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H^- INTRA-BEAM STRIPPING LOSS RATE IN ISOCHRONOUS CYCLOTRON*

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Abstract

Binary collisions inside a H^- bunch result in H^- stripping and subsequent particle loss. This phenomenon, called intra-beam stripping, was observed in LEAR and SNS superconducting linac. We mimic the derivation made for the linac to derive the intra-beam stripping loss rate for an isochronous cyclotron. And then, we apply this theory to the TRIUMF 500 MeV H^- cyclotron to estimate the loss.

INTRODUCTION

Beam loss is one of the major concerns for high power proton (and H^-) accelerators. With the growing of beam power, the fractional beam loss permitted by the machine radioactivation issue becomes smaller and thus more challenging. For H^- accelerators, the primary beam loss mechanisms include halo formation through beam dynamics problems (e.g. coupling resonance crossing) [1], residual gas stripping [2], electromagnetic (Lorentz) stripping [3], and intra-beam stripping [4]. In the TRIUMF 500 MeV H^- cyclotron, the total beam loss outside the central region is $< 10\%$ at present: $\sim 1\%$ by gas stripping (under 2×10^{-8} Torr residual pressure), $\sim 3\%$ by electromagnetic stripping (from 400 to 480 MeV), and $\leq 2\%$ by vertical halo growth due to resonance crossings. It was queried how much loss is caused by the intra-beam stripping. The intra-beam stripping arises from binary collisions inside a H^- bunch that cause loosely-bound electrons to be stripped off, leaving neutral H^0 particles, which are subsequently lost due to lack of focusing, steering, and acceleration. This phenomenon was first observed in LEAR [4] and afterwards at SNS superconducting linac [5]. To address this issue for the TRIUMF cyclotron [6], we mimic the derivation made for the linac to derive the intra-beam stripping loss rate for an isochronous cyclotron. And then, we apply the theory to the TRIUMF cyclotron to estimate the loss.

DERIVATION OF LOSS RATE

The particle loss rate due to the intra-beam stripping can be calculated by considering a differential volume $d\vec{r} = dxdydz$ in which the incident particles with velocities between $\vec{v}_1$ and $\vec{v}_1 + d\vec{v}_1$ impinge on the target particles in the same bunch at the same location with velocities between $\vec{v}_2$ and $\vec{v}_2 + d\vec{v}_2$. The number of particles scattered into a solid angle $d\Omega$ over unit time from this collision is the product of number of incident particles, the differential cross section, and the number of target particles. In the beam frame, the loss rate is represented as:

$$\begin{aligned}\frac{dN}{dt} &= -\frac{1}{2}\iint d\vec{r}d\vec{v}_1 N f(\vec{r},\vec{v}_1)\int d\vec{v}_2 N f(\vec{r},\vec{v}_2)|\vec{u}|\frac{d\sigma}{d\Omega}d\Omega \\ &= -\frac{N^2}{2}\iint d\vec{r}d\vec{v}_1 f(\vec{r},\vec{v}_1)\int d\vec{v}_2 f(\vec{r},\vec{v}_2)|\vec{u}|\sigma(|\vec{u}|), \quad (1)\end{aligned}$$

where the distribution function $f(\vec{r},\vec{v})$ is normalized to 1 and $f(\vec{r},\vec{v})d\vec{r}d\vec{v}$ gives the fraction of particles with coordinates and velocities in the range $\vec{r}$ to $\vec{r}+d\vec{r}$, and $\vec{v}$ to $\vec{v}+d\vec{v}$. N is the number of particles in the bunch, $\vec{u} = \vec{v}_1 - \vec{v}_2$ is the relative velocity between colliding particles, $\sigma(|\vec{u}|)$ is the total cross section for single electron stripping. The factor $1/2$ in the front of the integral removes the double counting of each collision in the integral.

Equation (1) is a general expression. In an isochronous cyclotron, particles in a bunch have no collision longitudinally. This is because any fast moving particle cannot surpass the slow moving ones. Unlike the synchrotron where there exists periodical phase oscillation longitudinally, isochronous cyclotron has no phase oscillation; instead, particles of different energies have the same revolution period. In this case, the particle's longitudinal velocity does not matter to the loss rate, only the transverse velocities involve. So, $f(\vec{r},\vec{v})$ can be written as a product of independent probability density as follows:

$$f(\vec{r},\vec{v}) = f(x,v_x)\, f(y,v_y)\, f(z). \quad (2)$$

The density distribution is assumed to be gaussian. For the x-x' plane (similar for the y-y' plane), it is:

$$f(x,x') = \frac{1}{2\pi\sigma_x\sqrt{\epsilon_x/\beta_x}}\exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{(x'+\alpha_x x/\beta_x)^2}{2\epsilon_x/\beta_x}\right], \quad (3)$$

where $\sigma_x = \sqrt{\beta_x\epsilon_x}$. This means that the velocity distribution at certain location x has a mean value $\bar{x'} = -\alpha_x x/\beta_x$ and a standard deviation $\sqrt{\epsilon_x/\beta_x}$. What matters to the intra-beam stripping is the local velocity (angular) spread $\sqrt{\epsilon_x/\beta_x}$ rather than the entire velocity (angular) spread $\sqrt{\gamma_x\epsilon_x}$. So, in the beam frame, $f(x,v_x)$ can be written as:

$$f(x,v_x) = \frac{1}{2\pi\sigma_x\sigma_{v_x}}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{v_x^2}{2\sigma_{v_x}^2}\right), \quad (4)$$

similar for the $f(y,v_y)$, while $f(z)$ is expressed as:

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right). \quad (5)$$

Equation (1) is then represented as:

$$\frac{dN}{dt} = -\frac{N^2}{2}I_x I_y I_z|\vec{u}|\sigma(|\vec{u}|), \quad (6)$$

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where

$$\begin{aligned} I_x &\equiv \iint f(x, v_{x_1}) dx dv_{x_1} \int f(x, v_{x_2}) dv_{x_2} \\ &= \frac{1}{\sqrt{4\pi}\sigma_x} \int f(v_{x_1}) dv_{x_1} \int f(v_{x_2}) dv_{x_2}, \end{aligned} \tag{7}$$

where

$$f(v_{x_{1,2}}) = \frac{1}{\sqrt{2\pi}\sigma_{v_x}} \exp\left(-\frac{v_{x_{1,2}}^2}{2\sigma_{v_x}^2}\right). \tag{8}$$

Similarly, we have

$$I_y \equiv \frac{1}{\sqrt{4\pi}\sigma_y} \int f(v_{y_1}) dv_{y_1} \int f(v_{y_2}) dv_{y_2}, \tag{9}$$

and

$$f(v_{y_{1,2}}) = \frac{1}{\sqrt{2\pi}\sigma_{v_y}} \exp\left(-\frac{v_{y_{1,2}}^2}{2\sigma_{v_y}^2}\right), \tag{10}$$

while

$$\begin{aligned} I_z &\equiv \int f^2(z) dz = \frac{1}{(\sqrt{2\pi}\sigma_z)^2} \int_{-\infty}^{+\infty} \exp\left(-\frac{z^2}{\sigma_z^2}\right) dz \\ &= \frac{1}{\sqrt{4\pi}\sigma_z}. \end{aligned} \tag{11}$$

So Eq. (6) becomes:

$$\begin{aligned} \frac{1}{N}\frac{dN}{dt} = &- \frac{N}{2(4\pi)^{3/2}\sigma_x\sigma_y\sigma_z} \int f(\vec{v}_1) d\vec{v}_1 \\ &\cdot \int f(\vec{v}_2) d\vec{v}_2 |\vec{u}| \sigma(|\vec{u}|), \end{aligned} \tag{12}$$

where

$$\begin{aligned} f(\vec{v}_{1,2}) &= \frac{1}{2\pi\sigma_{v_x}\sigma_{v_y}} \exp\left(-\frac{v_{x_{1,2}}^2}{2\sigma_{v_x}^2} - \frac{v_{y_{1,2}}^2}{2\sigma_{v_y}^2}\right), \\ d\vec{v}_{1,2} &= dv_{x_{1,2}} dv_{y_{1,2}}. \end{aligned} \tag{13}$$

In order to perform the Eq. (12) integration, we do variable transformations:

$$\vec{u} = \vec{v}_1 - \vec{v}_2, \quad \vec{w} = \vec{v}_1 + \vec{v}_2, \tag{14}$$

which are

$$\begin{aligned} u_x = v_{x_1} - v_{x_2}, \quad w_x = v_{x_1} + v_{x_2}, \\ u_y = v_{y_1} - v_{y_2}, \quad w_y = v_{y_1} + v_{y_2}. \end{aligned} \tag{15}$$

So we have

$$v_{x_1}^2 + v_{x_2}^2 = \frac{u_x^2 + w_x^2}{2}, \quad v_{y_1}^2 + v_{y_2}^2 = \frac{u_y^2 + w_y^2}{2}, \tag{16}$$

and

$$dv_{x_1} dv_{x_2} = \frac{1}{2} du_x dw_x, \quad dv_{y_1} dv_{y_2} = \frac{1}{2} du_y dw_y, \tag{17}$$

where $\frac{1}{2}$ is the Jacobian of transformation. Equation (12) thus becomes

$$\begin{aligned} \frac{1}{N}\frac{dN}{dt} = &- \frac{N}{2(4\pi)^{3/2}\sigma_x\sigma_y\sigma_z} \frac{1}{2\pi\sigma_{v_x}\sigma_{v_y}} \frac{1}{2\pi\sigma_{v_x}\sigma_{v_y}} \frac{1}{4} \\ &\cdot \iint_{-\infty}^{+\infty} \exp\left(-\frac{w_x^2}{4\sigma_{v_x}^2} - \frac{w_y^2}{4\sigma_{v_y}^2}\right) dw_x dw_y \\ &\cdot \iint_{-\infty}^{+\infty} \exp\left(-\frac{u_x^2}{4\sigma_{v_x}^2} - \frac{u_y^2}{4\sigma_{v_y}^2}\right) |\vec{u}| \sigma(|\vec{u}|) du_x du_y. \end{aligned} \tag{18}$$

Using the basic formula

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \quad (a > 0), \tag{19}$$

we can easily work out the integration over $\vec{w}$:

$$\iint_{-\infty}^{+\infty} \exp\left(-\frac{w_x^2}{4\sigma_{v_x}^2} - \frac{w_y^2}{4\sigma_{v_y}^2}\right) dw_x dw_y = 4\pi\sigma_{v_x}\sigma_{v_y}. \tag{20}$$

So we arrive at

$$\begin{aligned} \frac{1}{N}\frac{dN}{dt} = &- \frac{N}{64\pi^{5/2}\sigma_x\sigma_y\sigma_z\sigma_{v_x}\sigma_{v_y}} \iint_{-\infty}^{+\infty} \sigma(|\vec{u}|) \sqrt{u_x^2 + u_y^2} \\ &\cdot \exp\left(-\frac{u_x^2}{4\sigma_{v_x}^2} - \frac{u_y^2}{4\sigma_{v_y}^2}\right) du_x du_y. \end{aligned} \tag{21}$$

When the relative velocity dependence of the stripping cross-section is neglected, or, for estimation purpose, we could insert a maximum value σ_{max} of the cross-section and pull it out of the integral. Next, we denote

$$X \equiv \frac{u_x}{2}, \quad Y \equiv \frac{u_y}{2}, \tag{22}$$

Eq. (21) then reads

$$\begin{aligned} \frac{1}{N}\frac{dN}{dt} = &- \frac{N\sigma_{max}\sqrt{\sigma_{v_x}^2 + \sigma_{v_y}^2}}{8\pi^{5/2}\sigma_x\sigma_y\sigma_z} \iint_{-\infty}^{+\infty} \frac{\sqrt{X^2 + Y^2}}{\sqrt{\sigma_{v_x}^2 + \sigma_{v_y}^2}} \\ &\exp\left(-\frac{X^2}{\sigma_{v_x}^2} - \frac{Y^2}{\sigma_{v_y}^2}\right) \frac{dX dY}{\sigma_{v_x}\sigma_{v_y}}. \end{aligned} \tag{23}$$

We finally obtain

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N\sigma_{max}\sqrt{\sigma_{v_x}^2 + \sigma_{v_y}^2}}{8\pi^2\sigma_x\sigma_y\sigma_z} \cdot F(\sigma_{v_x}, \sigma_{v_y}), \tag{24}$$

where

$$F(a, b) = \frac{1}{\sqrt{\pi}} \iint_{-\infty}^{+\infty} \sqrt{\frac{X^2+Y^2}{a^2+b^2}} \exp\left(-\frac{X^2}{a^2} - \frac{Y^2}{b^2}\right) \frac{dX dY}{ab} \tag{25}$$

is a dimensionless form factor.

Should be noted that the above derivations are performed in the beam frame. Next, we have to do relativistic transformation from the beam frame to the lab frame:

$$dt \longrightarrow dt/\gamma, \quad \sigma_z \longrightarrow \gamma\sigma_s,$$

and

$$\sigma_{v_x} = \beta\gamma c\theta_x, \quad \sigma_{v_y} = \beta\gamma c\theta_y.$$

We end up getting the loss rate in the lab frame:

$$\frac{1}{N}\frac{dN}{dt} = -\frac{N\sigma_{max}\beta c\sqrt{\theta_x^2+\theta_y^2}}{8\pi^2\gamma\sigma_x\sigma_y\sigma_s}\cdot F(\theta_x,\theta_y), \qquad (26)$$

where β and γ are the relativistic factors, $\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon_{x,y}}$ are the rms beam sizes in x and y, $\theta_{x,y} = \sqrt{\epsilon_{x,y}/\beta_{x,y}}$ are the rms angular spreads, σ_s is the rms bunch length. Clearly, the loss rate is proportional to the density of particles in the real space.

The reason why we write the form factor as $F(\theta_x,\theta_y)$ instead of $F(\beta c\gamma\theta_x, \beta c\gamma\theta_y)$ is because $F(a,b)$ does not depend on the absolute values of its parameters (a,b) but only on their ratio, that is,

$$F(a,b) = \frac{E\left(1-b^2/a^2\right)}{\sqrt{1+b^2/a^2}}, \qquad (27)$$

where $E(k)$ denotes the complete elliptic integral of the second kind:

$$E(k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\psi}\,d\psi, \quad |\arg(1-k)| < \pi, \quad (28)$$

where $\arg(1-k)$ denotes the phase angle of $(1-k)$ in the complex plane. When k is real and $(1-k) \geq 0$, then $\arg(1-k) = 0 < \pi$ and $E(k)$ is real. This implies a condition $b^2/a^2 \geq 0$, which is always true in our case here.

LOSS ESTIMATE FOR TRIUMF CYCLOTRON

Since the stripping cross-section depends on the velocity of particles, first of all we have to find out the relative velocity in the beam frame. We use smooth approximation for the cyclotron, so we have for the radial direction

$$\sigma_{v_x} = \beta\gamma c\theta_x = \beta\gamma c\sqrt{\frac{\epsilon_x}{\beta_x}} = c\sqrt{\beta\gamma\epsilon_{xn}\frac{Q_x}{\overline{R}}}, \qquad (29)$$

and similarly for the vertical direction

$$\sigma_{v_y} = c\sqrt{\beta\gamma\epsilon_{yn}\frac{Q_y}{\overline{R}}}, \qquad (30)$$

where ϵ_{xn} and ϵ_{yn} are the normalized emittances of circulating beam, Q_x and Q_y denote the radial and vertical tunes along an equilibrium orbit of average radius $\overline{R}$.

We've calculated the Q_x, Q_y and $\overline{R}$ values for 1563 static equilibrium orbits with energy from 0.3 MeV (injection) to 500.14 MeV (extraction) in a step of 0.32 MeV. The 0.32 MeV energy step accounts for a rf amplitude of 92 kV and rf phase excursion of 30° in average. Besides, we take $\epsilon_{xn} = \epsilon_{yn} = 1\pi$mm-mrad. These parameters are pessimistic estimates, so we are slightly exaggerating the accumulated loss. Figure 1 shows the resulting relative velocity over the entire energy range. It's seen that the total relative velocity is between 3×10^{-4} and 7×10^{-4}. This is almost falling on the plateau of the cross section curve [5], shown in Fig. 1, where the stripping cross section is ~4×10^{-15} cm^2.

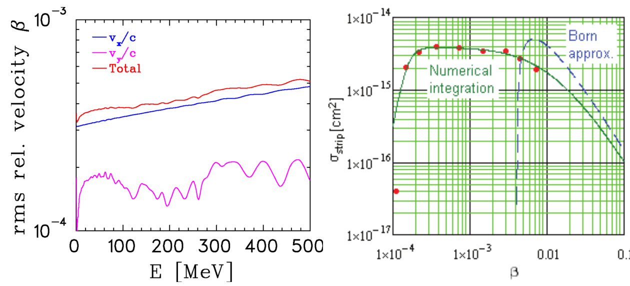

Figure 1: (Left) rms relative velocity of particles in the beam frame over the entire energy range of TRIUMF cyclotron. (Right) intra-beam stripping cross section vs. rms relative velocity.

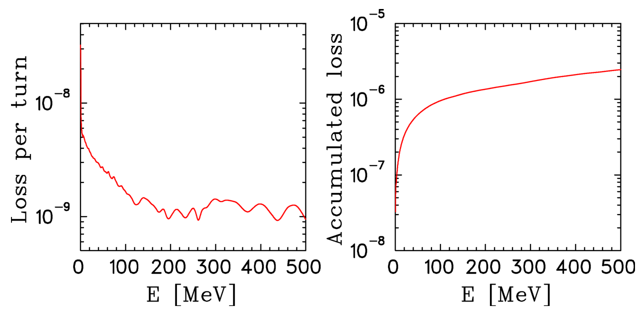

Figure 2: (Left) fractional loss per turn vs. energy. (Right) accumulated fractional loss vs. energy.

We assume a typical rf phase width of 40° for the bunch and a peak current of 300 µA. These, along with the nominal rf frequency of 23.055 MHz, give a bunch half length increasing from 1.8 cm to 55 cm, and particle density in the real space $(N/(\sigma_x\sigma_y\sigma_s))$ decreasing from 2.5×10^8/cm^3 to 1.2×10^7/cm^3. Figure 2 shows the resulting fractional loss per turn and the accumulated loss. The accumulated loss up to 500 MeV, in comparison with the electromagnetic stripping loss [7], is 3 to 4 order of magnitude lower. This is hardly measurable.

SUMMARY

The intra-beam stripping loss rate of H$^-$ in an isochronous cyclotron is derived. Estimate for the loss is then made for the TRIUMF 500 MeV cyclotron. The accumulated loss up to 500 MeV turns out to be 3 – 4 order of magnitude lower than the electromagnetic stripping loss. This is hardly measurable. Thus it is not a concern. Nevertheless, it's worthwhile to

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mention that the intra-beam stripping theory is applicable to other hydrogen molecular ions like H_2^+ and H_3^+. When the particle density reaches the order of $>10^{11}/\mathrm{cm}^3$, the loss rate might become a concern, depending on the stripping or dissociation cross section.

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