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DESIGN OF A 2 GeV CYCLOTRON WITH CONSTANT RADIAL AND VERTICAL TUNES

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Abstract

We demonstrate that a cyclotron can be made to have precisely constant betatron tunes over wide energy ranges. In particular, we show that the horizontal tune can be made constant and does not have to follow the Lorentz factor γ, while still perfectly satisfying the isochronous condition. To make this demonstration we developed a technique based on the calculation of the betatron tunes entirely from the geometry of realistic non-hard-edge closed orbits. The technique is applied to the particular case of a 800 MeV to 2 GeV proton cyclotron to produce a design that is presented here.

INTRODUCTION

This paper is a brief summary of a more detailed article [1]. There we demonstrate that, in a cyclotron with mid-plane symmetry, the transverse tunes are entirely determined by the geometry of its closed orbits. Capitalizing on this property, we have established a method to calculate tunes and produce isochronous field maps by starting from the geometry of realistic non-hard-edge orbits. The main advantage of this method is that it produces realistic and perfectly isochronous field distributions, and the corresponding transverse tunes, in a split second. We take advantage of this speed to explore the range of possible isochronous fields, which leads us to find solutions with simultaneously constant vertical and horizontal tunes over wide energy ranges.

We have applied this technique and proposed designs for several sets of cyclotron parameters [1, 2]. In this paper, we focus our effort on the case of a high-energy cyclotron, choosing a set of parameters that resembles that of the recently proposed 2 GeV CIAE machine [3, 4].

THEORY: TUNES OF ISOCHRONOUS ORBITS

Let's consider a fixed-(magnetic-)field accelerator with mid-plane symmetry. In this circular accelerator, let's consider one closed orbit at one particular energy, to which is attached the Frenet-Serret coordinate system (x, y, s). It is interesting to note that, as a result of the application of Maxwell's equations, the infinitesimal transverse motion of particles is fully determined by the curvature $\rho(s)$ and the local field index $n(s) = -\frac{\rho}{B_0}\frac{\partial B}{\partial x}\Big|_{x=y=0}$ of the corresponding closed orbit [5, 6]. This implies that it is sufficient to know the functions $\rho(s)$ and $n(s)$ over one period of the accelerator to know the value of the transverse betatron tunes.

Now, instead of starting from a magnetic field distribution to compute the properties of the orbits, which is the common practice for designing a cyclotron, let's start from the geometry of the orbits represented by some arbitrary function:

$$\begin{aligned} r:\ &\mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^+ \\ &(a, \theta) \mapsto r(a, \theta)\ , \end{aligned} \tag{1}$$

where r is the radius of the closed orbit, a is the orbit's average radius, and θ is the azimuth. The periodicity of the closed orbits imposes that:

$$r(a, \theta) = r(a, \theta + 2\pi/N) \text{ with } N \in \mathbb{N}^*\ , \tag{2}$$

where N is the lattice periodicity, i.e. number of sectors. The question now becomes: can we calculate both $\rho(s)$ and $n(s)$ directly from $r(a, \theta)$?

Firstly, the relation between s and θ is, first of all, given by:

$$\frac{\mathrm{d}s}{\mathrm{d}\theta} = \sqrt{r^2 + \left(\frac{\partial r}{\partial \theta}\right)^2}\ , \tag{3}$$

secondly, ρ is immediately obtained from the standard formula for polar coordinates:

$$\frac{1}{\rho} = \frac{r^2 + 2\left(\frac{\partial r}{\partial \theta}\right)^2 - r\frac{\partial^2 r}{\partial \theta^2}}{\left(r^2 + \left(\frac{\partial r}{\partial \theta}\right)^2\right)^{3/2}}\ . \tag{4}$$

Finally, obtaining $n(s)$ requires a little more work. We start by remarking that, in a cyclotron, orbits are isochronous which imposes a relation between the particle velocity and the orbit circumference:

$$\beta(a) = \frac{\mathcal{R}(a)}{\mathcal{R}_\infty}\ , \tag{5}$$

where $\mathcal{R}_\infty$ is a constant, corresponding to the value of $\mathcal{R}$ in the limit that the particles' speed is light speed; and $\mathcal{R}$ is the orbit circumference divided by 2π:

$$\mathcal{R}(a) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\mathrm{d}s}{\mathrm{d}\theta}\ \mathrm{d}\theta\ , \tag{6}$$

after imposing the isochronous condition, and applying the chain rule, we find an expression for n that depends only in $r(a, \theta)$ and its partial derivatives [1].

In conclusion, for any arbitrary continuum of closed orbits defined by the function $r(a, \theta)$, provided that r is smooth enough for the required partial derivatives to be defined, the transverse tunes of cyclotron depends on $r(a, \theta)$ and nothing else.

Corresponding Isochronous Field Map

So far we have needed only the knowledge of the shape $r(a, \theta)$ of the closed orbits to calculate tunes. The question

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that comes to mind is: what magnetic field distribution produces these closed orbits? The answer will depend on the choice of the particle mass m and charge q, and the scale of the solution will depend on the value of $\mathcal{R}_\infty$. The magnetic field within the median plane is given by:

$$B(r,\theta) = \frac{\beta(a)}{\sqrt{1-\beta^2(a)}}\frac{m}{q\rho(a,\theta)}, \quad (7)$$

where $\rho(a,\theta)$ is given by Eq. (4), $\beta(a)$ is given by Eq. (5), and $a = a(r,\theta)$ is calculated from $r(a,\theta)$ using numerical root finding. The magnetic field off the median plan can be obtained from extrapolation using Maxwell's equations, see for instance Ref. [7].

In the examples presented below, the transverse tunes and orbital frequency are crosschecked using particle tracking in 2-dimensional polar field map with the reference code CYCLOPS [8]. The field maps provided to CYCLOPS are generated using Eq. (7).

HIGH-ENERGY RING CYCLOTRON EXAMPLE

Let's impose some constraints on $r(a,\theta)$ so that the orbits have long straight sections, as in a ring cyclotron with drift spaces between sectors. For this example we chose to use the same basic parameters – number of sectors = 10, $\mathcal{R}_\infty = 28$ m, etc. – of the proposed 0.8 to 2 GeV CIAE ring cyclotron [3, 4].

Smooth-Orbits With Straight Sections

We cover the area between 0.8 GeV and 2 GeV using again 5 different orbits. Each orbit is defined as a spline, with periodic boundary conditions. The splines are made to pass through a few points that are aligned, forcing each orbit to be straight over some minimum distance. Each spline is parametrized with 4 constraints: the angular width and tilt of the straight section, plus the angular position and radius of the center of the reversed bend. We also introduce for each orbit an angular shift with respect to the innermost one. This sums up to a total of $5 \times 4 + 4 = 24$ parameters. We then calculate the Fourier transform of each of the 5 orbits truncating at order $j = 9$. The values of each harmonics C_j and S_j are interpolated for intermediate values of a using cubic splines.

We let the optimizer adjust these 24 free parameters, with the objective to minimize the RMS tune variation of both horizontal and vertical tunes. We found a large number of solutions, and picked one with the tunes away from low-order betatron resonance lines, see Fig. 1. The corresponding orbit shapes and field distribution are presented in Figs. 2 and 3.

Constant-Tune 2 GeV Cyclotron

Both vertical and horizontal tunes are constant to within 0.01, and the field is isochronous to a very high precision, see Fig. 4. Particle tracking using the cyclotron code CYCLOPS[8], shown with solid lines on Fig. 4, confirms this result. The corresponding magnetic field presents ~9-degree wide low-field sections, see Figs. 2 and 3. The entire tune spread is barely visible on the tune diagram around the working point (2.78, 3.16), see Fig. 1.

CONCLUSION

We have demonstrated that there exists magnetic field distributions, that satisfy Maxwell's equations, that are precisely isochronous and produce a constant transverse focusing (constant betatron tunes) in both transverse directions over a wide energy region. This allows for the design of

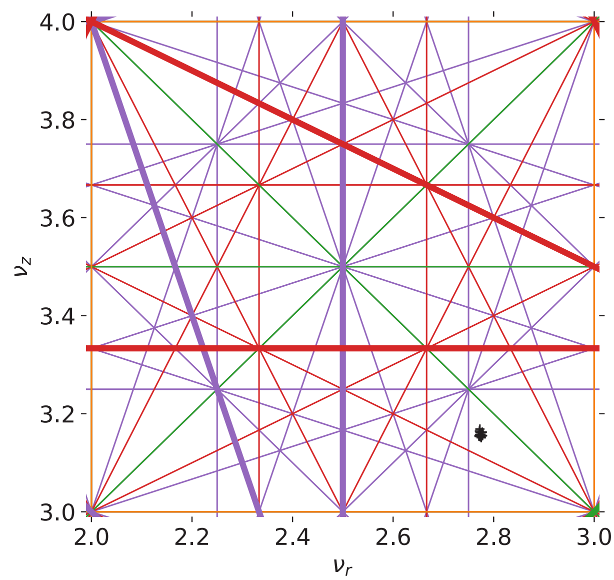

Figure 1: Tunes obtained using the field distribution shown in Fig. 2 are show in black, and appear as a small dot near the bottom right corner. Betatron resonance lines are shown up to 4$^{\text{th}}$ order, with the structural resonances shown with thick lines and non-structural with thin lines.

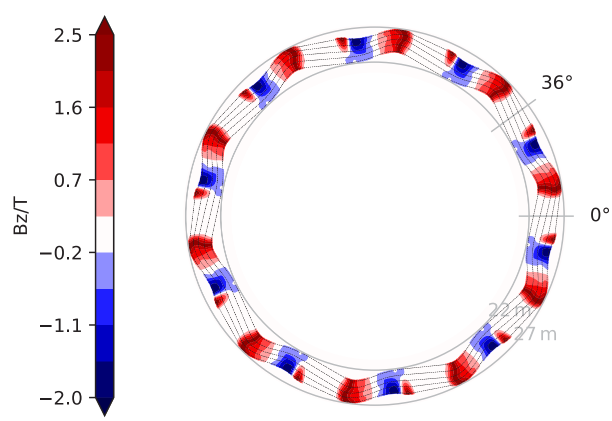

Figure 2: Magnetic field contours, with corresponding closed orbits, for a 10-sector 0.8 to 2 GeV proton cyclotron. Five closed obits are shown with thin dotted lines. The radii of the inner orbit and outer orbit are given for scale. Note the regions of low magnetic field are available for installation of rf cavities, injection and extraction systems, etc.

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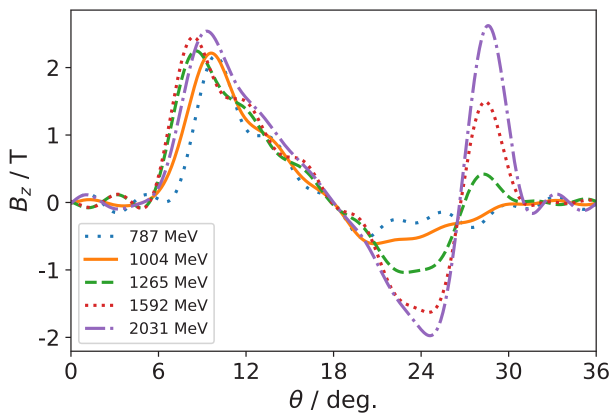

Figure 3: Magnetic field (B_z) along the 5 orbits shown in Fig. 2. The proton energy for each orbit is shown in the legend.

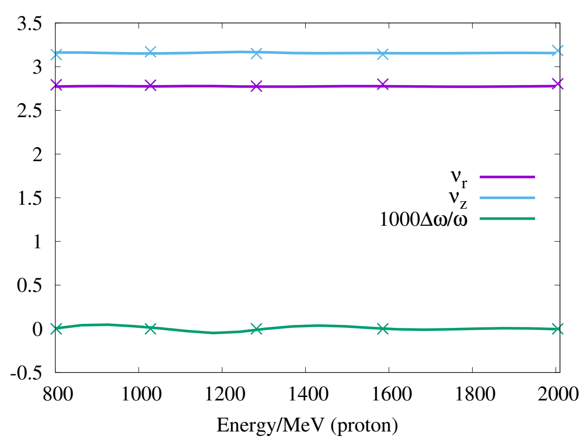

Figure 4: Transverse tunes, and relative variation of the orbital frequency, plotted as a function of the proton beam energy. Crosses show the results of calculation from `from_orbit`. Solid lines are results obtained after extracting a magnetic field map from `from_orbit`, and running it through the standard orbit code `CYCLOPS`.

high-energy cyclotrons that do not cross any betatron resonances, which could be a major advantage in the case of low-losses high-power machines.

The source code used to calculate tunes and generate fields maps for all the examples presented in this paper has been developed under a GPL-3 license and is available from: `https://gitlab.triumf.ca/tplanche/from-orbit`.

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