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CYCLOTRON BEAM EXTRACTION BY ACCELERATION

C. Baumgarten*, Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

Abstract

One of the decisive issues in the design of a new cyclotron is the choice of the beam extraction method. Typical methods are extraction by electrostatic extractors and by stripping. The former method requires DC high voltage electrodes which are notorious for high-voltage breakdowns. The latter method requires beams of atomic or molecular ions which are notorious for rest-gas- and Lorentz-stripping. We discuss the conditions to be met such that a charged particle beam will leave the magnetic field of an isochronous cyclotron purely by fast acceleration.

INTRODUCTION

One of the first decisions to be made in the conceptual design phase of a new cyclotron regards the method of beam extraction. The most frequently used methods are extraction by electrostatic septum extractors or by stripper foils. The latter requires the use of projectiles which are not fully stripped yet, i.e., for instance H^- or H_2^+ instead of bare protons (H^+). But stripping of H^- or H_2^+ is also known due to scattering of the projectiles with rest gas or by strong fields (i.e., Lorentz stripping). In order to avoid internal loss of particles and the ambient activation due to these losses, the field must be limited to rather low values (especially in case of H^-), and/or an excellent vacuum pressure is obligatory, typically 10^{-7} bar and below.

The use of electrostatic extractors (EEs) has the disadvantage that such devices are notorious for high-voltage breakdowns. Experience at PSI has shown that EEs are also sensitive to contamination by dust and dirt and to rf-power leaking off the accelerating structures.

So-called "self-extracting" cyclotrons have been suggested and built before [1], but the conditions to be met by such cyclotrons have, to the author's knowledge, never been fully clarified.

Here we investigate the question whether cyclotrons can be designed such that the beam leaves the cyclotron purely by fast acceleration. In order to answer this question we will first review the reasons why the beam does (usually) not leave the field of isochronous cyclotrons simply by acceleration.

THE SMOOTH ACCELERATION APPROXIMATION

For this purpose we use a simplified cyclotron model, i.e., we will ignore the requirement to use azimuthally varying fields (AVF) to obtain vertical stability. In cases where one is interested exclusively in the radial motion, this approximation is adequate since the AVF has only a weak influence on the radial tune.

Hence we assume rotational symmetry, that is, the field is a function of radius only $B = B(R)$. As well known, the field is isochronous for

$$B_{iso}(R) = B_0\,\gamma \tag{1}$$

where $\gamma = (1-\beta^2)^{-1}$ is the relativistic factor. Since the orbit length per revolution is $s = 2\pi\sqrt{R^2 + (\frac{dR}{d\theta})^2}$ and since $\frac{dR}{d\theta} \ll R$ usually holds, the velocity is in good approximation given by $v = \omega_{rev}\,R$ and hence we define the radial function

$$\gamma_R = \frac{1}{\sqrt{1 - R^2/a^2}} \tag{2}$$

with the so-called "cyclotron radius" $a = c/\omega_{rev}$. However any real world machine has a finite radius $R < a$ and is isochronous only up to the fringe field. A simple and reasonably realistic model of the fringe field is given by an Enge-type function [2, 3]

$$f(R) = (1 + \exp\left((R - R_h)/g\right))^{-1} \tag{3}$$

where R_h is the radius for $f = 1/2$ and g is a parameter which is approximately half of the pole gap. The magnetic field is then assumed to be

$$B(R) = B_{iso}\,f(R) = B_0\,\gamma_R\,f(R)\,. \tag{4}$$

When the beam enters the fringe field, the bunches will necessarily get out of sync with the accelerating rf voltage. The revolution frequency ω_{rev} is

$$\omega_{rev} = \frac{q}{m\,\gamma}\,B = \frac{q}{m\,\gamma}\,B_0\,\gamma_R\,f = \omega_0\,\frac{\gamma_R}{\gamma}\,f\,. \tag{5}$$

The velocity then is $v = \omega_{rev}\,R$ and the relativistic γ-factor

$$\gamma = 1/\sqrt{1 - \omega_{rev}^2\,R^2/c^2} = 1/\sqrt{1 - (\gamma_R^2 - 1)\,f^2/\gamma^2}\,. \tag{6}$$

Solving for γ gives

$$\gamma = \sqrt{1 + (\gamma_R^2 - 1)\,f^2}\,. \tag{7}$$

THE BENDING LIMIT

Any finite magnetic field has a bending limit, which can be obtained from

$$p = q\,R\,B(R)\,. \tag{8}$$

The momentum is maximal when $\frac{dp}{dR} = 0$ with

$$\frac{dp}{dR} = q\,B\,(1 + k) \tag{9}$$

where the field index k is defined by

$$k = \frac{R}{B}\,\frac{dB}{dR}\,. \tag{10}$$

* christian.baumgarten@psi.ch

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Hence the bending limit is reached for $k = -1$. Inserting $B(R)$ from Eq. (4) gives

$$k = \gamma_R^2 - 1 - (1 - f)\,\frac{R}{g} \tag{11}$$

so that the ultimate extraction point is characterized by

$$f = 1 - \gamma_R^2\,\frac{g}{R}\,. \tag{12}$$

Since γ_R is of order $O(1)$, and since the half-gap g holds usually $g \ll R$, the value of f at extraction is still close to one, and $1 - f$ is small.

THE PHASE SHIFT BY THE FRINGE FIELD

Usually the rf frequency is an integer multiple N_h (called *harmonic number*) of the orbital frequency ω_0: $\omega_{rf} = N_h\,\omega_0$. The phase shift per turn $\frac{d\phi}{dn}$ is given by

$$\frac{d\phi}{dn} = T_{rev}\,(\omega_{rf} - N_h\,\omega_{rev}) = 2\,\pi\,N_h\,(\frac{\omega_0}{\omega_{rev}} - 1)\,. \tag{13}$$

The phase shift per radius can therefore be written as

$$\frac{d\phi}{dR} = \frac{d\phi}{dn}/\frac{dR}{dn}\,. \tag{14}$$

The phase shift hence strongly depends on the radius gain per turn, which is known to be directly proportional to the energy gain per turn dE/dn:

$$\frac{dR}{dn} = \frac{dE}{dn}\,\frac{R\,\gamma}{E\,(\gamma+1)\,(1+k)}\,. \tag{15}$$

Most cyclotrons have an energy gain which is too small to prevent the phase from approaching 90° in the fringe field and from being decelerated back to the center. On the other hand, *if* the rf voltage of the accelerating structures ("Dees" or cavities) is high enough to approach the bending limit, then the beam will leave the magnetic field.

It follows, after some algebra [1], that

$$\frac{d\phi}{dR} = \frac{2\,\pi\,N_h\,m\,c^2}{dE/dn}\,(1 - \frac{\gamma_R\,f}{\gamma})\,(1+k)\,\gamma_R\,f\,\frac{R}{a^2}\,. \tag{16}$$

By the use of

$$\frac{df}{dR} = -\frac{1}{g}\,f\,(1-f) \tag{17}$$

one can change the independent variable from R to f so that

$$\frac{d\phi}{df} = -\frac{2\,\pi\,N_h\,m\,c^2}{dE/dn}\,(1 - \frac{\gamma_R\,f}{\gamma})\,(\frac{g\,\gamma_R^2}{1-f} - R)\,\gamma_R\,\frac{R}{a^2}\,. \tag{18}$$

In the region of interest, i.e., the fringe field region, the value of $1 - f$ changes rapidly over short distances, while the (relative) change of R and γ_R is small. Hence one can obtain a reasonable approximation of the term $(1 - \frac{\gamma_R\,f}{\gamma})$ by a first order Taylor series in f, located at $f = 1$, assuming that R and γ_R are constant:

$$(1 - \frac{\gamma_R\,f}{\sqrt{1 + (\gamma_R^2 - 1)\,f^2}}) = \frac{1-f}{\gamma_R^2} + \dots \tag{19}$$

With this approximation one obtains

$$\frac{d\phi}{df} \approx -\frac{2\,\pi\,N_h\,m\,c^2}{dE/dn\,\gamma_R}\,(g\,\gamma_R^2 - R\,(1-f))\,\frac{R}{a^2}\,. \tag{20}$$

With the energy gain given by

$$E = q\,V_{rf}\,\cos(\phi) \tag{21}$$

it follows that

$$d\sin(\phi) \approx -\frac{2\,\pi\,N_h\,m\,c^2}{q\,V_{rf}\,\gamma_R}\,(g\,\gamma_R^2 - R\,(1-f))\,\frac{R}{a^2}\,df\,. \tag{22}$$

the integration, again assuming that R and γ_R are approximately constant, from $f = 1$ to the extraction point (Eq. (12)) then gives

$$\sin(\phi_f) - \sin(\phi_i) = \frac{\pi\,N_h\,m\,c^2\,g^2\,\gamma_R\,(\gamma_R^2 - 1)}{q\,V_{rf}\,R^2}\,. \tag{23}$$

Finally we may approximate $\gamma_R \approx \gamma$ so that our main result becomes

$$\sin(\phi_f) - \sin(\phi_i) = \frac{\pi\,N_h\,E\,\gamma\,(\gamma+1)}{q\,V_{rf}}\,\frac{g^2}{R^2}\,. \tag{24}$$

If the initial phase ϕ_i is zero, and the final phase is supposed to be below 90°, $\sin(\phi_f) \le 1$ then

$$\frac{\pi\,N_h\,E\,\gamma\,(\gamma+1)}{q\,V_{rf}}\,\frac{g^2}{R^2} \le 1 \tag{25}$$

and hence

$$\frac{q\,V_{rf}}{E} \ge \pi\,N_h\,\gamma\,(\gamma+1)\,\frac{g^2}{R^2}\,. \tag{26}$$

EXAMPLES

COMET

The first example we consider is PSI's 250 MeV proton therapy cyclotron "COMET" [5, 6], which has a harmonic number $N_h = 2$, an extraction radius of $R = 810$ mm and a half-gap of about 22 mm. Inserting these numbers into Eq. (26) yields a minimal accelerating voltage of ≈ 3.25 MV per turn. The real voltage is about 380 kV, i.e., about an order of magnitude to low. Escape extraction would therefore be within reach, if the gap would be reduced by a factor of about 3, which is about 7 mm.

[1] More details are given in Ref. [4].

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Ring Cyclotron

The second example is the PSI's 590 MeV Ring cyclotron, an 8-sector machine with an extraction radius of 4.5 m, operated with $N_h = 6$. The gap has an elliptic shape, but at extraction it has almost the same gap as COMET, i.e., $g = 20$ mm. According to Eq. (26) the minimal voltage required for escape extraction is then $V_{rf} \geq 940$ kV. Since the average energy gain per turn of the PSI Ring is about 2.8 MeV, this machine already fullfills the condition for escape extraction.

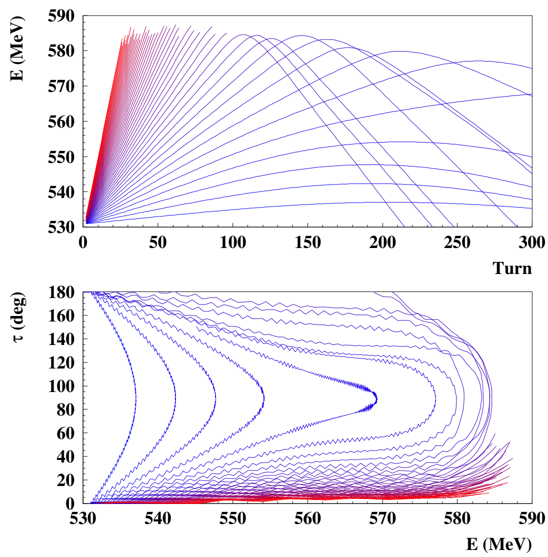

Figure 1: Top: Energy vs. turn-number for particles starting with phase $\phi_i = 0$ for various values of V_{rf} (increasing from blue to red). Bottom: Phase $\phi(E)$ for the same conditions.

Figure 1 shows some results of tracking calculations for the PSI Ring machine for various rf voltages, from zero (blue) to the actual voltage (red), without electrostatic extractor. The calculation start with initial phase $\phi_i = 0$ before extraction (at $E = 530$ MeV). For about 90 turns and less, i.e., for a voltage V_{rf} of more than (590 MeV – 560 MeV)/90 = 670 kV/turn, the beam does not reach 90° before extraction. As the red lines in Fig. 1 show, the change in phase due to the fringe field is very small and can almost be neglected.

However, even though the protons will definitely leave the field without electrostatic extractor, the strong negative gradient of the fringe field severely defocuses the beam. Without any measures - like for instance a magnetic gradient corrector - the protons would leave the field almost without preferred direction. Figure 2 shows that the final direction of a family of tracks with very similar starting conditions as some central orbit is almost arbitrary. Therefore extraction by acceleration (or "self-extraction"), requires a method to keep the beam radially compact. The simplest method to introduce a "gradient corrector" is the installation of a passive magnetic channel, i.e., of iron bars or stripes in the pole gap of the sector magnets which are magnetized by the main field itself. As orbit tracking calculations showed, the extraction is so fast that the field gradient is negative

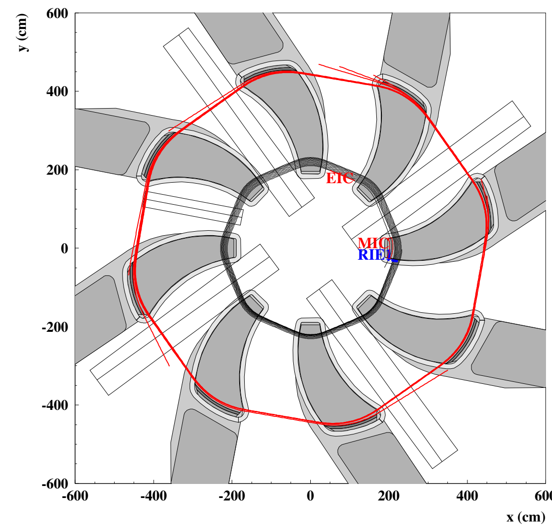

Figure 2: Top view the PSI Ring cyclotron (sector shapes and positions of the cavities) with a family of median plane orbits. The first few turns are indicated in black and the last turns in red.

only in the very last sector so that a gradient correction of the last sector should be sufficient to keep the beam radially compact (see Fig. 3). As shown in Fig. 4, this chosen family

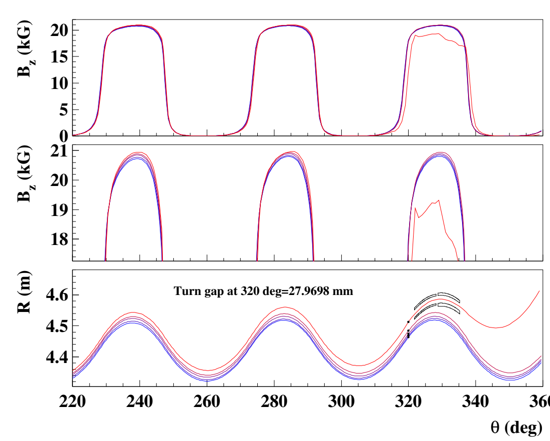

Figure 3: Top: Magnetic field along the last five turns (blue to red) in the Ring cyclotron with a magnetic gradient corrector installed on the last sector before extraction. Middle: Same plot with different scale. The last turn (red) "sees" the highest field up to the last sector. Bottom: Radius versus angle of the last five turns and the last sectors before extraction. The location of the main iron strips of the gradient corrector is indicated in black.

of tracks remains close together. An answer to the question whether this method could be used in the PSI Ring cyclotron with the required extraction efficiency of 99.99 %, requires further detailed studies. Nonetheless, the results show, as we believe, that extraction without electrostatic extractors

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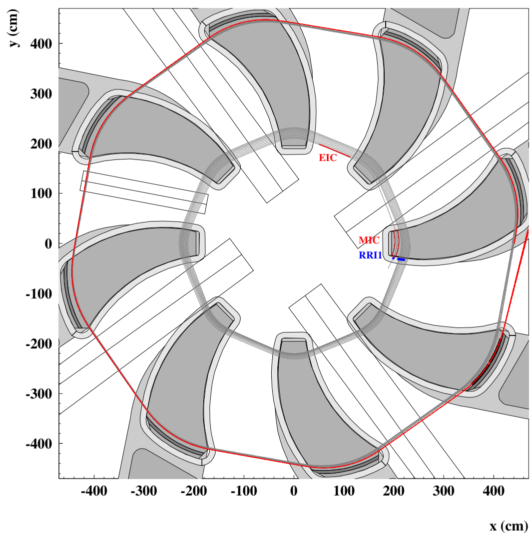

Figure 4: Median plane of the PSI Ring cyclotron with a family of median plane orbits. The first few turns are indicated in black and the last turns in red. The last sector (at $\theta \approx 315°$) before extraction is equipped with a passive magnetic channel.

is possible, specifically in cyclotrons with a small ratio g/R and a high energy gain per turn.

DREAM MACHINES

Both conditions, i.e., high radius and high energy gain per turn, are met in so-called "Dream machines" [7, 8], high current proton drivers for energies ≥ 800 MeV, which could be used for the transmutation of nuclear waste or for spallation neutron sources. The incredible increase of the beam current of the PSI Ring cyclotron from the originally specified 100 µA to 2.4 mA confirmed Joho's N^3-law [9], which states that the maximum beam current, assuming constant losses, increases with the third power of the energy gain per turn - or with the third power of the inverse turn number N^{-3}, respectively. Due to this law Dream machines require a large energy gain per turn.

The energy efficiency η_{acc}, one of the most important figures of this type of machines, is given by [10, 11]:

$$\eta_{acc} = \frac{P_{beam}}{P_{ohmic} + P_{beam}/\eta_{rf} + P_{aux}} \tag{27}$$

where $P_{ohmic} = N_{cav}\, V_{cav}^2/(2\, R_{cav})$ is the total ohmic loss in the walls of the cavities. Hence the ohmic loss increases linear with the number of cavities but quadratic with the voltage of the individual cavities. From the point of view of energy efficiency and reliability, dream machines have a large number of cavities, which naturally leads to a large number of sectors and a large radius. According to our result, these requirements reduce the phase shift caused by the lack of isochronism in the fringe field region and therefore facilitate extraction by acceleration.

ACCURACY

With respect to the accuracy of Eq. (26) one can distinguish two sources of uncertainty, firstly the error due to the use of the sectorless smooth acceleration approximation and secondly the error due to the approximations which have been used to derive Eq. (26). The former uncertainty can not be avoided and the latter depends on the accuracy of the fringe field model and the accuracy of the main parameter g.

The accuracy of Eq. (26) depends of the validity of it's main assumption, namely that the simplest Enge function with a single parameter g, which is proportional to the gap size, provides a reasonable description of the fringe field. As

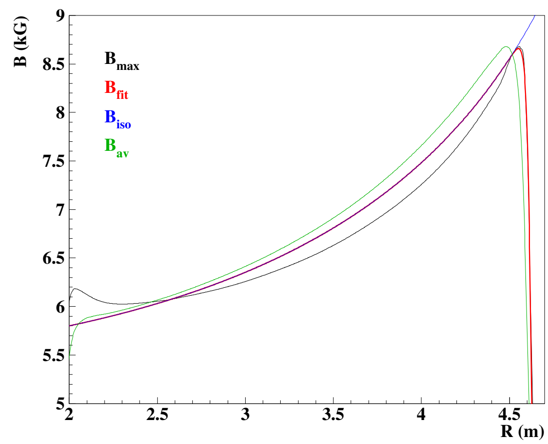

Figure 5: Fringe field of the PSI Ring cyclotron: Azimuthal average (green) vs. maximum field (black) and fit with Enge function to maximum field (red). The isochrounous field $B_{iso} = B_0\, \gamma_R$ is shown is blue.

shown in Fig. 5, the exact shape of the fringe field - and the parameter g - in a real cyclotron might depends significantly on the method of evaluation. Due to the azimuthal scalloping of the orbit, the azimuthal average of the magnetic field is usually less steep than the maximum field, taken as a function of radius. The attempt to obtain g from a fit to the average field therefore provides a too large value and a fit to the maximum field a slightly to low g.

CONCLUSION

We calculated an approximation for the integral phase shift of a cyclotron beam passing the simplest Enge-type fringe field. The result indicates that electrostatic extractors, though certainly helpful, are not intrinsically required for the beam extraction from high power cyclotrons of the kind of the PSI Ring cyclotron. Due to the high energy gain per turn, the beam leaves the magnetic field in any case. In order to extract a well-focused beam, measures to correct the gradient of the magnetic fringe field are required. However, even with electrostatic extractors, the gradient field of PSI Ring cyclotron requires an additional focusing magnet as

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part of extraction system [12, 13], so that the total number of extraction elements would not increase.

An extrapolation of the design principles of dream machines appears to facilitate extraction by acceleration in high power proton drivers. The possibility to avoid electrostatic elements in this kind of machines sheds new light on the possibility to use highly energy-efficient cyclotrons as drivers for accelerator driven systems.

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