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THE STUDY OF THE ISOCHRONOUS MAGNETIC FIELD AND THE EQUILIBRIUM ORBIT OF CS-30 CYCLOTRON

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Abstract

The CS-30 accelerator of the Institute of Nuclear Science and Technology of Sichuan University is a three-fan accelerator with constant angular width (45 degrees) at small radius and blade thickness increasing with radius at larger radius. In this paper, the magnetic field is analyzed, and the static equilibrium orbit, revolution frequency, oscillation frequencies and other data are calculated. These functions can be integrated to guide the accurate magnet numerical model setup of the existing CS-30 accelerator, which can be used in de education demonstration and experimental phenomena analysis. The optimization algorithm is innovatively introduced in the static equilibrium orbit calculation, which reduces the dependence of the results on the initial value and significantly improves the calculation speed. The calculation method presented in this paper is suitable for all cyclotron.

INTRODUCTION

CS-30 is produced by TCC Company in the United States and introduced by the State Science and Technology Commission in February 1984. It is the first cyclotron with compact structure introduced from abroad in China. In 2003, the cyclotron was moved from Beijing to the Institute of Nuclear Science and Technology of Sichuan University. After one year of installation and tuning, proton beams and α ions were accelerated to the internal target. After then it was routinely in operation and provide beams for experimental more than 100 hours per year. Now, it's mainly be used to produce isotopes, study irradiation effect of materials, and teaching demonstration device. Its application was limited by the absence of the external target system. The extraction elements are works well, what we need to extract the beam out of the cyclotron are just the beam line and the target station. Recently a beam line will be built and an external He^{2+} radiation target station will be built, which can significantly expend the application of the cyclotron. Since we never extract beam out before and lack of the data of the extraction elements settings, we need to study the beam dynamics properties to guide us in finding them. In order to do this, we need setup the numerical model of the cyclotron, so that we can get the detailed magnetic field distribution, which can be used in beam dynamics analysis. At the same time, the numerical model can also be used in the education.

METHODS AND MODEL

The 3D model of CS-30 magnet is shown in Fig. 1. The only thing we are not sure is the height of the steps on the fan, which are used to get the final isochronous magnetic field. In order to find the right number of the steps height, we need to analysis the magnetic field calculated from the model. If it is the isochronous, then the model is right; if it is not, we need to change the steps height until the isochronous field is obtained.

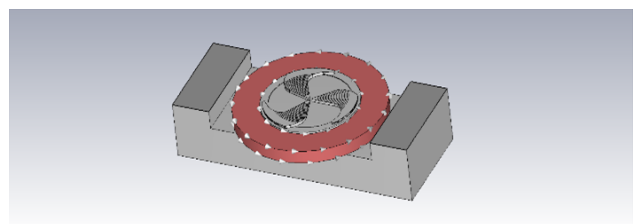

Figure 1: 3D model of the cyclotron magnet.

The magnetic induction intensity of CS-30 is shown in Fig. 2. To analyze the magnetic field. The most important thing is to calculate the equilibrium orbits. There are two main methods to calculate the equilibrium orbit. The first is the analytical method, which makes Fourier expansion of the magnetic field along the azimuth angle, and obtains the analytical solution under the first order approximation; the second is the numerical method, which is solved by the integral formula.

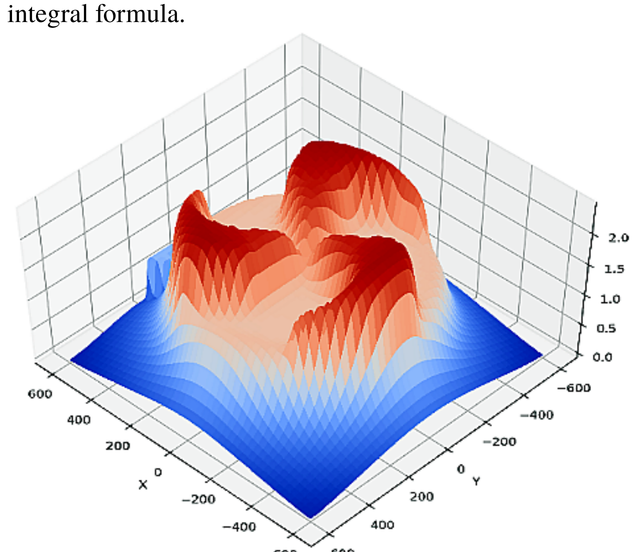

Figure 2: Magnetic induction intensity.

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Equilibrium Orbit

Analytical methods In the central plane of the synchrotron, the magnetic field distribution can be expressed as Eq. (1) [1]:

$$B(r,\theta) = \overline{B}(r)\,[1 + F(r,\theta)] \tag{1}$$

$F(r,\theta)$ is the magnetic field flutter function, which can be expanded by Fourier series:

$$F(r,\theta) = \sum_n A_n(r)\cos n\theta + B_n(r)\sin n\theta. \tag{2}$$

The motion of the central plane of the accelerator is:

$$\begin{aligned} \frac{dr}{d\theta} &= \frac{rp_r}{q}, \\ \frac{dp_r}{d\theta} &= q - \frac{ZerB_z}{p}, \\ \frac{dp_r}{d\theta} &= \sqrt{1-p_r^2}. \end{aligned} \tag{3}$$

We could set the momentum $p = Zer_0\overline{B}(r_0)$ of static equilibrium orbits of particles are as follows:

$$r_e = r_0(1 + x_e). \tag{4}$$

where r_0 is the radius of the orbit corresponding to the momentum, similar to what we did with the magnetic field, Fourier expansion:

$$x_e = \gamma + \sum_n \alpha_n \cos n\theta + \beta_n \sin n\theta \tag{5}$$

$$\begin{aligned} \gamma = &-\sum_n \frac{3n^2-2}{4(n^2-1)^2}(A_n^2 + B_n^2) \\ &+\frac{1}{2(n^2-1)^2}(A_nA_n' + B_nB_n') \end{aligned} \tag{6}$$

$$r_e = r_0\left(1 + \gamma + \sum_n \frac{A_n}{n^2+1}\cos n\theta + \frac{B_n}{n^2+1}\sin n\theta\right) \tag{7}$$

The equilibrium orbital in different radius are shown in Fig. 3.

Numerical methods From Eq. (3), the motion of a particle in a periodic field can be expressed as a matrix

$$Y(\theta) = MY(\theta_0) \tag{8}$$

$$Y(s) = \begin{pmatrix} y(\theta) \\ y'(\theta) \end{pmatrix}, \quad M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \tag{9}$$

For a given energy E, we can find the corresponding reference circle radius r_0 by the circular orbit approximation, set

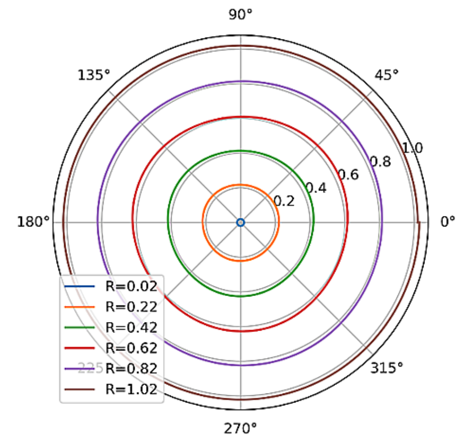

Figure 3: The equilibrium orbital computed analytically.

the initial position as (r_0, θ_0), there are three particles in the vicinity of (r_0, θ_0), their initial conditions are respectively

$$\begin{aligned} r = r_0 \quad p_r &= p_0, \\ r = r_0 + \delta r \; p_r &= p_0, \\ r = r_0 \quad p_r &= p_0 + \delta p, \end{aligned} \tag{10}$$

δr, δp are optional small quantities. Plug them into the equation of motion, integrate them and you get one cycle or one cycle. The latter solutions are respectively

$$\begin{aligned} r = r_{0f} \quad p_r &= p_{0f}, \\ r = r_{1f} \quad p_r &= p_{1f}, \\ r = r_{2f} \quad p_r &= p_{2f}, \end{aligned} \tag{11}$$

if (r_e, p_e) is the equilibrium orbit parameter at θ_0, then

$$y = r - r_e \quad y' = p - p_e \tag{12}$$

$$\begin{cases} m_{11}(r_0 - r_e) + m_{12}(p_0 - p_e) = r_{0f} - r_e \\ m_{21}(r_0 - r_e) + m_{22}(p_0 - p_e) = p_{0f} - p_e \\ m_{11}(r_0 + \delta r - r_e) + m_{12}(p_0 - p_e) = r_{1f} - r_e \\ m_{21}(r_0 + \delta r - r_e) + m_{22}(p_0 - p_e) = p_{1f} - p_e \\ m_{11}(r_0 - r_e) + m_{12}(p_0 + \delta p - p_e) = r_{2f} - r_e \\ m_{21}(r_0 - r_e) + m_{22}(p_0 + \delta p - p_e) = p_{2f} - p_e \end{cases} \tag{13}$$

Solve the equations above:

$$\begin{cases} m_{11} = \frac{r_{1f} - r_{0f}}{\delta r} \\ m_{12} = \frac{r_{2f} - r_{0f}}{\delta p} \\ m_{21} = \frac{p_{1f} - p_{0f}}{\delta r} \\ m_{22} = \frac{p_{2f} - p_{0f}}{\delta p} \end{cases} \tag{14}$$

Define:

$$\begin{aligned} m_{11}' &= m_{11} - 1 \\ m_{22}' &= m_{22} - 1, \\ A &= m_{11}'m_{22} - m_{12}m_{21}. \end{aligned} \tag{15}$$

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An approximation of (r_e, p_e) is obtained:

$$\begin{cases} r_e = r_0 + \frac{m_{22}'}{A}(r_0 - r_{0f}) - \frac{m_{12}}{A}(p_0 - p_{0f}) \\ p_e = p_0 + \frac{m_{11}'}{A}(p_0 - p_{0f}) - \frac{m_{21}}{A}(r_0 - r_{0f}) \end{cases} \tag{16}$$

By redoing the above calculation with (r_0, θ_0) instead of (r_0, θ_0), the exact value of (r_e, p_e) can be obtained after several iterations.

But this method relies heavily on the initial value, and it is easy to iterate over the answer because the initial value is not accurate enough.

This paper come up with a new method, which we call it optimization method. First, we set a particle with $[r_0, p_0]$. Put it in the equation of motion for 1/3 circle, then we could have $[r_e, p_e]$. Use sequential least squares to minimize D ($D = (r_e - r_0)^2 + (p_e - p_0)^2$) we can get pretty good equilibrium orbit.

There's a reason why this article uses 1/3 turn, because the magnetic field is triple symmetric, if you let the particles go all the way around. The particle will appear as shown in Fig. 4.

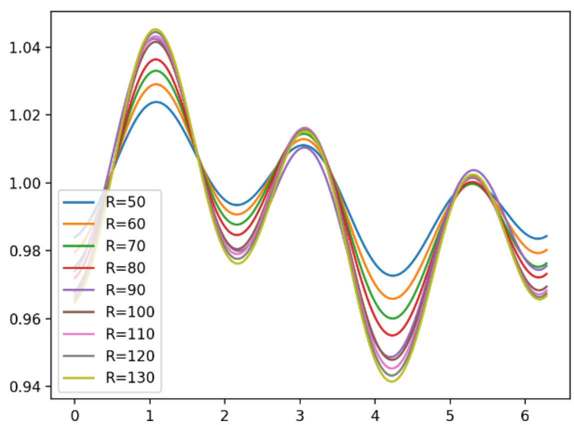

Figure 4: After one rotation.

Only if you set it to 1/3 turn can you get a triple symmetric equilibrium orbit (shown in Fig. 5).

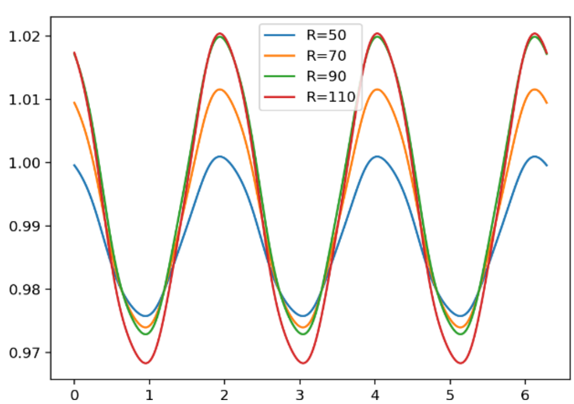

Figure 5: After one rotation.

Isochronous and Stable Evaluation

The oscillation frequency is shown in Fig. 6.

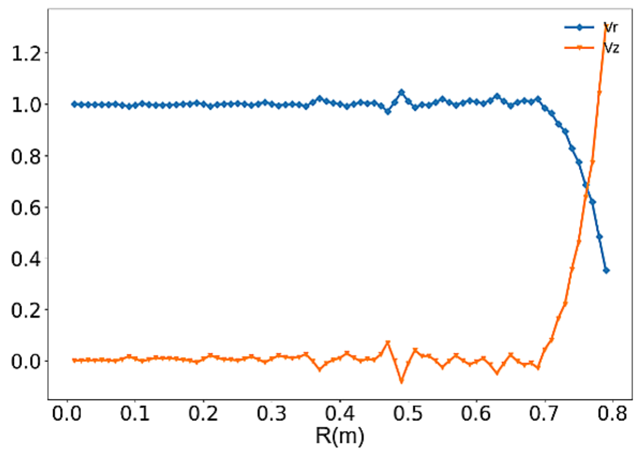

Figure 6: ν_r is the radial frequency, ν_z is the vertical frequency.

The period is shown is Fig. 7.

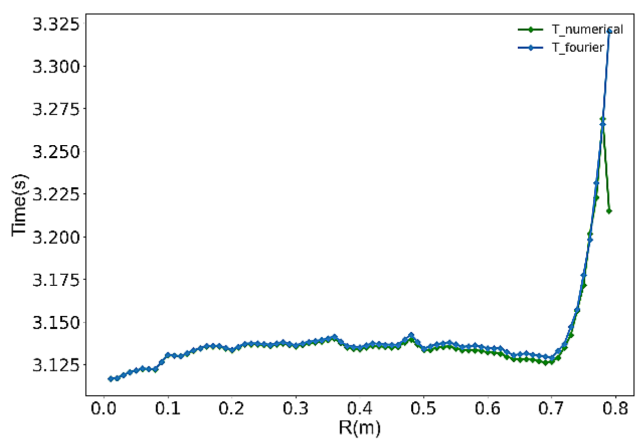

Figure 7: Period calculated by Numerical method and Analytical method.

CONCLUSION

The analysis of the magnetic field of the CS-30 is complete. A new method is adopted to calculate the equilibrium orbits. The magnetic field data is not accurate at large radius, which is speculated to be due to 1: inaccurate modelling, 2: The material of the yoke is unclear. I will solve these problems in my later work.

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