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ERROR MAGNETIC FIELD DUE TO THE MEDIAN PLANE ASYMMETRY AND ITS APPLICATIONS IN THE CYCLOTRON

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Abstract

Cyclotrons have a median plane symmetric structure. But the pole's geometric error and the unevenly magnetized soft iron give rise to non-zero asymmetrical fields in the geometric median plane. The asymmetric field can shift the vertical position of the beam. Moreover, The error of the tilted median plane can be the driving force when the tunes pass through coupling resonances. In this paper, we take the TRIUMF 500 MeV cyclotron as an example to study the asymmetric field resulting from imperfect median plane symmetry. An approach due to M. Gordon, and a highly accurate compact finite differentiation method are used to investigate the historical field survey data, which reveals redundancy in the survey data. The redundancy was used in this study to correct the error in the measurement data. Further, the median plane asymmetry field could be manipulated using trim coils or harmonic coils with top and bottom coil currents in opposite directions ('Br-mode'). Using the created asymmetric field, we improved the vertical tune measurement method to investigate the linear coupling resonance in TRIUMF 500 MeV cyclotron. Eventually, the coupling resonance is corrected and avoided using the available harmonic coils and trim coils.

INTRODUCTION

The median plane asymmetric field in a small gap cyclotron or cyclotron with a high magnetic field is usually negligible, and in that case, the surveyed or calculated axial field B_z in the median plane is usually sufficient for the beam dynamics study. But for a large gap magnet with low magnetic field, such as the TRIUMF 500 MeV cyclotron, the asymmetric field can significantly shift the vertical position of the beam. Moreover, the error of the tilted median plane can be the driving force when the tunes pass through coupling resonances [1]. To study the effect of the asymmetric field on the beam dynamics, only the field survey data can be used while the finite element analysis calculation can not reveal the pole machining errors and the variations of material properties of the steel. Using Gordon's field expansion technique [2], we have discovered self-consistency errors in these field data. However, these are easily and convincingly corrected, as will be shown. After correcting the errors in the measurement data, we recalculated the properties of static equilibrium orbits. Further, we have optimized the trim coils' settings to achieve a better beam vertical centering.

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FIELD EXPANSION OUT OF MEDIAN PLANE: GORDON'S APPROACH

The magnetic field map must satisfy Maxwell's equations to sufficient order. In a cyclotron, a typical way is to expand the field relative to the median plane. Gordon's approach is one of these; it is derived from the scalar potential Ψ that satisfies Laplace's equation. By solving the 3D Laplace equation using an operator trick, we get the potential Ψ expanded in powers of axial position z as follows [2]

$$\begin{aligned}\Psi &= \Psi_o + \Psi_e,\\ \Psi_o &= zB - \frac{z^3}{3!}\nabla_2^2 B + \frac{z^5}{5!}\nabla_2^4 B - \dots,\\ \Psi_e &= C - \frac{z^2}{2!}\nabla_2^2 C + \frac{z^4}{4!}\nabla_2^4 C - \dots,\end{aligned} \tag{1}$$

where $B = B(r,\theta)$ and $C = C(r,\theta)$, and where ∇_2^2 is the 2-dimensional Laplace operator

$$\nabla_2^2\Psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}. \tag{2}$$

Thus, for example, $\nabla^2\Psi = \frac{\partial^2\Psi}{\partial z^2} + \nabla_2^2\Psi = 0$.

The odd term Ψ_o produces a field with median plane symmetry, while the even term Ψ_e spoils this symmetry. The magnetic field is given by $\vec{B} = -\nabla\Psi$, that is

$$\begin{aligned}B_z &= -B + z\nabla_2^2 C + \frac{z^2}{2!}\nabla_2^2 B - \frac{z^3}{3!}\nabla_2^4 C - \frac{z^4}{4!}\nabla_2^4 B + \dots,\\ B_r &= -\frac{\partial C}{\partial r} - z\frac{\partial B}{\partial r} + \frac{z^2}{2!}\frac{\partial\nabla_2^2 C}{\partial r} + \frac{z^3}{3!}\frac{\partial\nabla_2^2 B}{\partial r} - \dots,\\ rB_\theta &= -\frac{\partial C}{\partial\theta} - z\frac{\partial B}{\partial\theta} + \frac{z^2}{2!}\frac{\partial\nabla_2^2 C}{\partial\theta} + \frac{z^3}{3!}\frac{\partial\nabla_2^2 B}{\partial\theta} - \dots.\end{aligned} \tag{3}$$

In most orbit programs, C is ignored and only the zero-order B_z value and the first-order B_r and B_θ values are used. This is acceptable only for z very small compared with the magnet gap since it violates $\nabla\cdot\vec{B} = 0$ and can therefore lead to non-physical results for finite z values. This can be remedied by including the z^2 term in B_z. In general, when B_r and B_θ are given to order z^n, then B_z should be given to order z^{n+1}.

Ignoring C is appropriate in the initial design stage of a cyclotron, but not for finding tolerances for manufacturing errors, nor for detailed investigations of orbit excursions and resonance crossings in an as-built cyclotron. In the TRIUMF cyclotron, the vertical closed orbit excursion is as large as ±1.3 cm even after correction, as shown below. In a synchrotron, the closed orbit distortion is corrected with

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separate small dipole magnets. But in cyclotrons, where the radial extent of the beam gap is orders of magnitude larger than that gap, this is not possible; instead, orbits are corrected vertically using trim coils that are placed above and below the median plane. Powered in opposition to each other, they create radial field components that can correct the beam vertical position.

REDUNDANCY IN THE ERROR FIELD SURVEY DATA

The symmetrical part of the cyclotron field is directly given by the measured axial field in the median plane, while the axial derivative of this field and the transverse components in the median plane all are derived from the function $C(r,\theta)$. We use this fact to correct errors in the measured asymmetric field components.

By substituting $z = 0$ in Eq. (3), the magnetic field in the median plane is written as

$$\begin{aligned} B_r &= -\frac{\partial C}{\partial r}, \\ rB_\theta &= -\frac{\partial C}{\partial \theta}, \\ B_z &= -B. \end{aligned} \tag{4}$$

when the median plane symmetry is broken, the axial derivative of the axial field dB_z/dz in the median plane is non-zero and is expressed as

$$\frac{dB_z}{dz} = \nabla_2^2 C = \frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2}. \tag{5}$$

The asymmetric components B_r, B_θ and dB_z/dz were measured at $z = 0$ on a polar grid with 1° intervals in azimuth and 3 inch intervals in radius, but the final data are given as Fourier series in azimuth, up to 29 harmonics. Since different harmonics are orthogonal, every harmonic individually satisfies the Eqs. (4) and (5). The n^{th} harmonic of the map B_{rn}, $B_{\theta n}$ and dB_{zn}/dz satisfies

$$\begin{aligned} B_{rn} &= -\frac{\partial C_n}{\partial r}, \\ rB_{\theta n} &= -\frac{\partial C_n}{\partial \theta}, \\ \frac{dB_{zn}}{dz} &= \frac{\partial^2 C_n}{\partial r^2} + \frac{1}{r}\frac{\partial C_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 C_n}{\partial \theta^2}, \end{aligned} \tag{6}$$

where C_n is the n^{th} harmonic of the scalar potential of the asymmetric field. Writing the harmonic in complex form, Eqs. (6) are simplified to ordinary differential equations (ODE) which have only the radius r as variable

$$\begin{aligned} B_{rn} &= -\frac{dC_n}{dr}, \\ B_{\theta n} &= -jn\frac{C_n}{r}, \\ \frac{dB_{zn}}{dz} &= \frac{d^2 C_n}{dr^2} + \frac{1}{r}\frac{dC_n}{dr} - n^2\frac{C_n}{r^2}. \end{aligned} \tag{7}$$

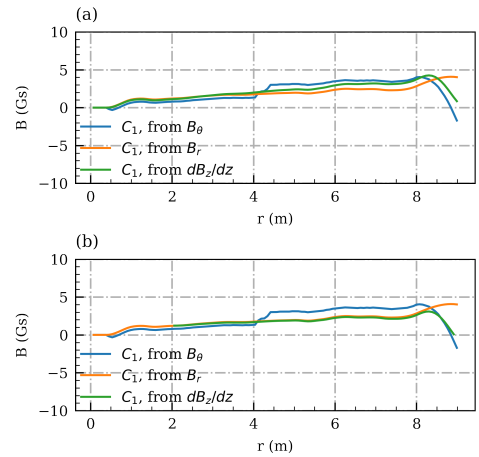

Figure 1: First harmonic of the asymmetric field potential map C_1. (a) Calculated using Eq. (7) starting from the center of the cyclotron, where the field is homogeneous and thus the ODE's initial conditions are $C_1(0) = -jB_\theta(0) = 0$ and $C_1'(0) = -B_r(0) = 0$. (b) Calculated using Eq. (7) starting from the radius of 2 m in the third ODE.

In the complex form of the n^{th} harmonic, n could be either sign. So Eq. (7) should be solved from -29^{th} to 29^{th} harmonics. By solving Eq. (7) numerically, we get three versions of the C map, from the survey data of B_r, B_θ and dB_z/dz respectively. As an example, Fig. 1 compares the obtained first harmonic of the C map. From B_θ, there are two regions of first derivative discontinuities, occurring at ~0.5 and 4 m respectively. This suggests the existence of some systematic error in the measurement data of B_θ. The difference between the C_1 as calculated from B_r and dB_z/dz grows with the radius, but this is easily corrected by changing the initial condition within uncertainty. As a result of the finite size of the flip coils, the survey data has a relatively larger error at a smaller radius. Thus, if integrating from a larger radius with a relatively constant slope in the field, the difference becomes smaller, as shown in Fig. 1(b).

APPLICATION I: CORRECTING THE ERROR IN THE FIELD SURVEY DATA

The redundancy in the survey data makes it possible to correct the error in the measurement data. Using Eq. (7), we can generate a full map from a single potential map C. To reduce numerical errors due to the interpolation while solving Eq. (7), we directly use the CFD method [3] to calculate the new maps. The equations used for the correction are obtained by substituting C_n with $jrB_{\theta n}/n$ in Eq. (7)

$$\begin{aligned} jnB_{rn} &= \frac{d(rB_{\theta n})}{dr}, \\ jn\frac{dB_{zn}}{dz} &= -\frac{d^2(rB_{\theta n})}{dr^2} - \frac{1}{r}\frac{d(rB_{\theta n})}{dr} + n^2\frac{rB_{\theta n}}{r^2}. \end{aligned} \tag{8}$$

The resulting first harmonic field is compared in Fig. 2. The B_θ survey data seems to be shifted upward at radii before 4 m and thereafter shifted downward, in comparison with

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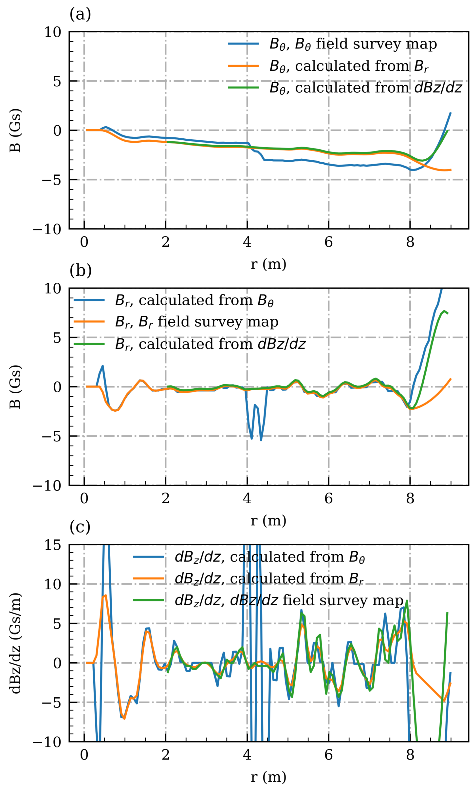

Figure 2: Comparison of the first harmonic of the asymmetric field components B_θ (a), B_r (b) and dB_z/dz (c), constructed with different survey data.

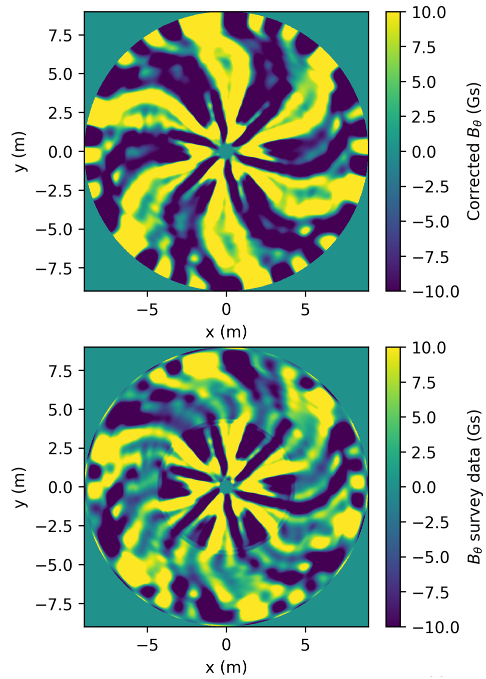

Figure 3: Azimuthal field map in the median plane. The survey map (lower) displays an obvious discontinuity at radius of ~4 m, and thereafter a blurred edge of the sector structure. The corrected map (upper) shows a sharper image of the sector edges of the main magnet.

those calculated from the other two survey maps. The B_r survey map agrees with the calculated ones except for the spike at ~4 m occurring in the one calculated from B_θ (see Fig. 2 (b)). B_θ is too noisy to give a usable dB_z/dz map, while the calculated dB_z/dz from B_r agrees with the survey result (see Fig. 2 (c)) and is even smoother than the survey data because high-frequency components are filtered out.

Since the magnet gap is so large at ~0.5 m, it is inconceivable that the radial field can step discontinuously on a scale of the flip coil separation (< 0.08 m). We, therefore, choose the radial field B_r survey data to correct the error of the azimuthal field B_θ; this is also because the reconstruction is less sensitive to the initial conditions than using dB_z/dz as shown in Fig. 1. Figure 3 compares the corrected B_θ map with the survey result.

APPLICATION II: MEASURING VERTICAL TUNE

TRIUMF cyclotron is equipped with 54 pairs of trim coils, each consisting of two circular loops, one above and one below the mid-plane, and spaced at 15 cm intervals radially. They could be used with top and bottom coils in opposite currents (referred to as B_r-mode), to create a radial field and thereby move the beam vertically.

The relation between the vertical displacement of the orbit Δz and the radial field is given in smooth approximation by

$$\Delta z = \frac{\overline{R}}{\overline{B}_z}\frac{\Delta\overline{B_r}}{\nu_z^2}, \tag{9}$$

where $\overline{B}_z = m\gamma\omega/q$, $\overline{R} = \beta c/\omega$ for isochronism, m is particle mass and ω is orbital frequency. Thus, the vertical tune can be found in the vertical displacement produced by creating a radial field using the B_r-mode of trim coil(s).

A detailed study of the tune measurement and adjustment is presented in our previous study [4]. Figure 4 shows the result. The measured results from different trim coil pairs agree well in the field overlap regions, meaning that both the trim coil field and the probe's radial position are consistently calibrated with the orbital average radius. The measured results reproduce several bumps of the CYC581 data, which is calculated from field survey data, but differ significantly in some areas. This could be because the trim coils were powered in a different pattern during the field survey than they are now, but this information has been lost over time.

APPLICATION III: CORRECTING THE LINEAR COUPLING RESONANCE

In the TRIUMF 500 MeV H$^-$ cyclotron, the linear coupling resonance $\nu_r - \nu_z = 1$ is crossed multiple times as shown by the CYC581 tune diagram in Fig. 5. This is calculated from the historical magnetic field survey data of 1974. It suggests that this resonance is crossed first around

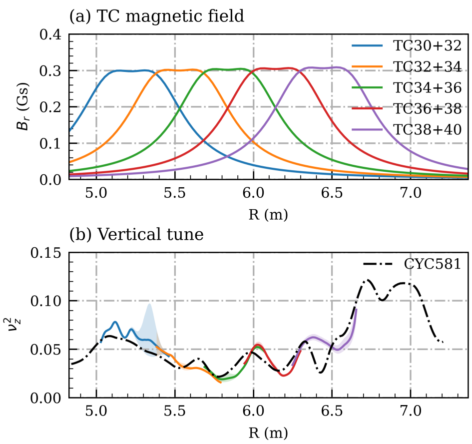

Figure 4: (a) The radial field profiles of indicated trim coil pairs, and (b) (with the same colour code) the measured vertical tune using these pairs, along with the CYC581 data.

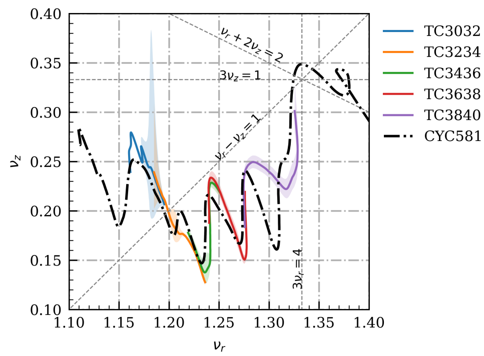

Figure 5: Tune diagram (coloured) experimentally measured using the indicated trim coil pairs, and (dot-dashed) obtained from the CYC581.

166 MeV and then again twice near 291 MeV. This resonance induces the exchange of betatron oscillation amplitudes between the radial and vertical directions. The usual technique would be to correct the resonance by applying a compensating radial first harmonic magnetic field of appropriate phase and amplitude. The TRIUMF cyclotron is equipped with 13 such 'harmonic coils'. These each has radial widths of 60 cm and consist of six sectors, 60° wide, on top and bottom of the vacuum tank. Powered in opposite directions, and following a 6-part segmented sine wave, they create a first harmonic radial field in the geometric median plane. A detailed resonance correction is studied by Yi-Nong [5], we present part of the results in this paper (Figs. 6 and 7).

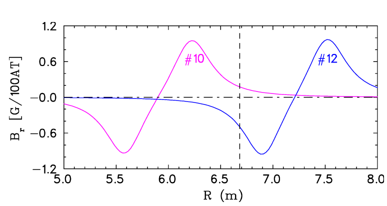

Figure 6: The B_r amplitude measured in the geometrical median plane due to harmonic correction coils HC10 and HC12 separately.

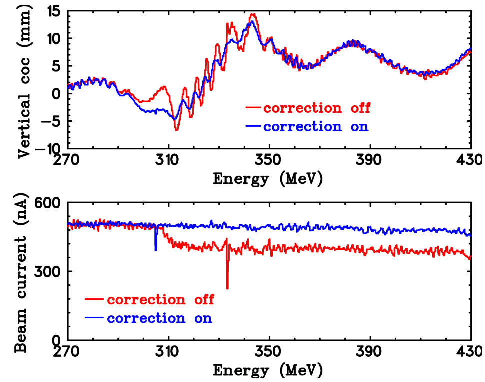

Figure 7: Radial probe measured vertical centre of charge (upper) and transmission (lower) vs. energy, before and after correction of the resonance. Here a coherent radial centring error of the beam orbit was intentionally introduced by detuning the HC2 B_z first harmonic amplitude.

CONCLUSION

By using Gordon's approach, the redundancy in the field survey maps B_θ, B_r and dB_z/dz, resulting from the magnetic median plane tilt error, is revealed. Using the redundancy, we crosschecked the field survey data of TRIUMF cyclotron. A systematic error in the B_θ survey data was found and has been corrected. The median plane error field could also be used to measure the vertical tune and correct the coupling resonance. We discussed the examples of these applications in the TRIUMF cyclotron.

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