H^- Intra-Beam Stripping Loss Rate in Isochronous Cyclotron

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INTRODUCTION

Beam loss is one of the major concerns for high power proton (and H^-) accelerators. With the growing of beam power, the fractional beam loss permitted by the machine radioactivation issue becomes smaller and thus more challenging. For H^- accelerators, the primary beam loss mechanisms include halo formation through beam dynamics problems (e.g. coupling resonance crossing), residual gas stripping, electromagnetic stripping, and intra-beam stripping. In the TRIUMF 500 MeV H^- cyclotron, the total beam loss outside the central region is $< 10\%$ at present: $\sim 1\%$ by gas stripping (under 2×10^{-8} Torr residual pressure), $\sim 3\%$ by electromagnetic stripping (from 400 to 480 MeV), and $\leq 2\%$ by vertical halo growth due to resonance crossings. It was queried how much loss is caused by the intra-beam stripping. The intra-beam stripping arises from binary collisions inside a H^- bunch that cause loosely-bound electrons to be stripped off, leaving neutral H^0 particles, which are subsequently lost. To address this issue for the TRIUMF cyclotron, we derive intra-beam stripping loss rate for an isochronous cyclotron. And then, we apply the theory to the TRIUMF cyclotron to estimate the loss.

DERIVATION OF LOSS RATE

The particle loss rate due to the intra-beam stripping can be calculated by considering a differential volume $d\vec{r} = dxdydz$ in which the incident particles with velocities between $\vec{v}_1$ and $\vec{v}_1 + d\vec{v}_1$ impinge on the target particles in the same bunch at the same location with velocities between $\vec{v}_2$ and $\vec{v}_2 + d\vec{v}_2$. The number of particles scattered into a solid angle over unit time from this collision is the product of number of incident particles, the differential cross section, and the number of target particles, that is,

$$\frac{dN}{dt} = -\frac{N^2}{2}\int d\vec{r}d\vec{v}_1 f(\vec{r},\vec{v}_1)\int d\vec{v}_2 f(\vec{r},\vec{v}_2)|\vec{u}|\sigma(|\vec{u}|)\,, \quad (1)$$

where the distribution function $f(\vec{r},\vec{v})$ is normalized to 1 and $f(\vec{r},\vec{v})d\vec{r}d\vec{v}$ gives the fraction of particles with coordinates and velocities in the range $\vec{r}$ to $\vec{r}+d\vec{r}$, and $\vec{v}$ to $\vec{v}+d\vec{v}$. N is the number of particles in the bunch, $\vec{u} = \vec{v}_1 - \vec{v}_2$ is the relative velocity between colliding particles, $\sigma(|\vec{u}|)$ is the total cross section for single electron stripping. In an isochronous cyclotron, particles in a bunch have no collision longitudinally because any fast moving particle cannot surpass the slow moving ones. In this case, only the transverse velocities matter to the loss rate. So, $f(\vec{r},\vec{v})$ can be written as a product of independent probability density:

$$f(\vec{r},\vec{v}) = f(x,v_x)\,f(y,v_y)\,f(z).$$

The density distribution is assumed to be gaussian. For the x-x' plane (similar for the y-y' plane), it is:

$$f(x,x') = \frac{1}{2\pi\sigma_x\sqrt{\epsilon_x/\beta_x}}\exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{(x'+\alpha_x x/\beta_x)^2}{2\epsilon_x/\beta_x}\right],$$

where $\sigma_x = \sqrt{\beta_x\epsilon_x}$. This means that the velocity distribution at certain location x has a mean value $\bar{x}' = -\alpha_x x/\beta_x$ and a standard deviation $\sqrt{\epsilon_x/\beta_x}$. What matters to the intra-beam stripping is the local velocity (angular) spread $\sqrt{\epsilon_x/\beta_x}$ rather than the entire velocity (angular) spread $\sqrt{\gamma_x\epsilon_x}$. So, in the beam frame, $f(x,v_x)$ can be written as:

$$f(x,v_x) = \frac{1}{2\pi\sigma_x\sigma_{v_x}}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{v_x^2}{2\sigma_{v_x}^2}\right),$$

similar for the $f(y,v_y)$, while $f(z)$ is expressed as:

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right).$$

Eq. 1 is then represented as:

$$\frac{dN}{dt} = -\frac{N^2}{2}I_x I_y I_z|\vec{u}|\sigma(|\vec{u}|), \quad (2)$$

where

$$I_x \equiv \frac{1}{\sqrt{4\pi}\sigma_x}\int f(v_{x_1})dv_{x_1}\int f(v_{x_2})dv_{x_2},$$

$$f(v_{x_{1,2}}) = \frac{1}{\sqrt{2\pi}\sigma_{v_x}}\exp\left(-\frac{v_{x_{1,2}}^2}{2\sigma_{v_x}^2}\right).$$

Similar for I_y and $f(v_{y_{1,2}})$, while

$$I_z \equiv \int f^2(z)dz = \frac{1}{\sqrt{4\pi}\sigma_z}.$$

So Eq. 2 becomes:

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N}{2(4\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\int f(\vec{v}_1)d\vec{v}_1\cdot\int f(\vec{v}_2)d\vec{v}_2|\vec{u}|\sigma(|\vec{u}|), \quad (3)$$

where

$$f(\vec{v}_{1,2}) = \frac{1}{2\pi\sigma_{v_x}\sigma_{v_y}}\exp\left(-\frac{v_{x_{1,2}}^2}{2\sigma_{v_x}^2} - \frac{v_{y_{1,2}}^2}{2\sigma_{v_y}^2}\right), d\vec{v}_{1,2} = dv_{x_{1,2}}dv_{y_{1,2}}.$$

In order to perform the Eq. 3 integration, we do variable transformations:

$$\vec{u} = \vec{v}_1 - \vec{v}_2,\ \vec{w} = \vec{v}_1 + \vec{v}_2,$$

So we have

$$v_{x_1}^2 + v_{x_2}^2 = \frac{u_x^2 + w_x^2}{2},\ v_{y_1}^2 + v_{y_2}^2 = \frac{u_y^2 + w_y^2}{2},$$

and

$$dv_{x_1}dv_{x_2} = \frac{1}{2}du_x dw_x,\ dv_{y_1}dv_{y_2} = \frac{1}{2}du_y dw_y,$$

where $\frac{1}{2}$ is the Jacobian of transformation. Eq. 3 thus becomes

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N}{2(4\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\frac{1}{2\pi\sigma_{v_x}\sigma_{v_y}}\frac{1}{2\pi\sigma_{v_x}\sigma_{v_y}}\frac{1}{4}$$

$$\cdot\int_{-\infty}^{+\infty}\exp\left(-\frac{w_x^2}{4\sigma_{v_x}^2} - \frac{w_y^2}{4\sigma_{v_y}^2}\right)dw_x dw_y$$

$$\cdot\int_{-\infty}^{+\infty}\exp\left(-\frac{u_x^2}{4\sigma_{v_x}^2} - \frac{u_y^2}{4\sigma_{v_y}^2}\right)|\vec{u}|\sigma(|\vec{u}|)du_x du_y.$$

Using the basic formula

$$\int_{-\infty}^{+\infty}e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}\,(a>0),$$

we can easily work out the integration over $\vec{w}$:

$$\int_{-\infty}^{+\infty}\exp\left(-\frac{w_x^2}{4\sigma_{v_x}^2} - \frac{w_y^2}{4\sigma_{v_y}^2}\right)dw_x dw_y = 4\pi\sigma_{v_x}\sigma_{v_y}.$$

So we arrive at

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N}{64\pi^{5/2}\sigma_x\sigma_y\sigma_z\sigma_{v_x}\sigma_{v_y}}\int_{-\infty}^{+\infty}\sigma(|\vec{u}|)\sqrt{u_x^2+u_y^2}$$

$$\cdot\exp\left(-\frac{u_x^2}{4\sigma_{v_x}^2} - \frac{u_y^2}{4\sigma_{v_y}^2}\right)du_x du_y.$$

When the relative velocity dependence of the stripping cross-section is neglected, or, for estimation purpose, we could insert a maximum value σ_{max} of the cross-section and pull it out of the integral. Next, we denote

$$X \equiv \frac{u_x}{2},\ Y \equiv \frac{u_y}{2}.$$

We finally obtain

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N\sigma_{max}\sqrt{\sigma_{v_x}^2 + \sigma_{v_y}^2}}{8\pi^2\sigma_x\sigma_y\sigma_z}\cdot F(\sigma_{v_x},\sigma_{v_y}), \quad (4)$$

where

$$F(a,b) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\sqrt{\frac{X^2+Y^2}{a^2+b^2}}\exp\left(-\frac{X^2}{a^2} - \frac{Y^2}{b^2}\right)\frac{dXdY}{ab} \quad (5)$$

is a dimensionless form factor.

The above derivations are performed in the beam frame. Next, we do relativistic transformation from the beam frame to the lab frame:

$$dt \longrightarrow dt/\gamma,\ \sigma_z \longrightarrow \gamma\sigma_s,$$

$$\sigma_{v_x} = \beta\gamma c\theta_x,\ \sigma_{v_y} = \beta\gamma c\theta_y.$$

We end up getting the loss rate in the lab frame:

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N\sigma_{max}\beta c\sqrt{\theta_x^2+\theta_y^2}}{8\pi^2\gamma\sigma_x\sigma_y\sigma_s}\cdot F(\theta_x,\theta_y), \quad (6)$$

where β and γ are the relativistic factors, $\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon_{x,y}}$ are the rms beam sizes in x and y, $\theta_{x,y} = \sqrt{\epsilon_{x,y}/\beta_{x,y}}$ are the rms angular spreads, σ_s is the rms bunch length. Clearly, the loss rate is proportional to the density of particles in the real space. The form factor $F(a,b)$ does not depend on the absolute values of its parameters (a,b) but only on their ratio, that is,

$$F(a,b) = \frac{E\left(1-b^2/a^2\right)}{\sqrt{1+b^2/a^2}}, \quad (7)$$

where $E(k)$ denotes the complete elliptic integral of the second kind:

$$E(k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\psi}\,d\psi,\ |\arg(1-k)| < \pi.$$

Loss Estimate for TRIUMF Cyclotron

Since the stripping cross-section depends on the velocity of particles, first of all we have to find out the relative velocity in the beam frame. We use smooth approximation for the cyclotron, so we have for the radial direction

$$\sigma_{v_x} = \beta\gamma c\theta_x = \beta\gamma c\sqrt{\frac{\epsilon_x}{\beta_x}} = c\sqrt{\beta\gamma\epsilon_{xn}\frac{Q_x}{\bar{R}}},$$

and similarly for the vertical direction

$$\sigma_{v_y} = c\sqrt{\beta\gamma\epsilon_{yn}\frac{Q_y}{\bar{R}}},$$

where ϵ_{xn} and ϵ_{yn} are the normalized emittances of circulating beam, Q_x and Q_y denote the radial and vertical tunes along an equilibrium orbit of average radius $\bar{R}$. We've calculated the Q_x, Q_y and $\bar{R}$ values for 1563 static equilibrium orbits with energy from 0.3 MeV (injection) to 500.14 MeV (extraction) in a step of 0.32 MeV. Fig. 1 shows the resulting relative velocity over the entire energy range. It's seen that the total relative velocity is between 3×10^{-4} and 7×10^{-4}. This is almost falling on the plateau of the cross section curve, shown in Fig. 1, where the stripping cross section is $\sim 4\times10^{-15}\,\mathrm{cm}^2$.

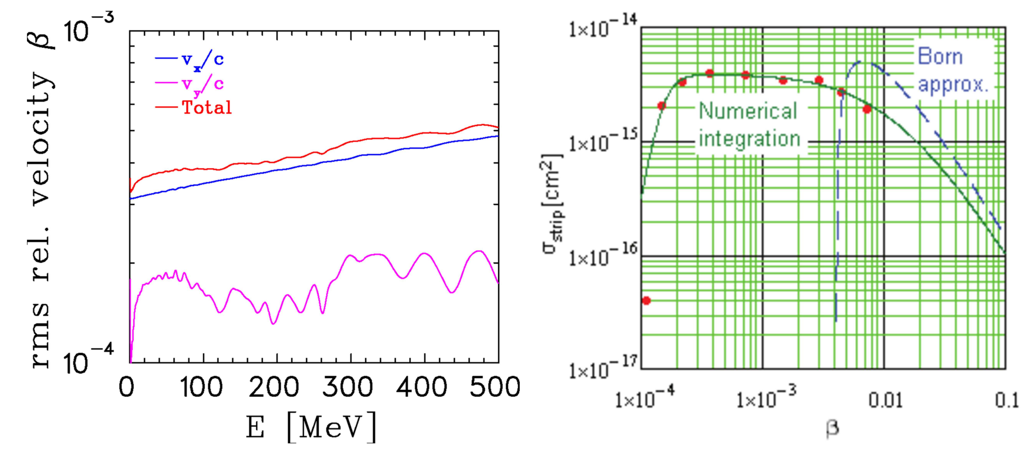

Figure 1: *(Left) rms relative velocity of particles in the beam frame over the entire energy range of TRIUMF cyclotron. (Right) intra-beam stripping cross section vs. rms relative velocity.*

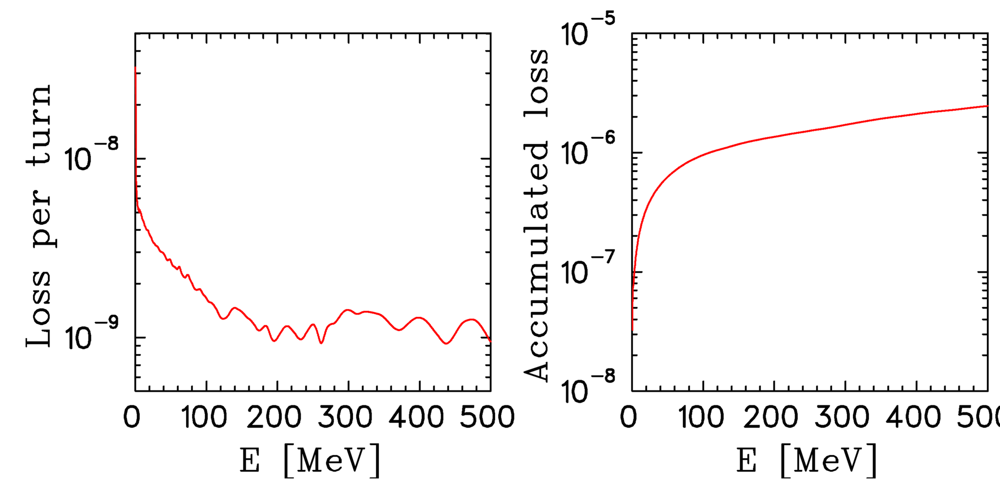

Figure 2: *(Left) fractional loss per turn vs. energy. (Right) accumulated fractional loss vs. energy.*

We assume a typical rf phase width of 40° for the bunch and a peak current of 300 μA. These, along with the nominal rf frequency of 23.055 MHz, give a bunch half length increasing from 1.8 cm to 55 cm, and particles density in the real space ($N/(\sigma_x\sigma_y\sigma_s)$) decreasing from $2.5\times10^8/\mathrm{cm}^3$ to $1.2\times10^7/\mathrm{cm}^3$. Fig. 2 shows the resulting fractional loss per turn and the accumulated loss. The accumulated loss up to 500 MeV, in comparison with the electromagnetic stripping loss, is 3 to 4 order of magnitude lower. This is hardly measurable. Thus it is not a concern.