$$\Psi_e = C - \frac{z^2}{2!}\nabla_2^2 C + \frac{z^4}{4!}\nabla_2^4 C - \ldots,$$

The odd term Ψ_o produces a field with median plane symmetry, while the even term Ψ_e spoils this symmetry.

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The magnetic field is given by the gradient of the scalar potential

$$B_z = -B + z\nabla_2^2 C + \frac{z^2}{2!}\nabla_2^2 B - \frac{z^3}{3!}\nabla_2^4 C - \frac{z^4}{4!}\nabla_2^4 B + \ldots,$$

$$B_r = -\frac{\partial C}{\partial r} - z\frac{\partial B}{\partial r} + \frac{z^2}{2!}\frac{\partial \nabla_2^2 C}{\partial r} + \frac{z^3}{3!}\frac{\partial \nabla_2^2 B}{\partial r} - \ldots,$$

$$rB_\theta = -\frac{\partial C}{\partial \theta} - z\frac{\partial B}{\partial \theta} + \frac{z^2}{2!}\frac{\partial \nabla_2^2 C}{\partial \theta} + \frac{z^3}{3!}\frac{\partial \nabla_2^2 B}{\partial \theta} - \ldots.$$

Redundant in the error field survey data

Writing the magnetic field on the median plane in complex harmonic form of the azimuthal direction, the field expressions are simplified to ordinary differential equations (ODE) which have only the radius *r* as variable

$$B_{rn} = -\frac{dC_n}{dr},$$

$$B_{\theta n} = -jn\frac{C_n}{r},$$

$$\frac{dB_{zn}}{dz} = \frac{d^2C_n}{dr^2} + \frac{1}{r}\frac{dC_n}{dr} - n^2\frac{C_n}{r^2}.$$

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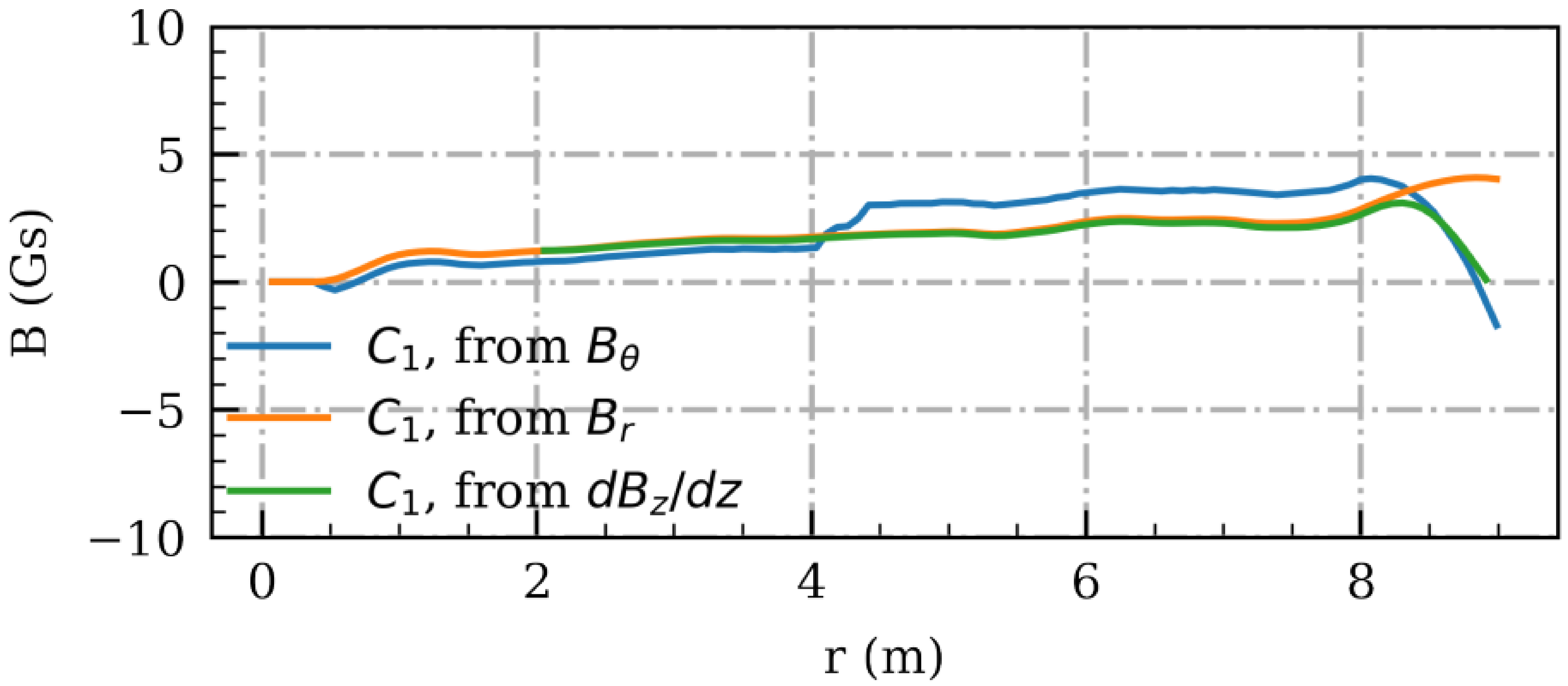
First harmonic of the asymmetric field potential map



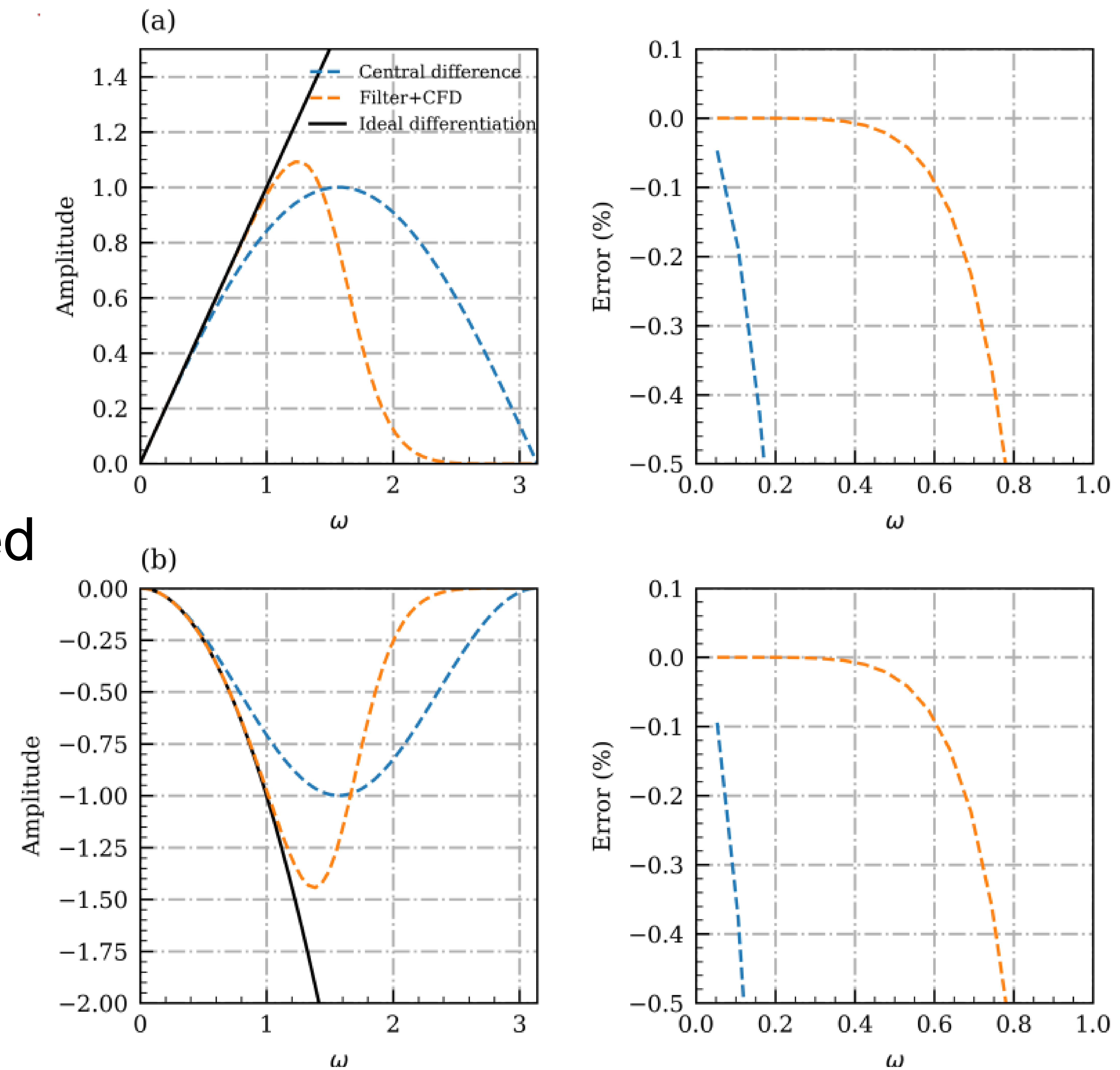
Correcting the error in the field survey data

Second-order central difference scheme

$$f_i' = \left(f_{i+1} - f_{i-1}\right)/2d,$$
$$f_i'' = \left(f_{i+2} + f_{i-2} - 2f_i\right)/4d^2,$$

Compact finite difference method (CFD) was used to improve the accuracy of the calculated derivatives

$$\frac{1}{3}f_{i-1}' + f_i' + \frac{1}{3}f_{i+1}' = \frac{14}{9}\frac{f_{i+1} - f_{i-1}}{2d} + \frac{1}{9}\frac{f_{i+2} - f_{i-2}}{4d},$$
$$\frac{2}{11}f_{i-1}'' + f_i'' + \frac{2}{11}f_{i+1}'' = \frac{12}{11}\frac{f_{i+1} + f_{i-1} - 2f_i}{d^2} + \frac{3}{11}\frac{f_{i+2} + f_{i-2} - 2f_i}{4d^2}.$$



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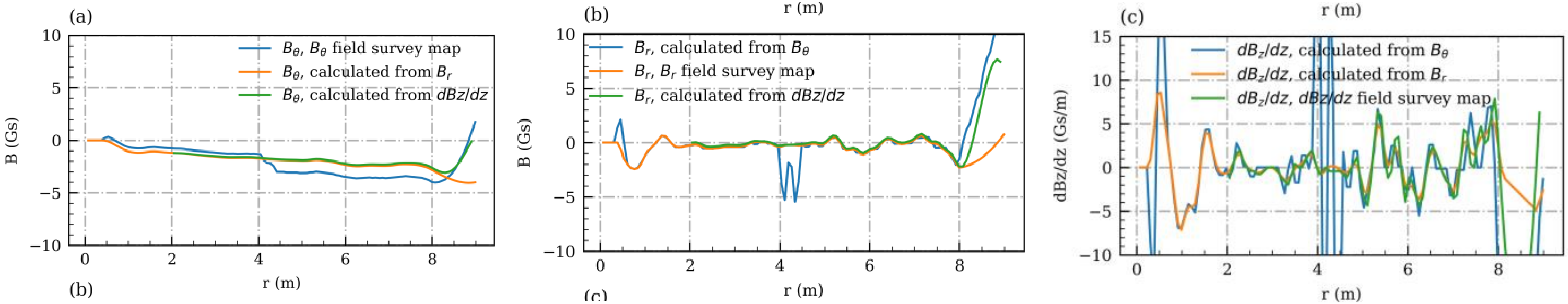

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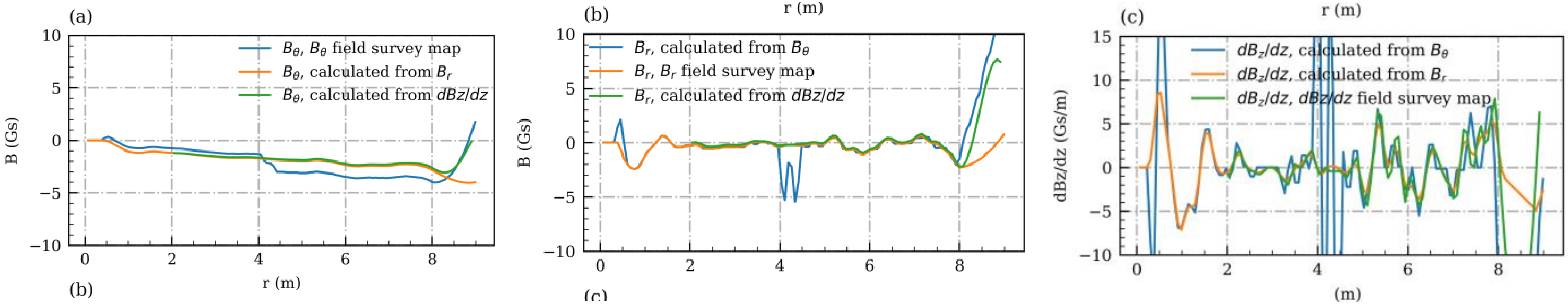

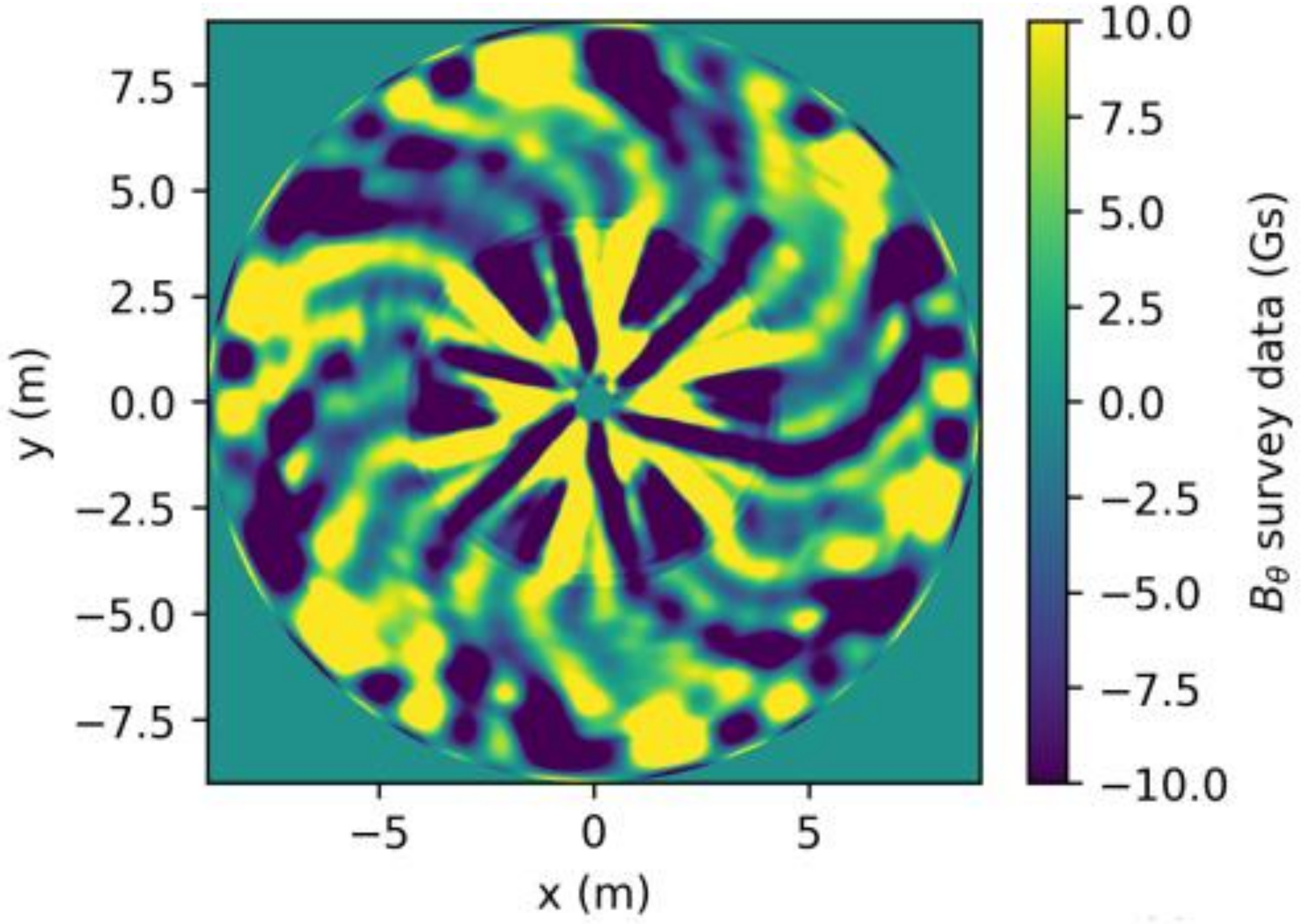

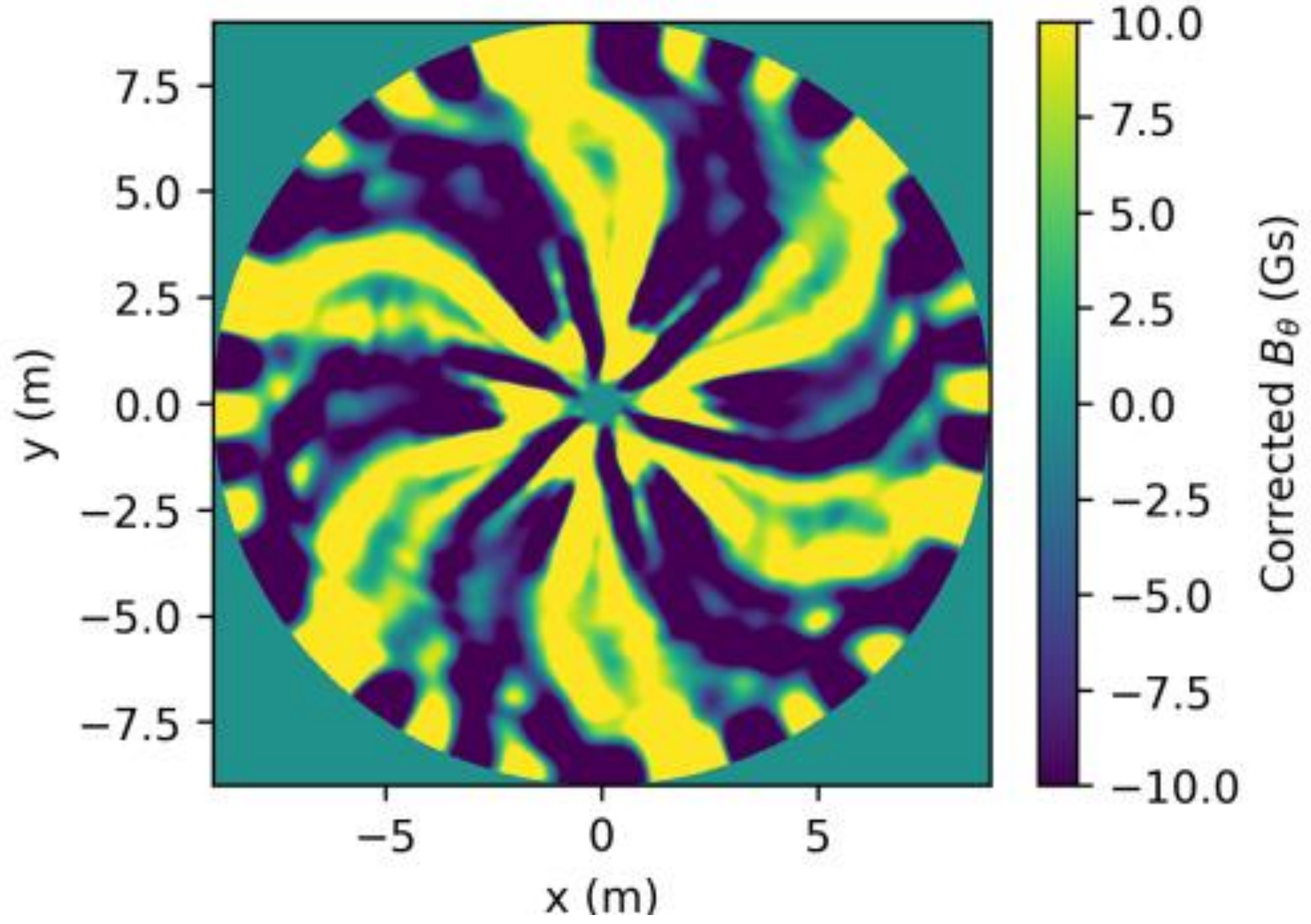

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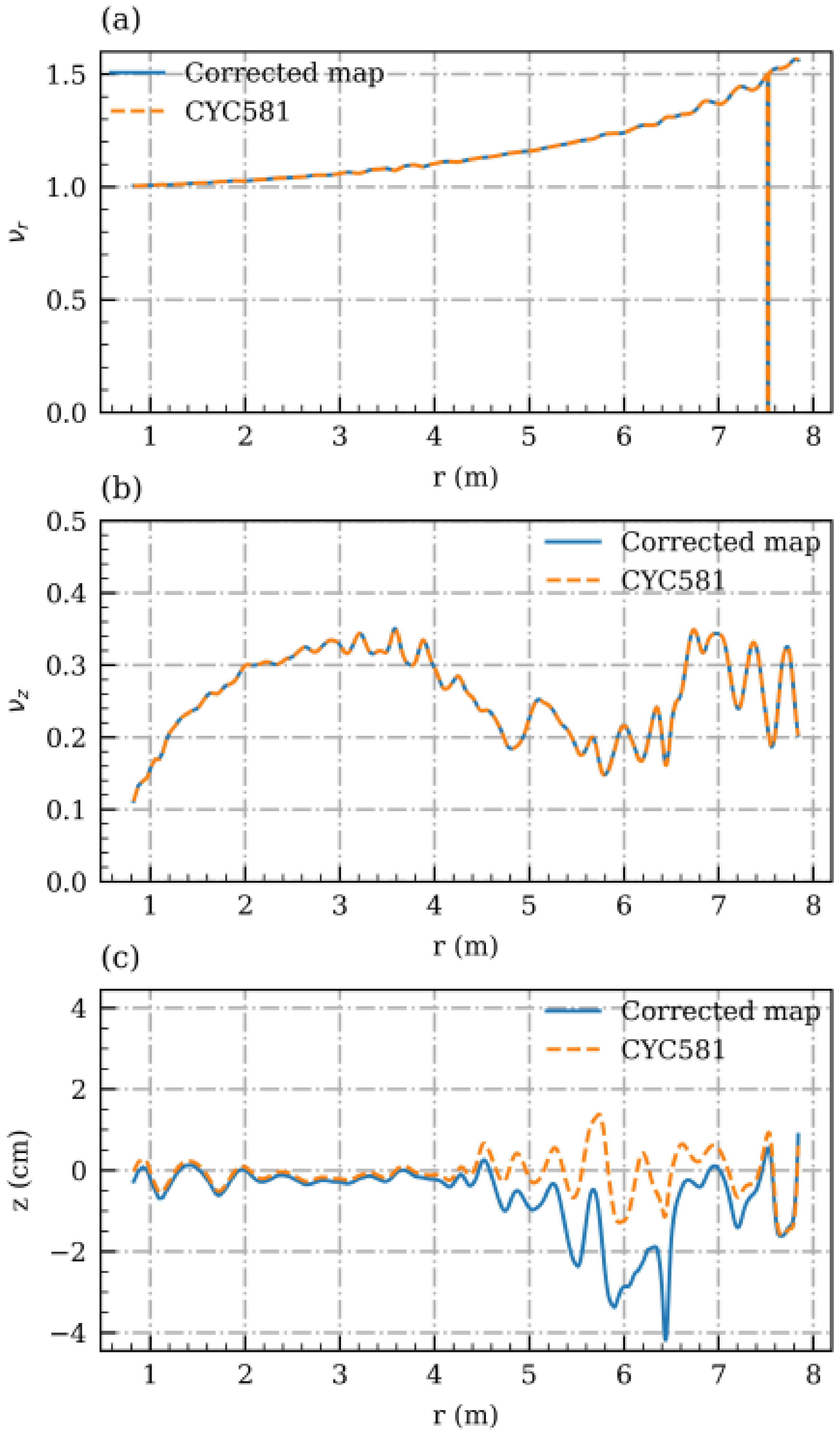

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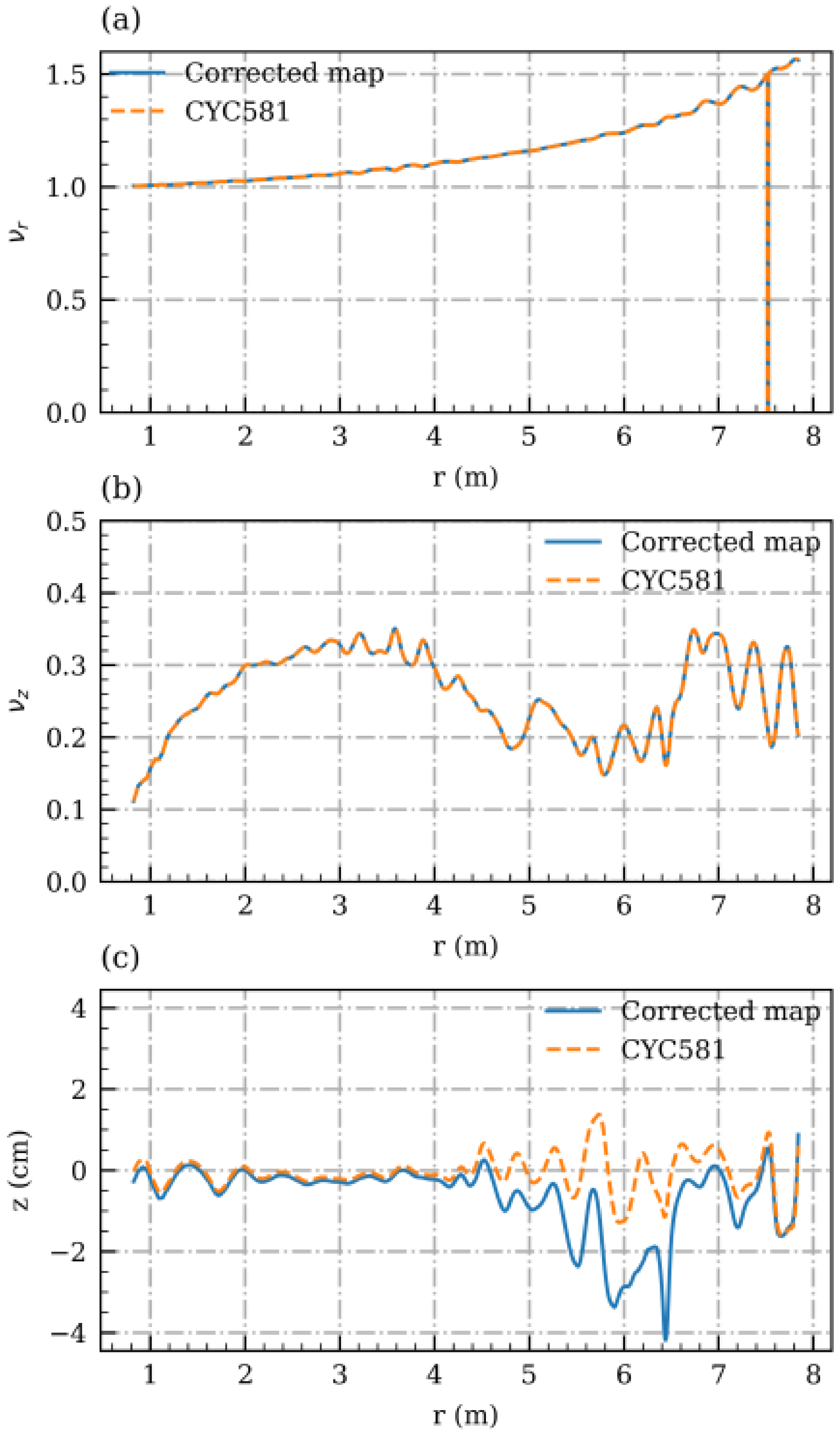
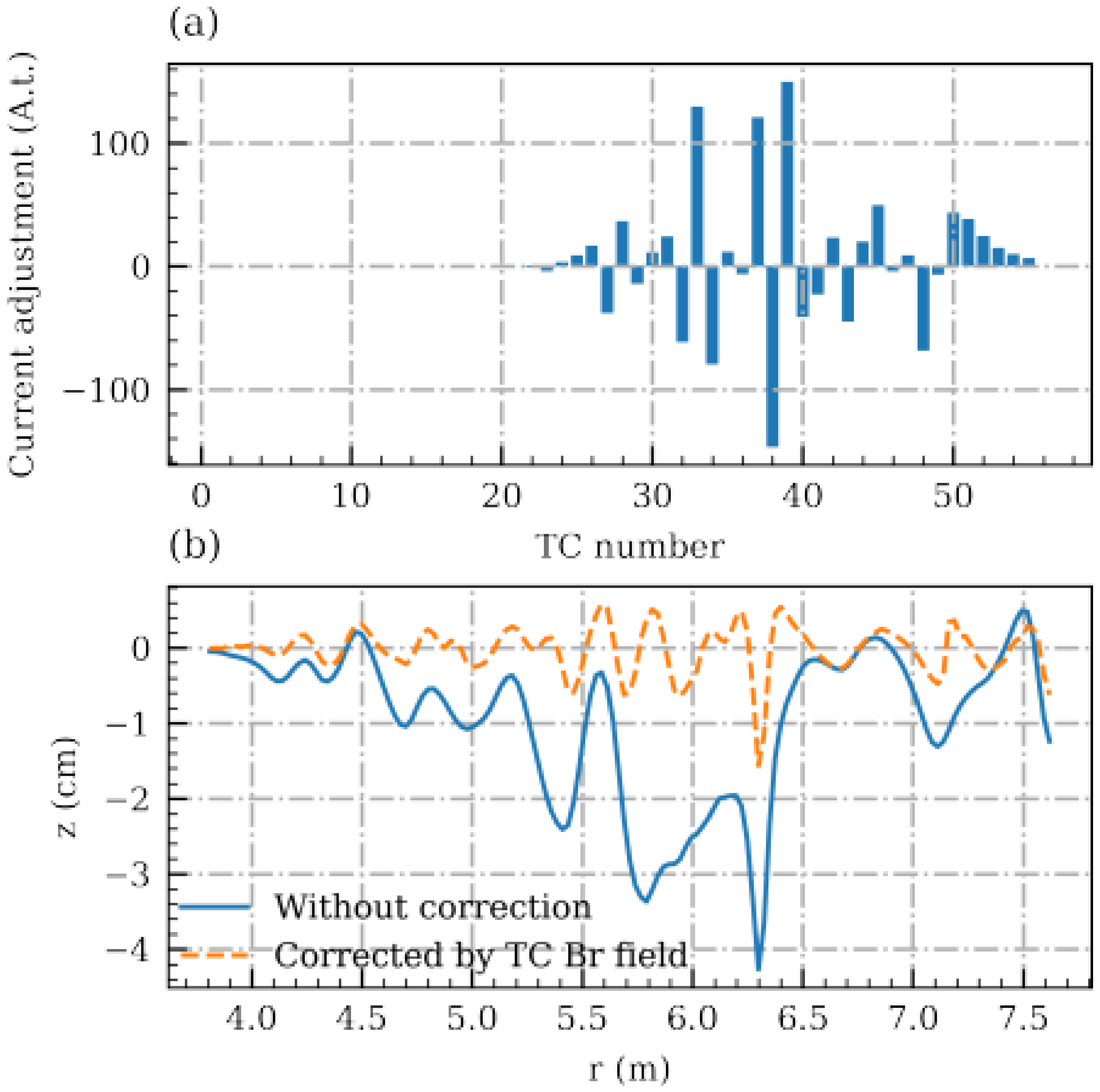


The vertical displacement Δz

$$\Delta z = \frac{\overline{R}}{\overline{B}_z}\frac{\Delta\overline{B_r}}{\nu_z^2},$$



Measuring the vertical tune

The vertical displacement Δz as a function of average radius, magnetic field and ν_z is written as

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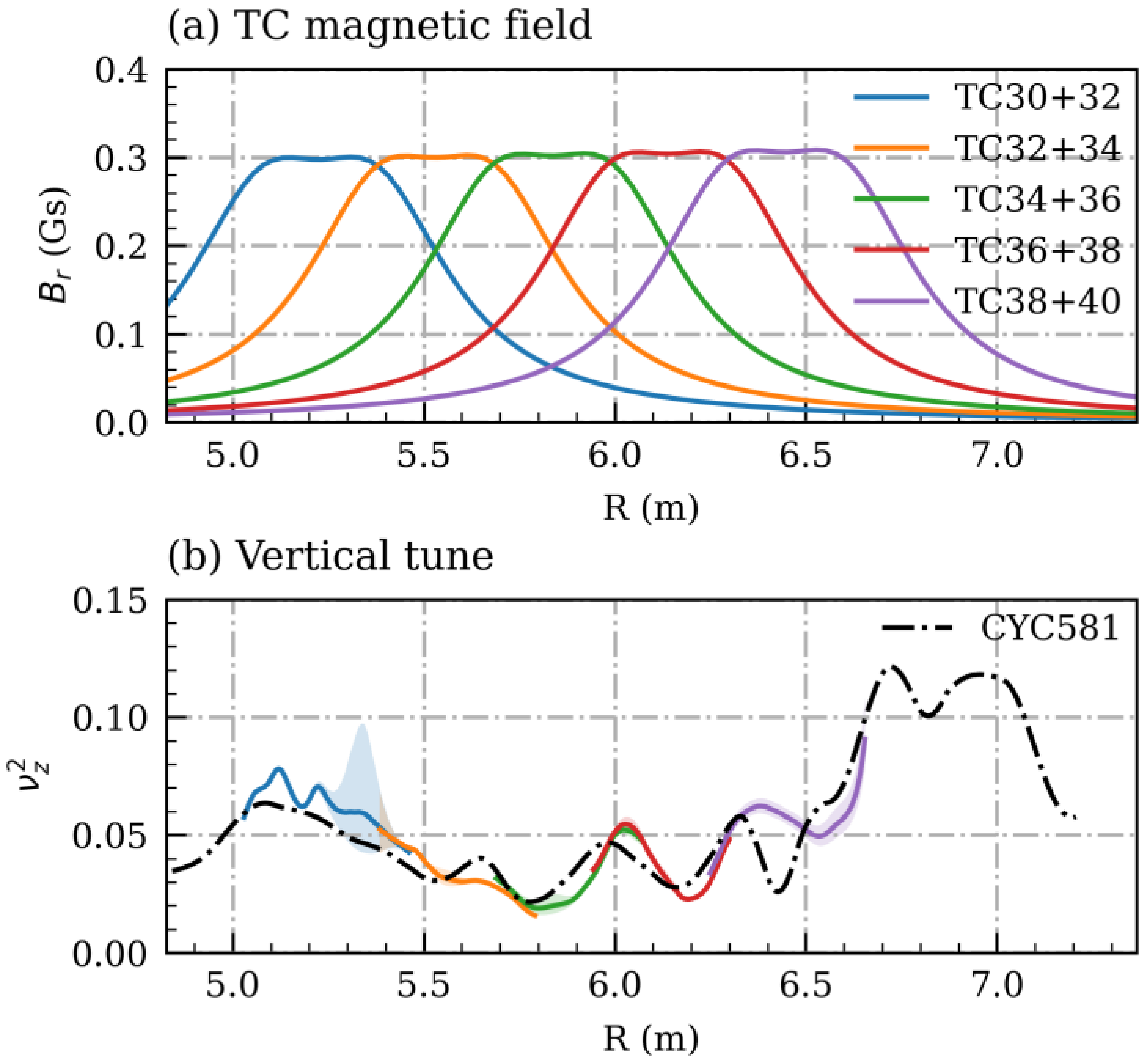

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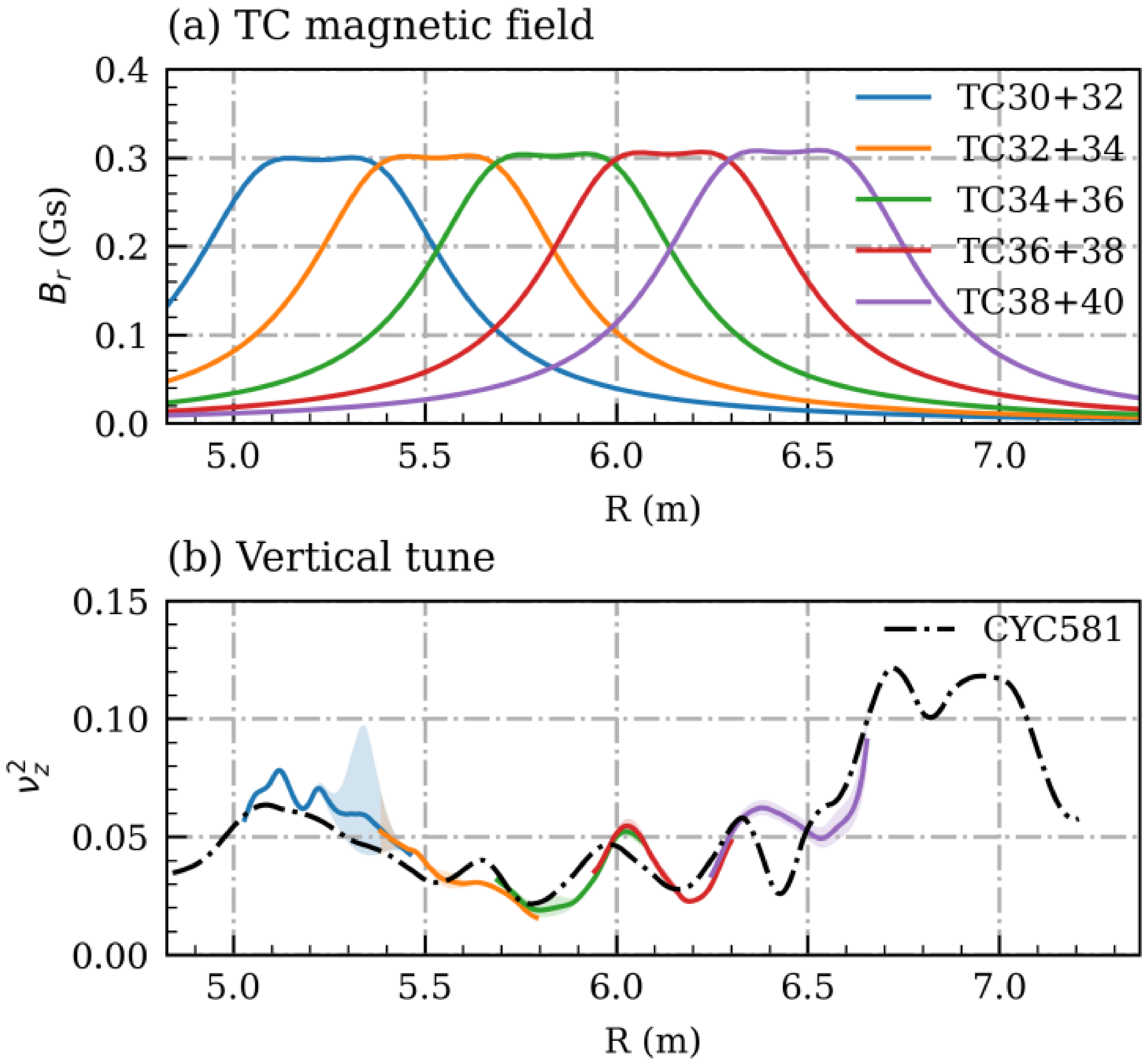
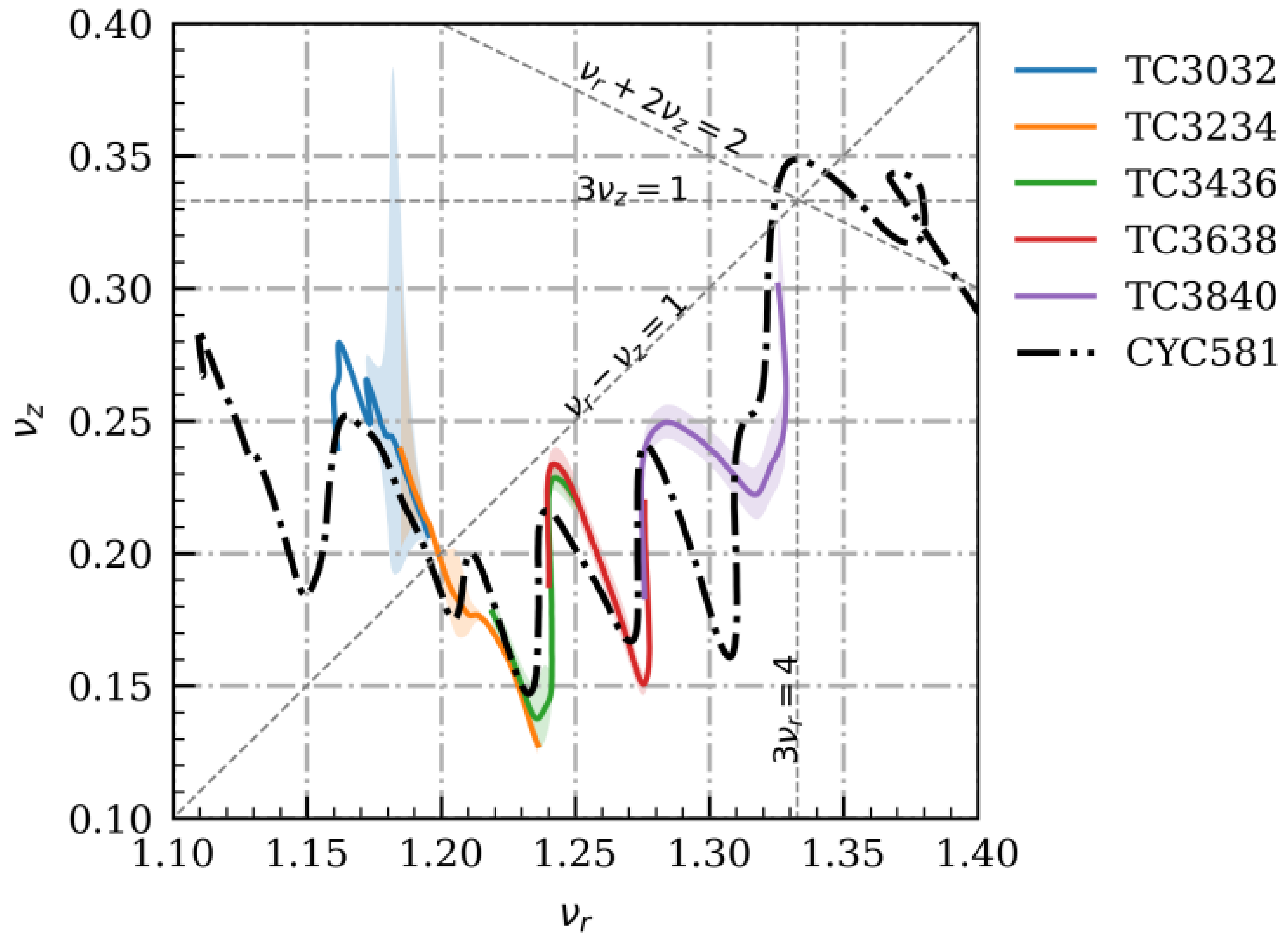
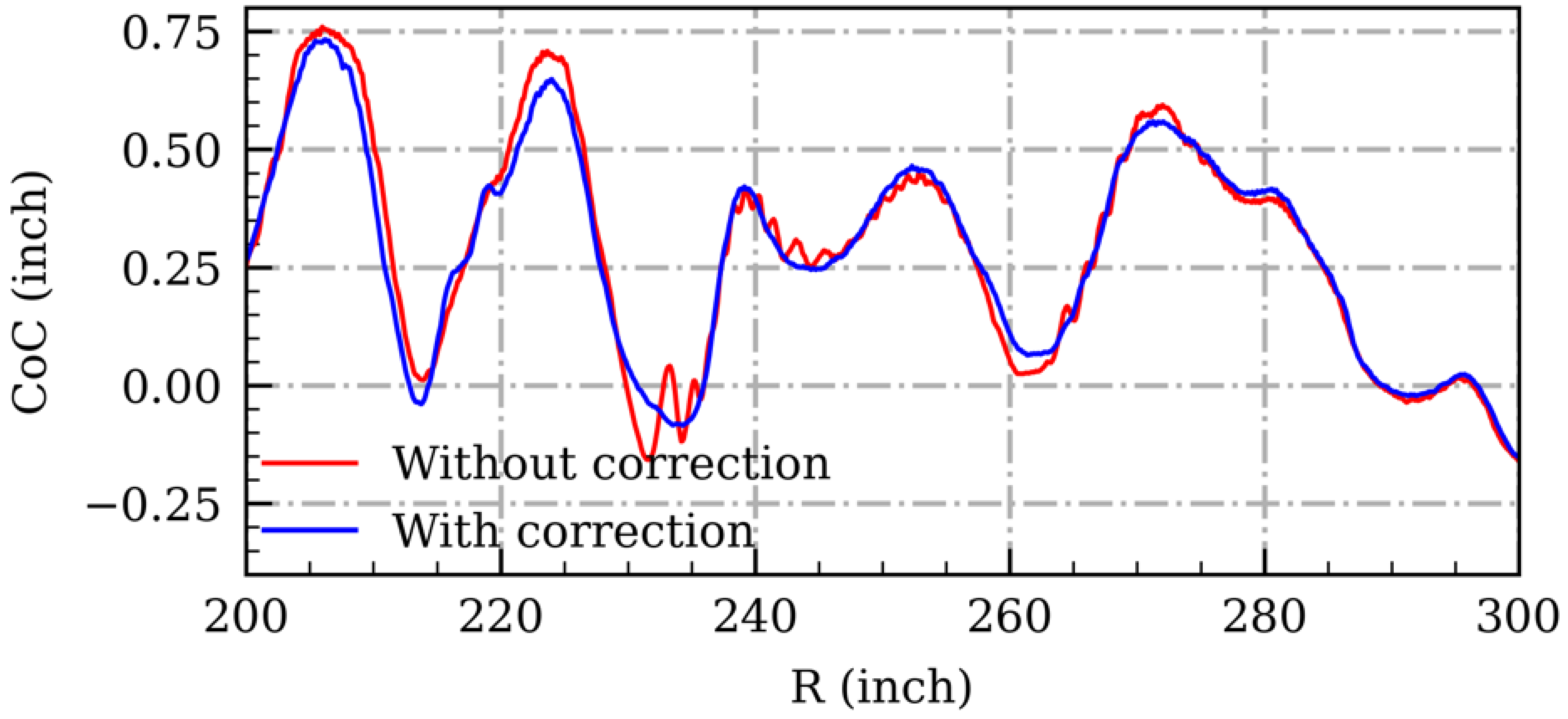
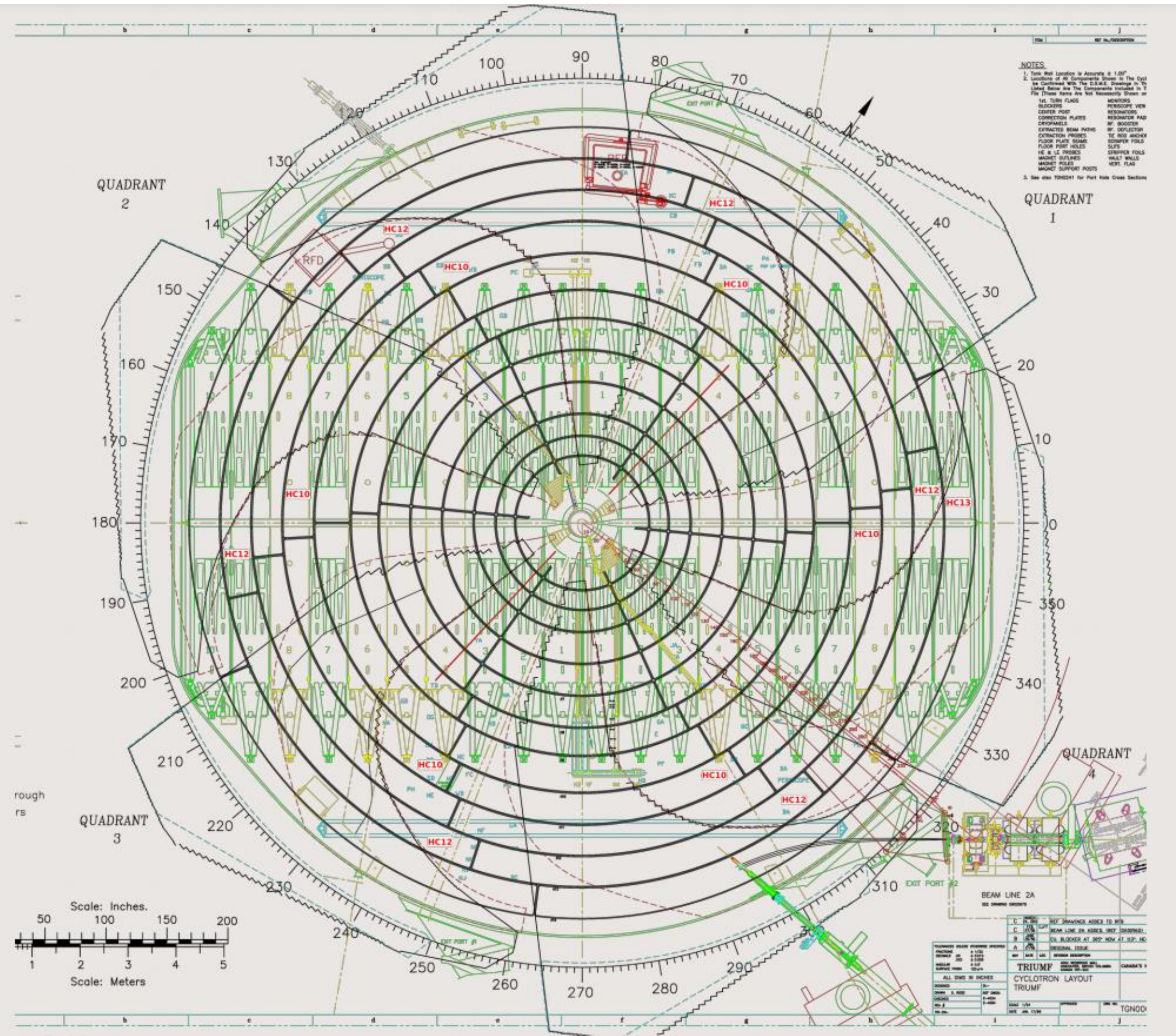
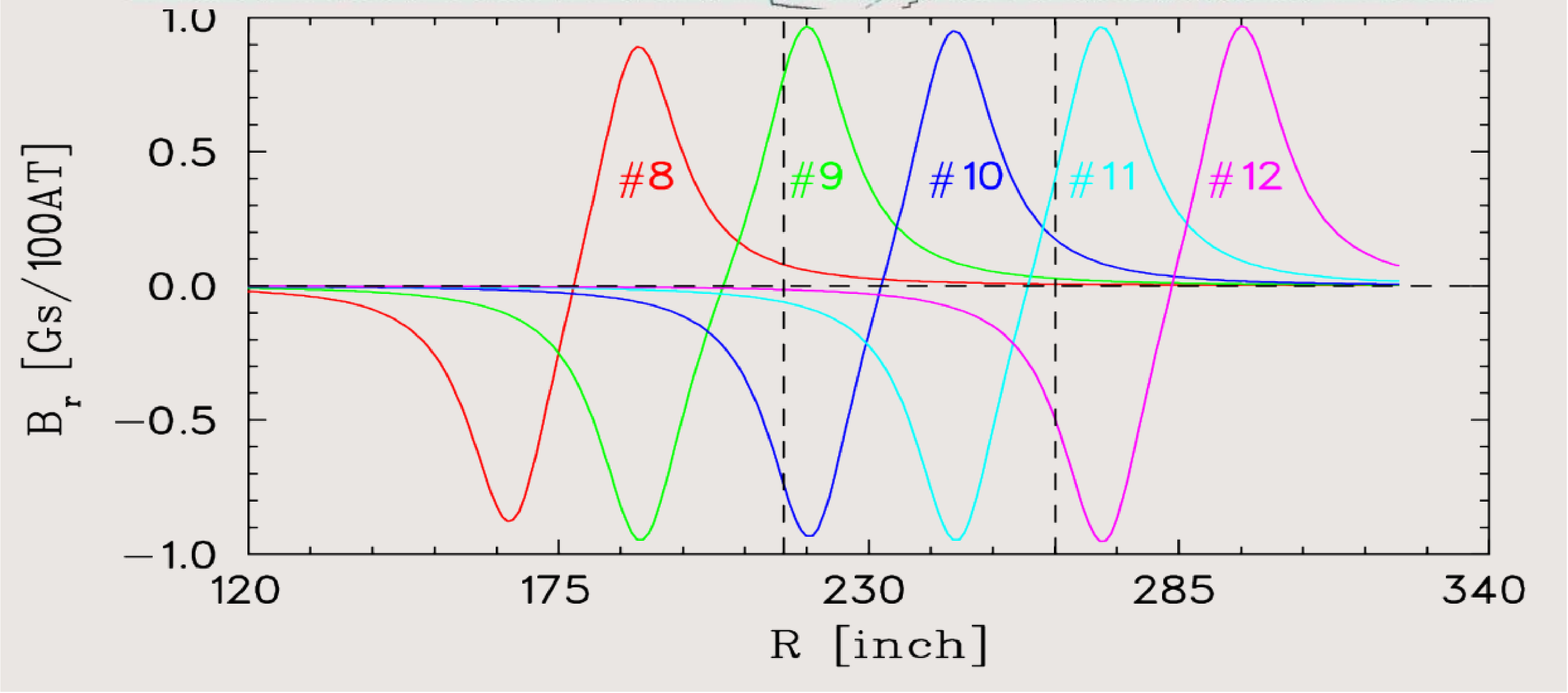






Correcting the linear coupling resonance





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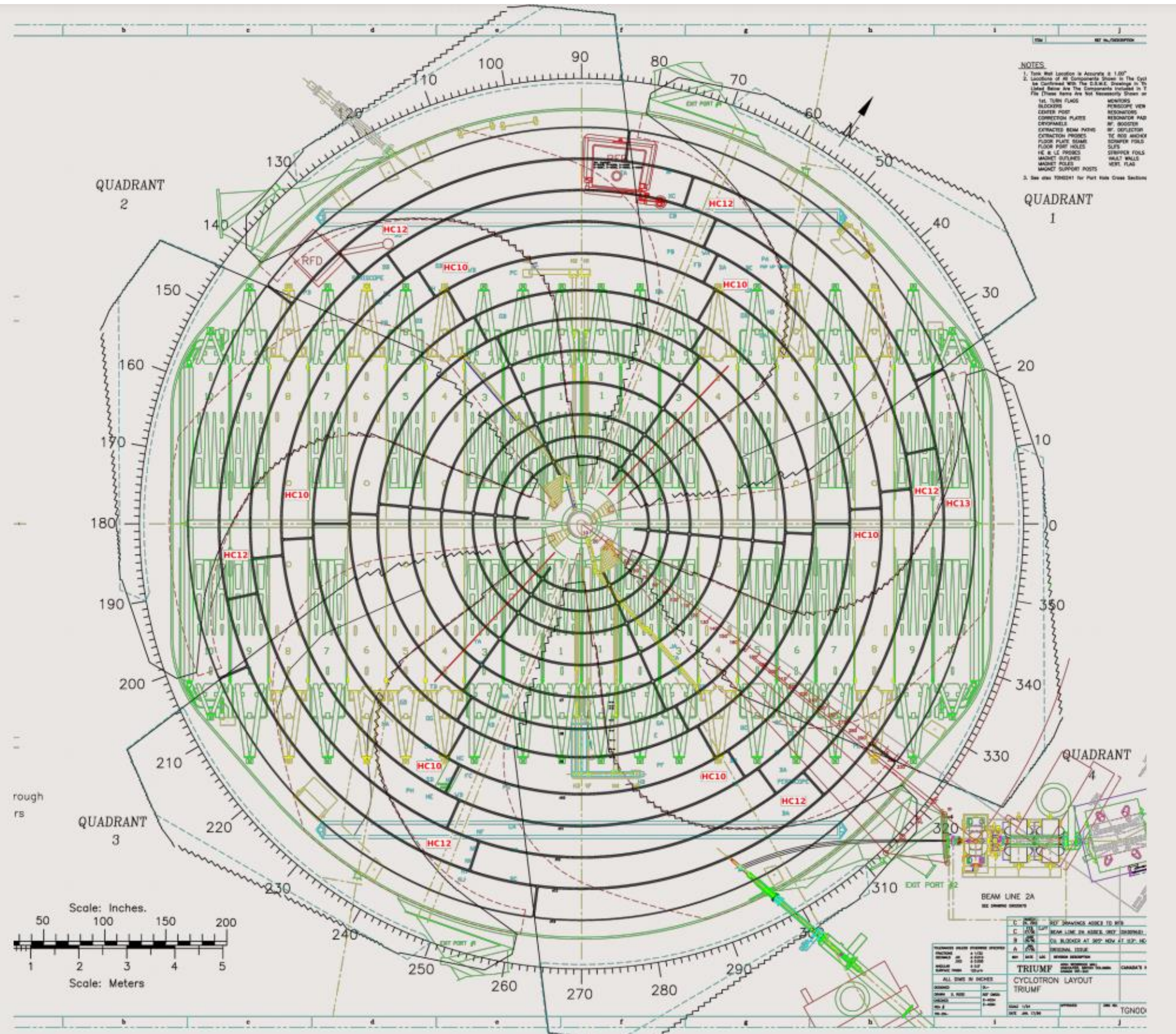

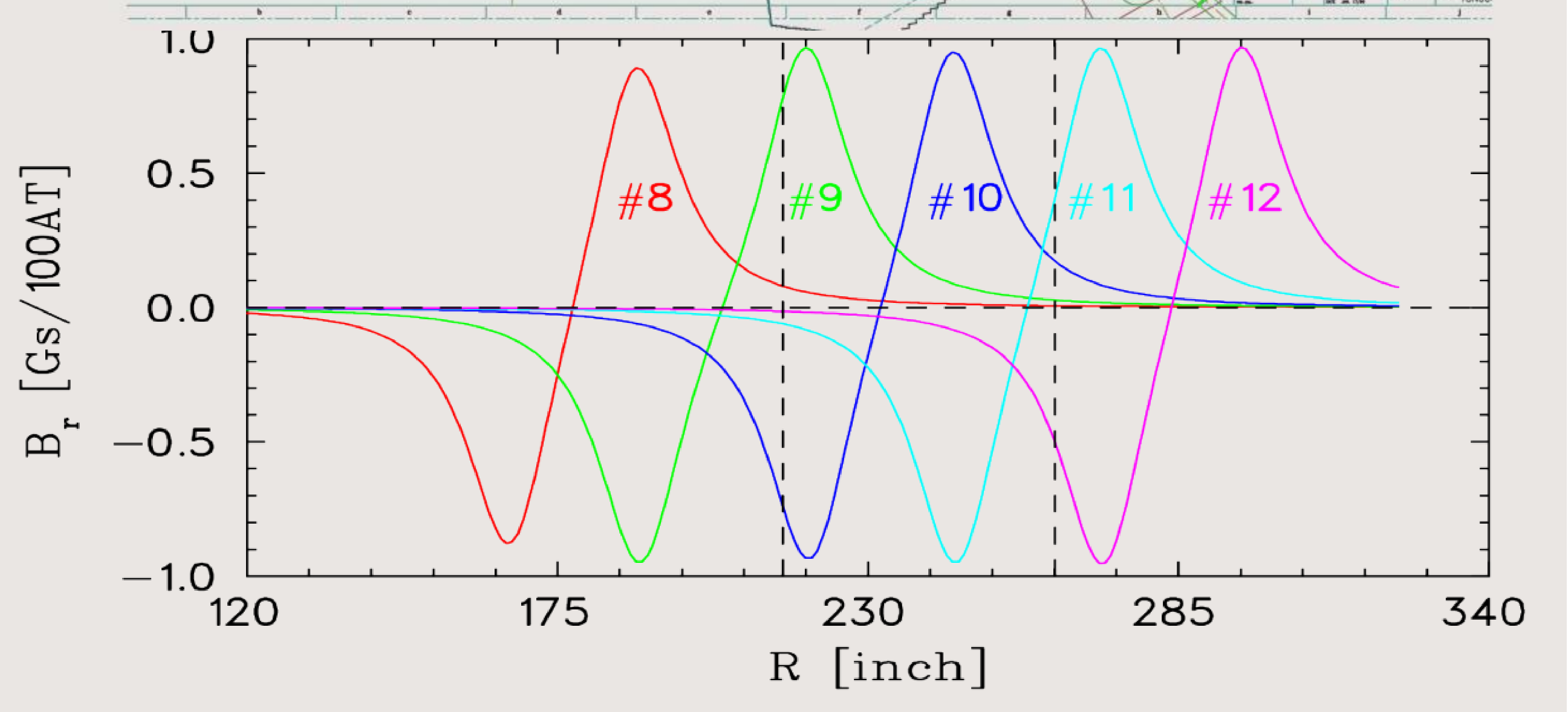

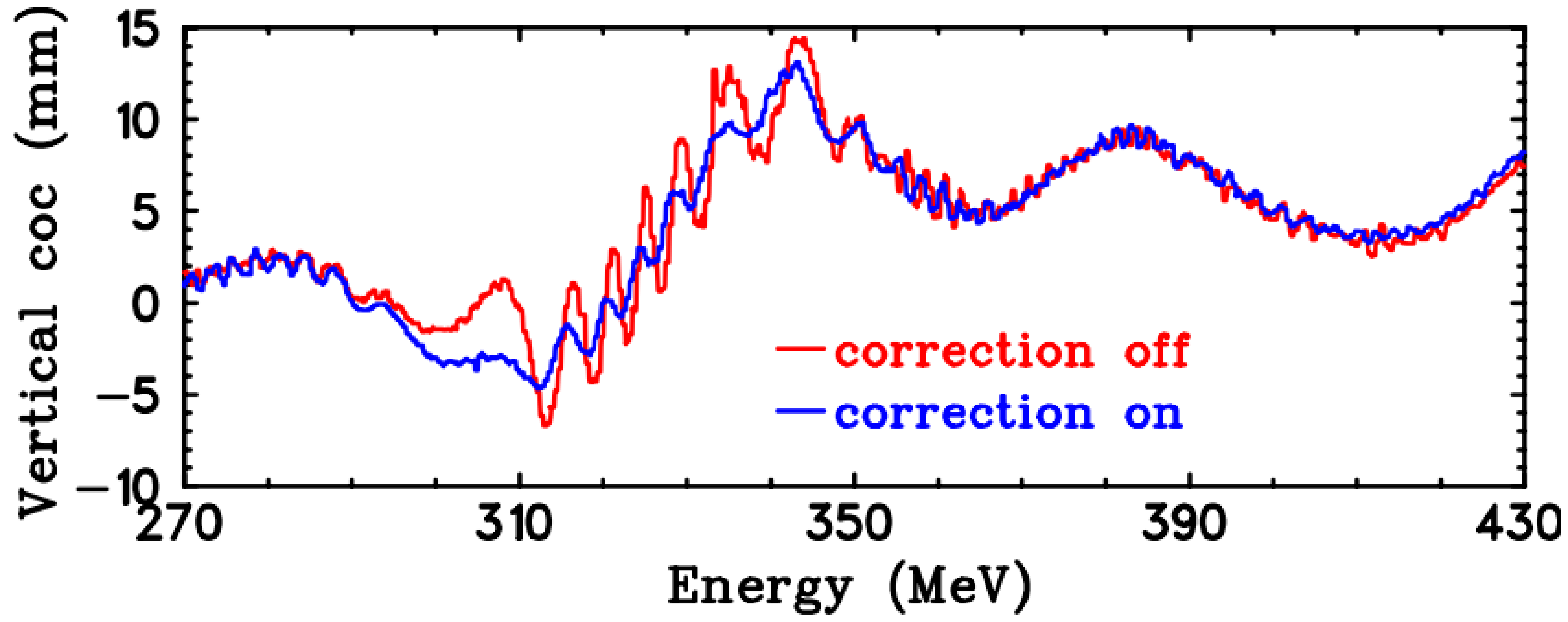

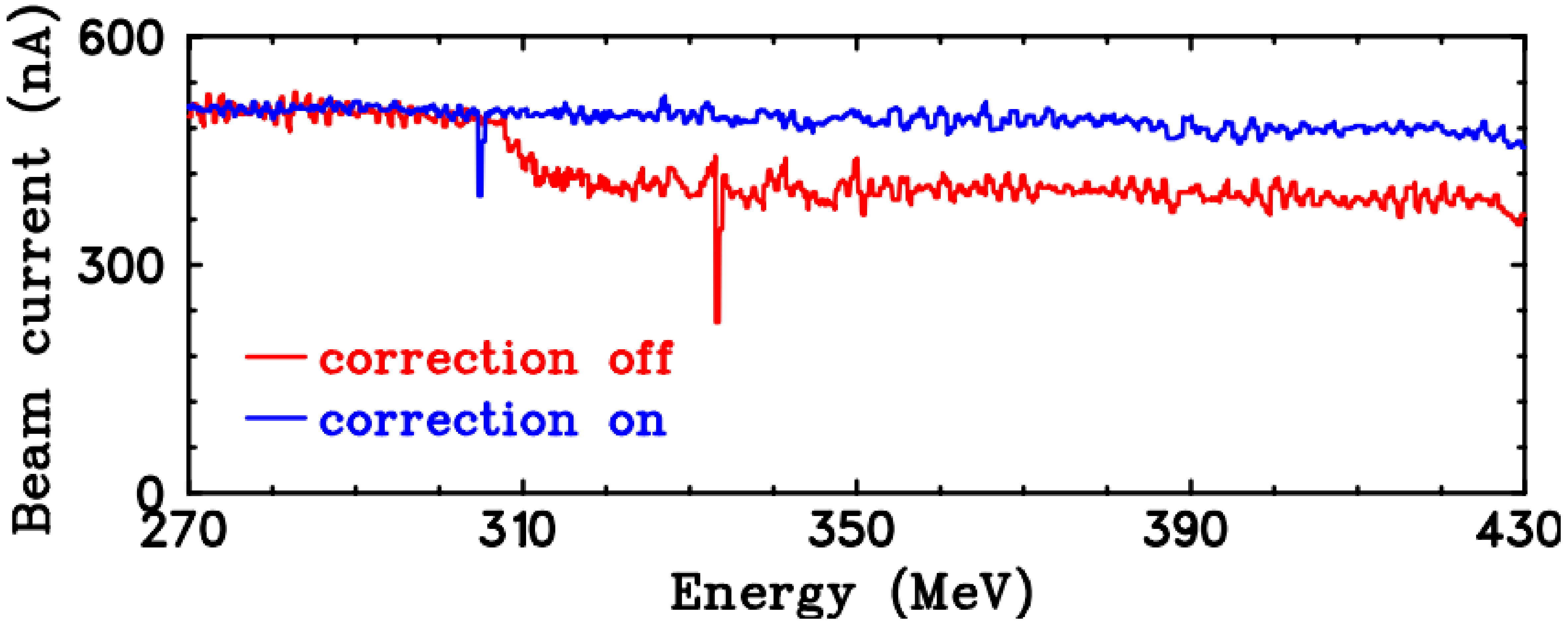

Conclusion

- Gordon's approach gives a clear physical insight into the asymmetric field resulting from a tilted median plane.

- The redundancy in the cyclotron median plane asymmetric field revealed by Gordon's approach could be used to correct the error in the field survey data.

- By manipulating the median plane asymmetric field, we could measure the vertical tune and correct the coupling resonance in cyclotron, which improves the running of TRIUMF cyclotron.

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Thank you
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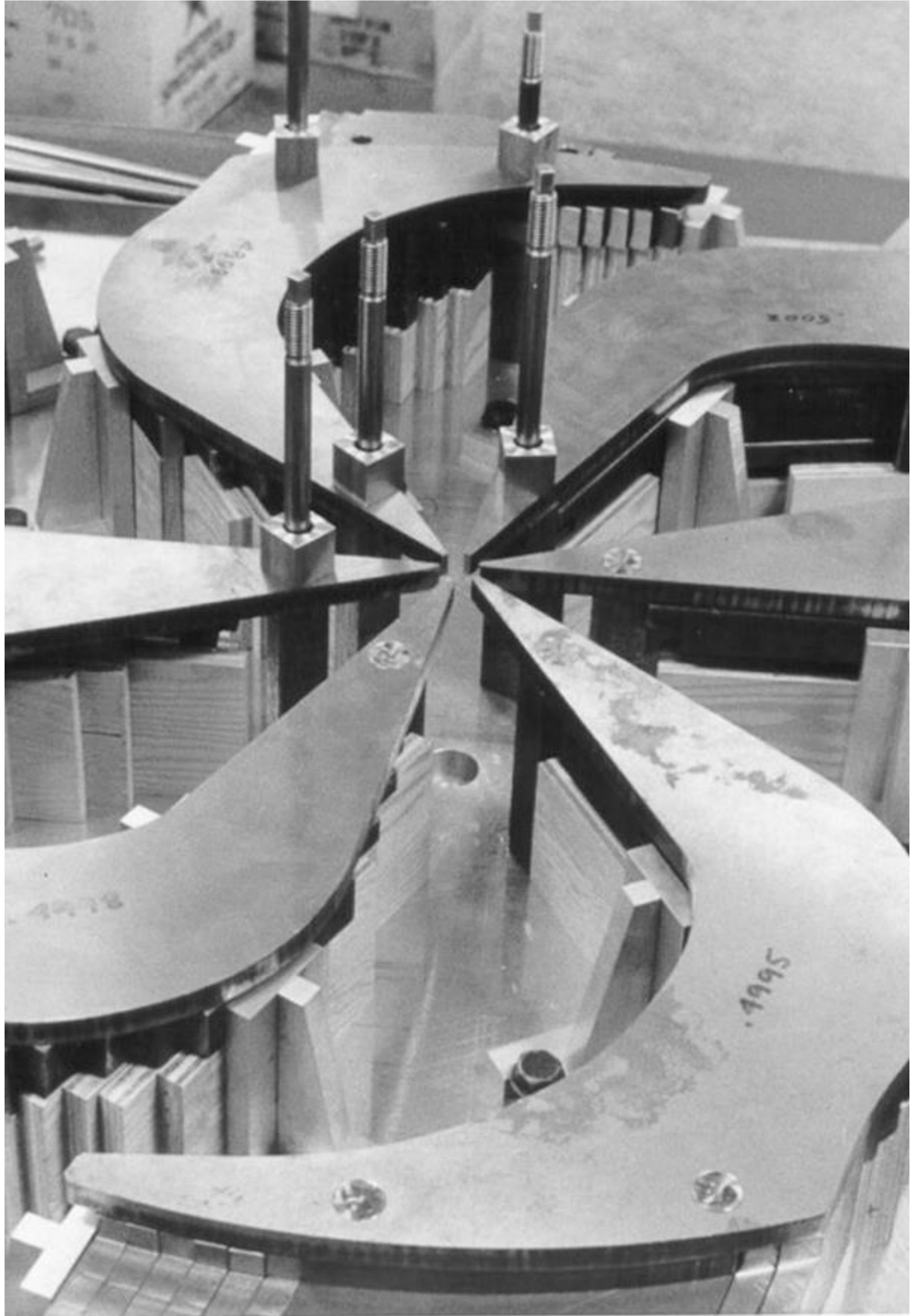

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