ON THE ENERGY LIMIT OF COMPACT ISOCHRONOUS CYCLOTRONS

W. Kleeven

Ion Beam Applications (IBA), Louvain-La-Neuve, Belgium

Abstract

Existing analytical models for transverse beam dynamics in isochronous cyclotrons are often not valid or not precise for relativistic energies. The main difficulty in developing such models lies in the fact that cross-terms between derivatives of the average magnetic field and the azimuthally varying components cannot be neglected at higher energies. Taking such cross-terms rigorously into account results in an even larger number of terms that need to be included in the equations. In this paper, a method is developed which is relativistically correct and which provides results that are practical and easy to use. We derive new formulas, graphs, and tables for the radial and vertical tunes in terms of the flutter, its radial derivatives, the spiral angle and the relativistic gamma γ . Using this method, we study the $2\nu_r = N$ structural resonance (N is number of sectors) and provide formulas and graphs for its stopband. Combining those equations with the new equation for the vertical tune, we find the stability zone and the energy limit of compact isochronous cyclotrons for any value of N. We confront the new analytical method with closed orbit simulations of the IBA C400 cyclotron for hadron therapy.

INTRODUCTION

In this paper we derive the maximum energy that can be realized in compact isochronous cyclotrons. This limit is determined by two competing requirements namely the need for sufficient vertical focusing on the one hand and the need to avoid the stopband of the half-integer resonance $2v_r = N$ on the other hand (v_r is the radial tune). With increasing energy, the isochronous field index increases rapidly and more and more azimuthal field variation f is needed to remain vertically stable; but with higher f, the stopband of the resonance broadens, and the energy limit associated with it rapidly reduces. The energy limit depends on N and on the spiral angle ξ of the sectors. We assume that the magnetic field is perfectly N-fold rotational and median plane symmetric and therefore do not consider other, than the half-integer linear resonance. We derive practical formulas which are useful especially in the cyclotron design phase. Our main assumption/approximation is that f is not too large. Results are derived up to $O(f^2)$ (equivalent to O(F), where F is the flutter). For compact cyclotrons F is generally well below 1 and for these machines we expect our results to be precise. For separate sector cyclotrons, care should be taken, however. The special case of such cyclotrons with radial sectors (no spiraling) has been studied by Gordon [1], by assuming a hard-edge model where in the magnet sections the orbits are perfectly circular and in the empty straight sections the magnetic field is zero. In Gordon's model, there is no need to assume a small flutter, but on the other hand, his assumptions will probably not be valid for compact cyclotrons and

maybe also less accurate for coil-dominated superconducting ring cyclotrons where the magnetic field has the tendency to spread out more smoothly and non-uniformly. For separate sector cyclotrons with a larger magnetic filling factor, the flutter drops quickly ($F \approx 0.25$ for a filling factor of 80%) and we expect our results to become more accurate. Another interesting derivation of the isochronous cyclotron energy limit has been made by Danilov et al. from the JINR [2]. In their analysis however, they consider only the first dominant Fourier component of the field and they further assume that its amplitude is independent on radius and its phase increases linearly with radius. Also, contributions due to higher order radial derivatives of the average magnetic field are ignored. A similar approach was used by King and Walkinshaw [3]. We closely follow the Hamiltonian approach that has been firstly introduced by Hagedoorn and Verster [4]; in this paper we wish to pay tribute to them. The derivation is too elaborate and complex to show in detail and therefore in this paper, we present a strongly compressed version. The full derivation and results can be found in reference [5].

METHOD OF DERIVATION

We study the static (non-accelerated) motion near a given constant radius r_0 defined by $P_0 = qr_0\bar{B}(r_0)$, where P_0 is the particle kinetic momentum. The reduced magnetic field $\mu(r, \theta) = B(r, \theta)/B(r_0)$ is represented by a Fourier series with respect to the azimuth θ . The radial dependence of the average field $\bar{\mu}(r)$ and of the Fourier components $A_n(r), B_n(r)$ of the azimuthally varying field profile $f(r, \theta)$, are Taylor expanded relative to the same radius r_0 . The magnitude of f is approximately equal to the magnitude of the dominant Fourier component $C_N = (A_N^2 + B_N^2)^{1/2}$ and the flutter F is approximately equal to $C_N^2/2$. In all our derivations we use a perturbation analysis where |f| serves as the measure for precision. In general any quantity of interest $g(\theta)$ can be split in its average part $\bar{g} = \frac{1}{2\pi} \oint g(\theta) d\theta$ and its oscillating part $osc(g) = g(\theta) - \overline{g}$. Oscillating parts of O(f) can be moved to the next higher order by a properly constructed canonical transformation. In doing so, new average contributions of $O(f^2)$ are generated. Our goal is to derive results up to $O(f^2)$. The reason for this is that the first significant terms in the expressions for the isochronous magnetic field and the radial and vertical tunes are of $O(f^2)$. In line with the HV-paper [4], we keep the average part of any azimuthally varying term up to $O(f^2)$, but neglect oscillating terms $O(f^2)$ as they would generate new terms of $O(f^3)$ when transforming them to higher order. However, we make one important generalization/improvement as compared to the HV-paper. Hagedoorn and Verster assumed that radial derivatives of the average magnetic field $(\bar{\mu}', \bar{\mu}'', \bar{\mu}''', \dots)$ are small quantities of $O(f^2)$ and therefore neglected cross-terms between those derivatives and

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the Fourier content in all expansions (note that we define the prime-operator for radial derivatives of a function h(r) as $h' = r \frac{dh}{dr}$, $h^{(n)} = r^n d^n h/dr^n$). This is valid at low but not at relativistic energies, as is shown in Fig. 1.



Figure 1: Derivatives of the isochronous magnetic field.

As we are interested in the cyclotron energy limits we must consider the derivatives $\bar{\mu}', \bar{\mu}'', \bar{\mu}''', \dots$ as terms of $O(f^0)$. This makes the derivation (and the final results) considerably more complex as many more terms need to be kept in the Hamiltonian expansion and certain canonical transformations used in the HV-paper, must be modified/generalized.

Our derivation starts from a general cyclotron Hamiltonian in polar coordinates. We introduce reduced canonical coordinates (by normalizing with respect to r_0), reduced canonical momenta (by normalizing with respect to P_0) and the reduced magnetic field (by normalizing with respect to $\overline{B}(r_0)$). The two transverse motions are decoupled by assuming that the horizontal motion is in the median plane and the projection of the vertical motion follows the Equilibrium Orbit (EO). This gives two separate Hamiltonians: H_x for the horizontal and H_z for the vertical motion. The EO is found by the requirement that it must be a periodic solution of H_x , and therefore can be expressed as a Fourier series. New canonical variables w.r.t. the EO are introduced and the Hamiltonian H_x is Taylor expanded with respect to these variables. Only terms up to second degree in the canonical variables need to be kept. This corresponds to linear motion and is sufficient as the half-integer resonance $2v_r = N$ is linear. By a second canonical transformation, H_x is brought to the normal form of the structure $H(p,x,\theta) = \frac{1}{2}p^2 + \frac{1}{2}(v_{x0}^2 + f_x(\theta))x^2 \text{ (and similar for } H_z).$ Here the parameters v_{x0} and $f_x(\theta)$ depend only on the reduced magnetic field quantities. The term v_{x0} is $O(f^0)$ and the term $f_x(\theta)$ is an oscillating term (zero average) of $O(f^1)$. We design a third linear canonical transformation, that moves the oscillating part $f_x(\theta)$ to new terms of the next higher order $O(f^2)$. Within our required level of approximation, we only need to keep the average of these new terms. This solves the motion $(O(f^2))$ as H becomes independent of θ . The betatron tune of the motion becomes $v_x^2 = v_{x0}^2 + \frac{1}{2} \sum \frac{c_n^2}{n^2 - 4v_x^2}$. Here the c_n are the Fourier amplitudes of the function $f_x(\theta)$.

For derivation of the $2\nu_r = N$ resonance we first introduce action-angle variables in an horizontal phase space that rotates with frequency N/2 and then design a canonical transformation that moves the oscillating part of O(f) to the next higher order $O(f^2)$. This gives us the following general expression for the lower and upper limits of the stopband:

$$v_{x0}(1,2) = \frac{N}{2} \mp \frac{c_N}{2N} - \frac{3}{8} \frac{c_N^2}{N^3} - \frac{1}{2N} \sum_{n > N} \frac{c_n^2}{n^2 - N^2} \,.$$

Here v_{x0} and c_n must be expressed in terms of reduced magnetic field parameters $\bar{\mu}', \bar{\mu}'', \ldots, C_n, C'_n, \ldots, \varphi'_n$.

The $O(f^2)$ contributions to the final results all have a similar structure of the following general form:

$$R^{(2)} = \sum_{n} \alpha_n(\bar{\mu}', \dots) C_n^2 + \beta_n(\bar{\mu}', \dots) C_n^2 \varphi_n'^2$$
$$+ \gamma_n(\bar{\mu}', \dots) C_n C_n' + \delta_n(\bar{\mu}', \dots) C_n'^2,$$

where the summation runs over all Fourier components n, the coefficients α_n, \ldots depend on $\bar{\mu}', \bar{\mu}'', \bar{\mu}''', \ldots$ and the variables C_n , φ_n are the amplitude and phase of harmonic n. To simplify this structure, we make a few approximations. Firstly, we assume a perfectly isochronous magnetic field. In this case the coefficients α_n, \ldots will depend on γ only. Secondly, we assume that the phase-derivatives φ'_n do not depend on n. In practice this is accurately true for the first several (often up to 5) Fourier components. Since contributions of higher components rapidly drop with increasing *n*-value, this approximation will be accurate. In this way the variable $\varphi'_n = \varphi'$ can be taken out of the series summations (note that φ' is related to the frequently used spiral angle ξ as $\varphi' = \tan \xi$). Thirdly, we introduce a method where the higher Fourier harmonics (n > N) are expressed in terms of the dominant harmonic (n = N). For this we assume a symmetrical hard-edge profile of the azimuthally varying field with equal hill and valley angle. This approximation is reasonable because the optical quantities are dominantly determined by the principal harmonic; it allows us to approximate the higher harmonic content and therefore is expected to be more accurate than only considering the dominant harmonic. For such a profile we have:

$$F(r) = \frac{\left\langle B^2(\theta, r) \right\rangle - \left\langle B(\theta, r) \right\rangle^2}{\left\langle B(\theta, r) \right\rangle^2} ,$$

$$C_n^2 = \frac{N^2}{n^2} C_N^2 = \frac{16F}{\pi^2 (2k+1)^2} , \quad k = 1, 2, \dots$$

The n-dependence of the harmonic amplitudes C_n can now be included in the coefficients α_n, \ldots and the dominant component C_N can be expressed in terms F and taken outside of the summation. Finally, we sum the series analytically and express the results in elementary functions of γ and N. The $O(f^2)$ terms are thus transformed to the simpler form:

$$R^{(2)} \approx F\left(a_N(\gamma) + b_N(\gamma)\varphi'^2 + c_N(\gamma)\frac{F'}{F} + d_N(\gamma)(\frac{F'}{F})^2\right) .$$

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RESULTS: SOME EXAMPLES

Following the approach as outlined in the previous paragraph, we find the following expression for the radial tune:

$$v_x^2 = 1 + \bar{\mu}'_{rel} + \frac{8N^2F}{\pi^2} \left[\tilde{a}_N + \tilde{b}_N \varphi'^2 + \tilde{c}_N \frac{F'}{F} + \tilde{d}_N \left(\frac{F'}{F}\right)^2 \right] .$$

Here $1 + \bar{\mu}'_{rel} = \gamma^2$ and the coefficient $\tilde{a}_N, \tilde{b}_N, \tilde{c}_N, \tilde{d}_N$ depend on γ as follows:

$$\begin{split} \tilde{a}_{N}(\gamma) &= \tilde{q}_{1}t_{2} + \tilde{q}_{2}t_{1} + (\tilde{q}_{3} + \tilde{q}_{5}t_{2})(1 + t_{2}^{2}) + \tilde{q}_{4} ,\\ \tilde{b}_{N}(\gamma) &= \frac{\pi}{8\gamma N} \left(t_{1} - 2t_{2} \right) ,\\ \tilde{d}_{N}(\gamma) &= \frac{\pi}{128\gamma^{3}N} \left(\frac{3\pi\gamma}{N} + t_{1} - 8t_{2} \right) ,\\ \tilde{c}_{N}(\gamma) &= \frac{\pi}{96\gamma^{3}N} \left[(11 - 9\gamma^{2}) \frac{3\pi\gamma}{N} + 3(\gamma^{2} + 1)t_{1} \right. \\ &+ 24(2\gamma^{2} - 3)t_{2} - \frac{12\pi\gamma}{N} (\gamma^{2} - 1)t_{2}^{2} \right] , \end{split}$$

where $t_1 = \tan(\pi \gamma/N)$, $t_2 = \tan(\pi \gamma/2N)$ and the coefficients \tilde{q}_i are defined as:

$$\begin{split} \tilde{q}_0(\gamma) &= \frac{1}{4\gamma^4} (4 + (\gamma^2 - 1)(\gamma^2 + 10))(\gamma^2 - 1)^2 ,\\ \tilde{q}_1(\gamma) &= \frac{-\pi}{8N\gamma^3} [6 - \tilde{q}_6 + 15\tilde{q}_0] , \quad \tilde{q}_2(\gamma) = \frac{+\pi}{32N\gamma^3} \tilde{q}_6 ,\\ \tilde{q}_3(\gamma) &= \frac{\pi^2}{16N^2\gamma^2} [4 - \tilde{q}_6 + 7\tilde{q}_0] , \quad \tilde{q}_5(\gamma) = \frac{-\pi^3\tilde{q}_0}{16N^3\gamma} ,\\ \tilde{q}_4(\gamma) &= \frac{+\pi^2}{32N^2\gamma^2} [4 - \tilde{q}_6 + 16\tilde{q}_0] , \quad \tilde{q}_6(\gamma) = (\gamma^2 + 1)^2 . \end{split}$$

Above equations are elaborate but not difficult as they depend on elementary mathematical functions only. We developed a cyclotron design template in excel in which the first three columns (radius r, flutter F(r), magnetic sector center-azimuth $\varphi(r)$) must be pasted by the user and only three more constants (N, revolution frequency f_{rev} and the central field B_0) must be defined. The template then calculates all results derived in the paper such as the tunes, the resonance stopband limits, the isochronous field profile $B_{iso}(r)$ and the relation $\gamma = \gamma(r)$.

To validate the derivations, we compare our results with the C400 hadron therapy cyclotron [6–8]. This K=1600 machine is now in construction by the company Normandy Hadron Therapy (NHa) based in Caen, France, in collaboration with IBA. Figure 2 shows the Fourier properties and a histogram plot of the C400 magnetic field. The upper left shows the flutter *F* and the normalized amplitudes of the first five structural components and the lower left their spiral angles ξ_n . It is seen that the ξ_n are all closely the same and the flutter is roughly equal to $C_4^2/2$ as stated earlier.

The lower right figure shows different alternatives for the definition of the spiral angle. The first one uses the azimuth at which the magnetic field around a circle reaches its maximum. The second and third alternatives use the azimuth at



Figure 2: C400 harmonics and spiral angle comparison.

which the azimuthal derivative of the magnetic field reaches its maximum (at sector entrance) or minimum (at sector exit) respectively. The fourth alternative uses the azimuth at which the basic harmonic component C_4 reaches its maximum. The first alternative is not a good choice, because it deviates too much at high radii. For the radial tune and the $v_r = N/2$ stopband, the other three alternatives give closely the same results. However, as we will see later, for the vertical tune the average of the second and third alternatives give the best match with the C400 closed orbit simulations. This makes sense because it is at the sector edges where the strong vertical focusing takes place. We therefore use this definition in our paper. Figure 3 compares our analytical radial tune (black curve) with the numerical closed orbit tune (blue curve). In the left figure the relativistic contribution to the tune $(=\gamma)$ is also shown separately (red curve). At extraction, this contribution accounts for almost 75% of the total. The right of Figure 3 shows the part of the radial tune that is due to the flutter only. Here there is good agreement between the analytical and the closed orbit results. There is a small difference because in the derivation of the tune, the approach towards the half-integer resonance was ignored (as this is treated separately). The dashed curve in the right of Figure 3 show the flutter contribution to the radial tune that is obtained if the energy-dependence of the tune-coefficients would have been ignored (simulated by evaluating the coefficients at the value $\gamma = 1$). This is equivalent to a derivation in which the cross-terms between the average field radial derivatives and the magnetic field Fourier terms are neglected (as was done in the HV-paper). This approximation becomes inaccurate at relativistic energies.

The derivation and final equations of the vertical tune are very similar to those of the radial tune. The vertical tune depends critically on the definition of the spiral angle. The reason for this is that v_z^2 is obtained as a difference between two larger numbers (negative field index $\bar{\mu}'$ and positive flutter-terms), which to a large extent cancel each other.

This is illustrated (for the C400) in Figure 4. The numerical CO result is shown in blue. The red curve (Bmax) uses the spiral angle obtained from the azimuth where the

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Figure 4: C400 vertical tune comparison.

magnetic field around a circle is maximum. This model fits well up to a radius of about 1.2 m (\approx 125 MeV/u), but beyond that immediately collapses. The orange curve (H4), based on the phase φ_4 of the basic harmonic, gives some improvement but is still not satisfactory. The green curve (edges), based on the average of the sector-in and sector-out azimuths, shows a further improvement but still deviates substantially from the numerical curve at higher energies. The black curve shows our best result. Here the spiral angle (from the previous case - edges) is corrected for the fact that the EO is not circular and therefore enters and exits from the sector with a non-zero radial momentum. We find the following formula for the correction [5]:

$$\varphi_{corr}'=\varphi'\left(1+\frac{\pi^2 F}{4N^2}(1+\varphi'^2)\right)+O(f^4)\;.$$

For the lower and upper limits $\gamma_{1,2}$ of the $2\nu_r = N$ resonance stopband we find the following equation:

$$\begin{split} \gamma_{1,2} &= \frac{N}{2} \mp \frac{2\sqrt{F}}{\pi N} \sqrt{(1 + \frac{N^2}{4} + \frac{F'}{2F})^2 + N^2 \varphi'^2} \\ &- \frac{F}{\pi^2 N^3} \left(\bar{a}_N - \bar{b}_N \varphi'^2 - \bar{c}_N \frac{F'}{F} + \bar{d}_N (\frac{F'}{F})^2 \right) \end{split}$$

Its right-hand-side consists of contributions of $O(f^0)$ (first term), $O(f^1)$ (second term) and $O(f^2)$ (third term). Table 1 shows the values of the $O(f^2)$ -term coefficients $\bar{a}_N, \bar{b}_N, \bar{c}_N, \bar{d}_N$ for a range of *N*-numbers.

Table 1:	Stopband	Coefficients
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N	a_N	\boldsymbol{b}_N	c_N	d_N
3	312.4	41.1	52.8	1.164
4	877.2	73.1	92.9	1.164
5	2043.8	114.2	144.4	1.164
6	4145.6	164.4	207.4	1.164
8	12856.6	292.2	367.6	1.164
10	31149.8	456.6	573.7	1.164
12	64346.3	657.6	825.6	1.164



Figure 5: Stability diagram of the isochronous cyclotron.

Figure 5 shows in one plot the resonance limits (solid lines) and the vertical focusing limits (dashed lines) as function of the flutter *F* and for several spiral angles ξ . The focusing limit is determined by the condition $v_z = 0$. It increases monotonically with increasing *F* and ξ , whereas the normalized resonance limit $(\gamma - 1)/(\frac{N}{2} - 1)$ decreases monotonically with increasing *F* and ξ . To have a stable cyclotron, the operating point as defined by *F*, ξ and γ must be below the corresponding solid lines and the corresponding dashed lines (the green-colored zone in the middle figure). The lines itself represent extreme limits of stability and in practice sufficient distance must be taken. For the vertical tune one could require for example a minimum value $v_{min} > 0$. If γ_0 is the focusing limit found from Figure 5 then the dashed line

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in the plot will shift down by the amount $\Delta \gamma \approx -v_{min}^2/(2\gamma_0)$. At the intersect between solid and dashed lines the highest achievable energy is found for a given spiral angle. These points are shown as black dots. Table 2 shows these energy limits as a function of the design spiral ξ and several N-numbers. The table also shows the corresponding flutter values that are required to achieve these limits. The energy limits are the absolute limits for the isochronous cyclotron as dictated by the beam dynamics of these machines. In practice there are of course other limits determined by technology.

Table 2: Energy Limits of Isochronous Cyclotrons

	N=3		N=4		N=6	
ξ	F	Т	F	Т	F	Т
deg	-	MeV/u	-	MeV/u	-	MeV/u
0	0.1384	75.7	0.2934	151	0.5063	243
45	0.0858	125	0.1995	263	0.387	464
60	0.0495	157	0.1245	352	0.272	668
70	0.0257	180	0.0686	418	0.168	850
75	0.0153	190	0.0424	450	0.111	950
80	0.0072	198	0.0204	477	0.056	1045
	N=8		N=10		N=12	
0	0.6324	294	0.7135	326	0.7697	348
45	0.5180	587	0.6085	673	0.6741	732
60	0.3945	891	0.4880	1049	0.5607	1170
70	0.2653	1193	0.3510	1465	0.4250	1677
75	0.1852	1380	0.2571	1738	0.3231	2034
80	0.1000	1572	0.1488	2047	0.1975	2471

Figure 6 shows the tunes for a H_2^+ cyclotron with symmetry N=3, that has been studied at IBA. The upper figure shows the radial tune and vertical tune (2x) obtained from a numerical closed orbit code (black-solid and red solid respectively), and the radial tune (black-dashed) and vertical tune (2x, red-dashed) calculated analytically. In this example, the half-integer resonance hits at the radius of 48.2 cm, corresponding with an energy of 187.5 MeV/u and a vertical tune value of $v_7 = 0.27$. The lower figure shows the flutter F and the spiral angle ξ of the magnetic field. At the resonance energy they are F=0.0074 and ξ =79.5° respectively. Table 2 shows an extreme energy for ξ =80° of 198 MeV/u. Correcting this value for the non-zero vertical tune (=0.27). we obtain the stopband energy at E=186.5 MeV/u. This is extremely close to the numerical result of 187.5 MeV/u. Our result for N = 3 agrees very well also with the result found by King and Walkinshaw [3] who report, in their study on the conversion of the Harwell synchrocyclotron, a proton energy limit of 190 MeV for a spiral angle of about 75°.

CONCLUSION

This work determines the minimum number of sectors that are needed for obtaining a given top energy with a compact isochronous cyclotron. It also provides the best trade off between spiral angle and flutter. The provided equa-



Figure 6: Example for a 230 MeV/u H_2^+ cyclotron.

tions, graphs and tables may facilitate the conceptual and/or preliminary design of a new cyclotron.

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