Some examples of recent progress of beam-dynamics studies for cyclotrons

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remark:
The references that are quoted in this presentation are not always explicitly mentioned, but are listed at the end of this PPT-file

I will limit myself to two main subjects

1. The **space charge problem:** it is actually very relevant: one would like to know the beam intensity limits in view of possible applications of (compact) cyclotrons, for example:
   i. For generating high fluxes of neutrinos for experiments such as IsoDAR or Daeδalus (see talk of Dr. Winklehner),
   ii. for RIB facilities (see talk of Dr. Okune of RIKEN, or talk of Dr. Zhang of CIAE)
   iii. as drivers for ADS or nuclear waste transmutation (see talk of Dr. Sakurai of RIKEN)

- An overview of the increasing understanding and the use (semi-) analytical models is given
- And the latest developments and achievements of numerical simulations is discussed

2. I will then discuss recent progress and tools for simulations for industrial and medical cyclotrons at IBA.

Three intensity limiting effects of space charge in cyclotrons

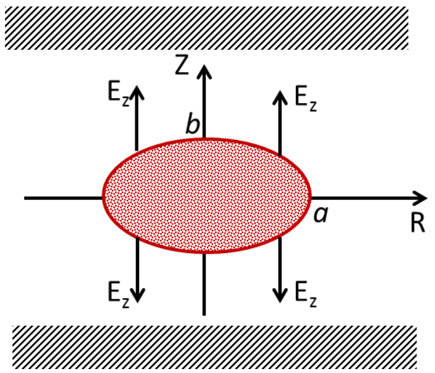
1. The problem of the beam blow-up due to the weak vertical focusing in the cyclotron center.
2. The problem of destruction of turn-separation due to space charge induced energy-spread.
3. The problem of space-charge induced halo for beams that are not well matched.

Space charge effects in the cyclotron center(1)

Here, I refer to papers of Rick Baartman, Thomas Planche and Yi-Nong Rao [1-3] from Triumf.

In the central region of a (compact) cyclotron the flutter goes to zero and the magnetic vertical focusing becomes very weak. The beam current is limited by the space charge defocusing and the resulting vertical losses on the central region. A measure for the depression of the vertical tune is the Laslett tune shift:

Laslett incoherent vertical tune-shift $$\Delta \upsilon_z{}^2 = -\frac{2}{\pi}\frac{NRr_p}{\boldsymbol{\beta}^2\gamma^3 B_f}\left(\frac{1}{b(a+b)}\right)$$

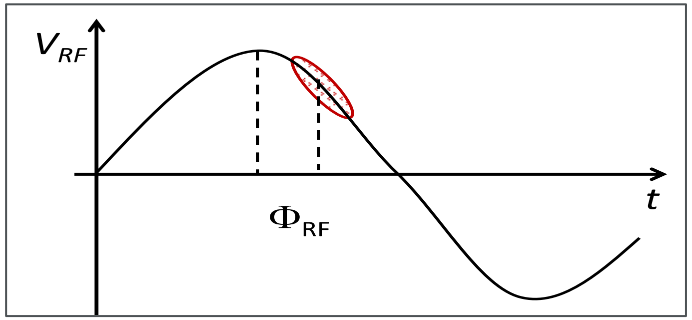


The formula shows a $1/E_{kin}$ dependence. So increasing the injection energy E_{kin} helps to increase the current limit. This is done for example at PSI (870 keV) and at Triumf (300 keV). Drawback is that losses of the high injected beam power can already be harmful. Furthermore such high injection energies can not be done in smaller machines.

In some cases a small magnetic field bump in the cyclotron center may be used to somewhat increase the vertical focusing (at the cost of some RF phase slip)

Space charge effects in the cyclotron center(2)

For compact cyclotrons one mainly has to rely on the weak electric focusing of the RF gaps. This focusing is phase dependent. The center of the bunch must cross the gap on the falling RF slope . Then the head of the bunch is weaker focused than the tail. Space charge causes progressive loss of the head of the bunch[1].

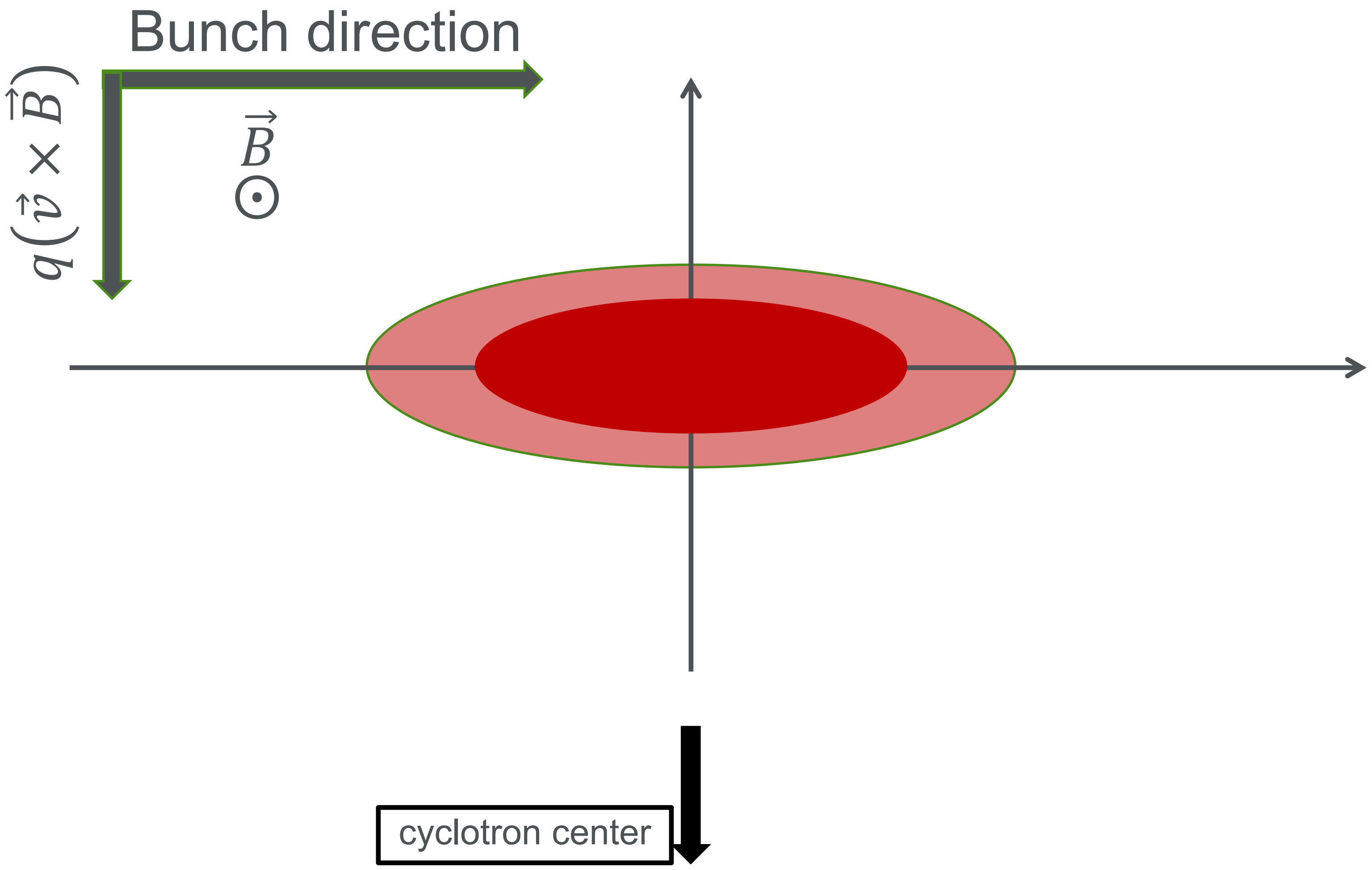


When injecting with a (spiral) inflector, the beam needs to be strongly compressed in all three dimensions (by bunching and transverse focusing). The beam at the inflector exit is strongly mismatched with respect to the weak vertical focusing. The complicated spiral inflector optics strongly correlates the 6D phase space. Both effects result in strong emittance growth. An elegant way to analyze this problem is by the beam σ-matrix approach (see Baartman ECPM 2015)

The bunching, as well as the beam transport along the cyclotron axis, through the spiral inflector and the acceleration in the central region during the first few turns, is a very complex 3D space charge problem. There is a need for a precise particle tracking approach.

Vortex motion ⇒ space charge induced energy spread

Intuitive understanding of the vortex motion in an isochronous cyclotron



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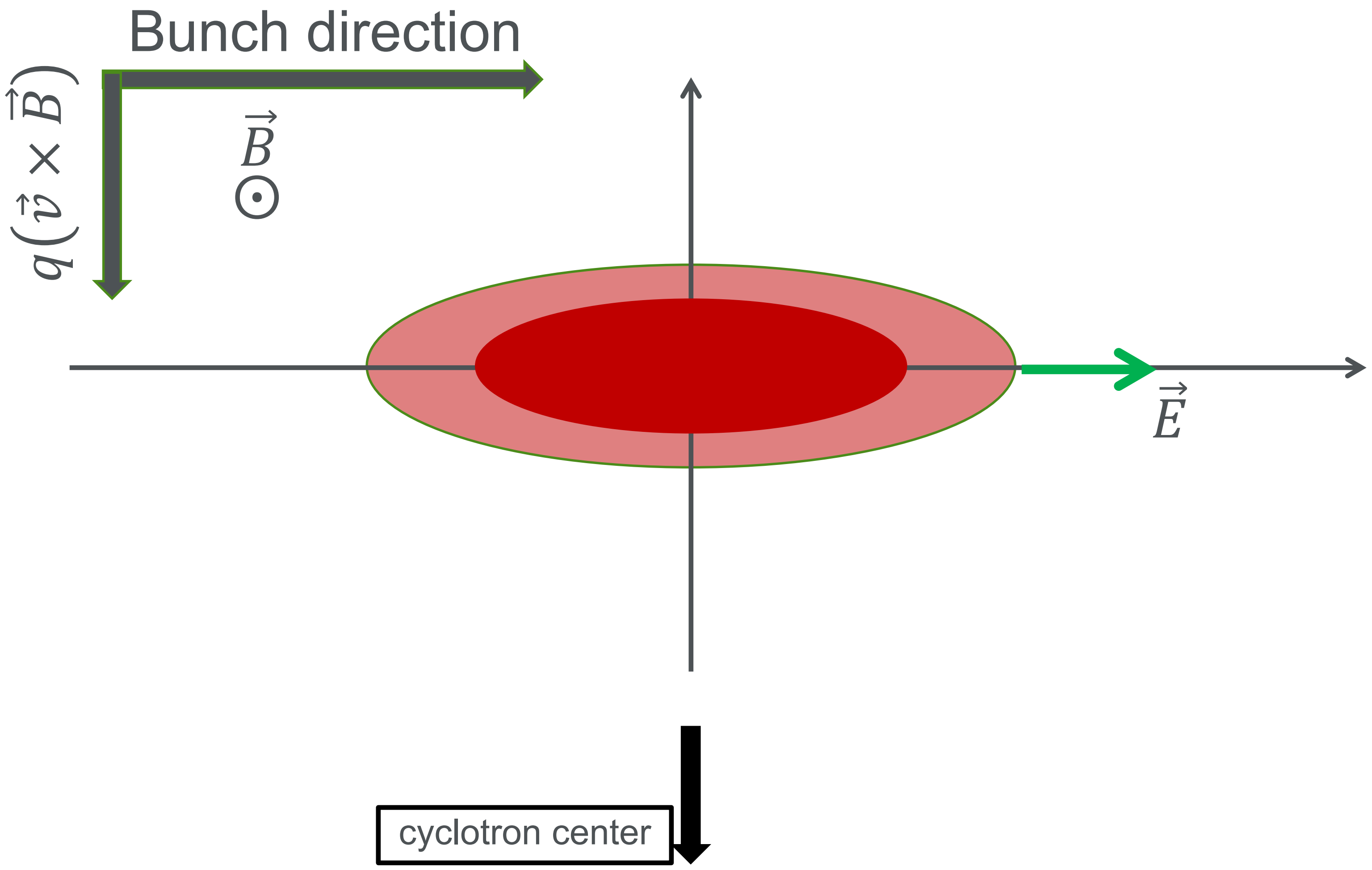

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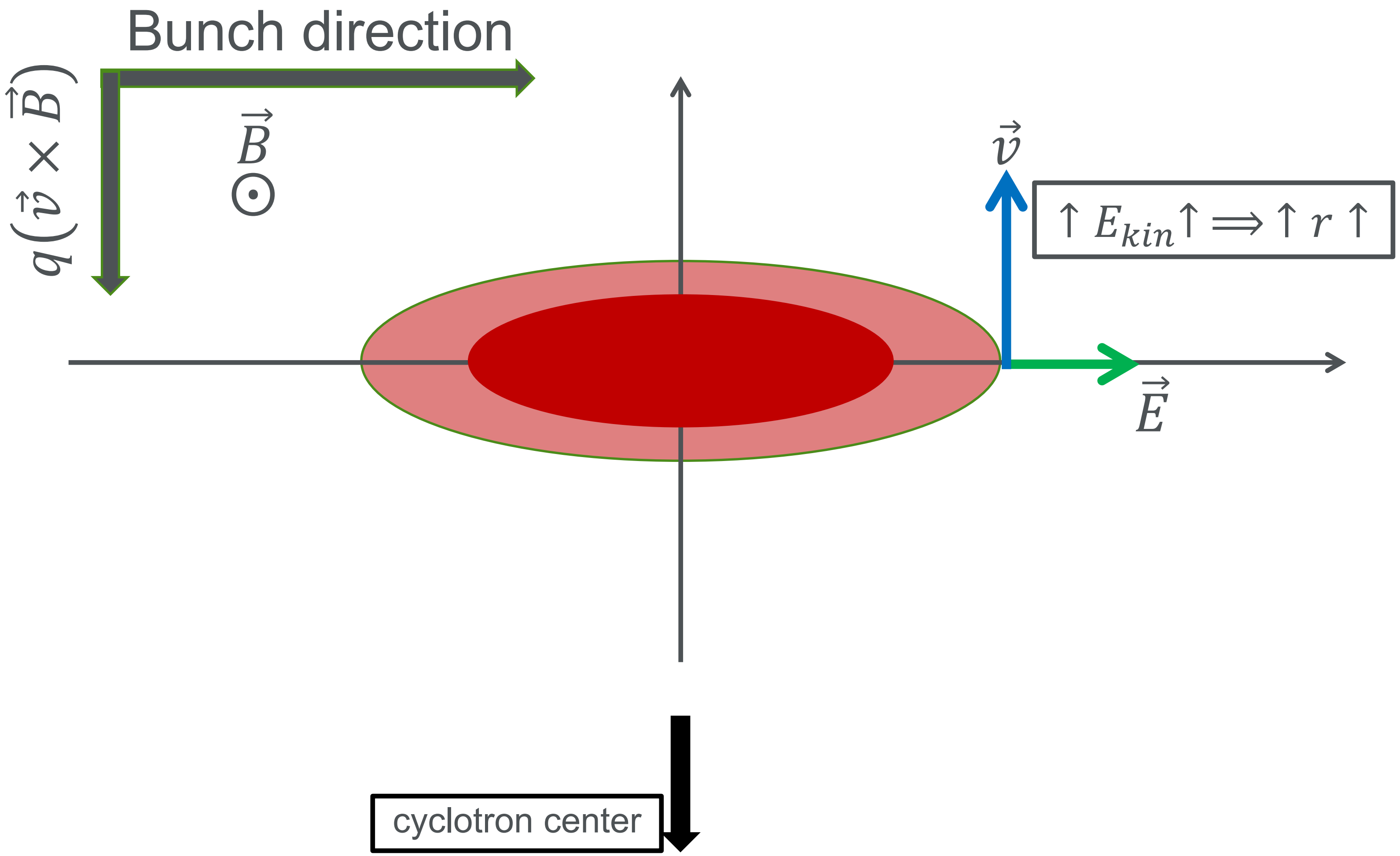

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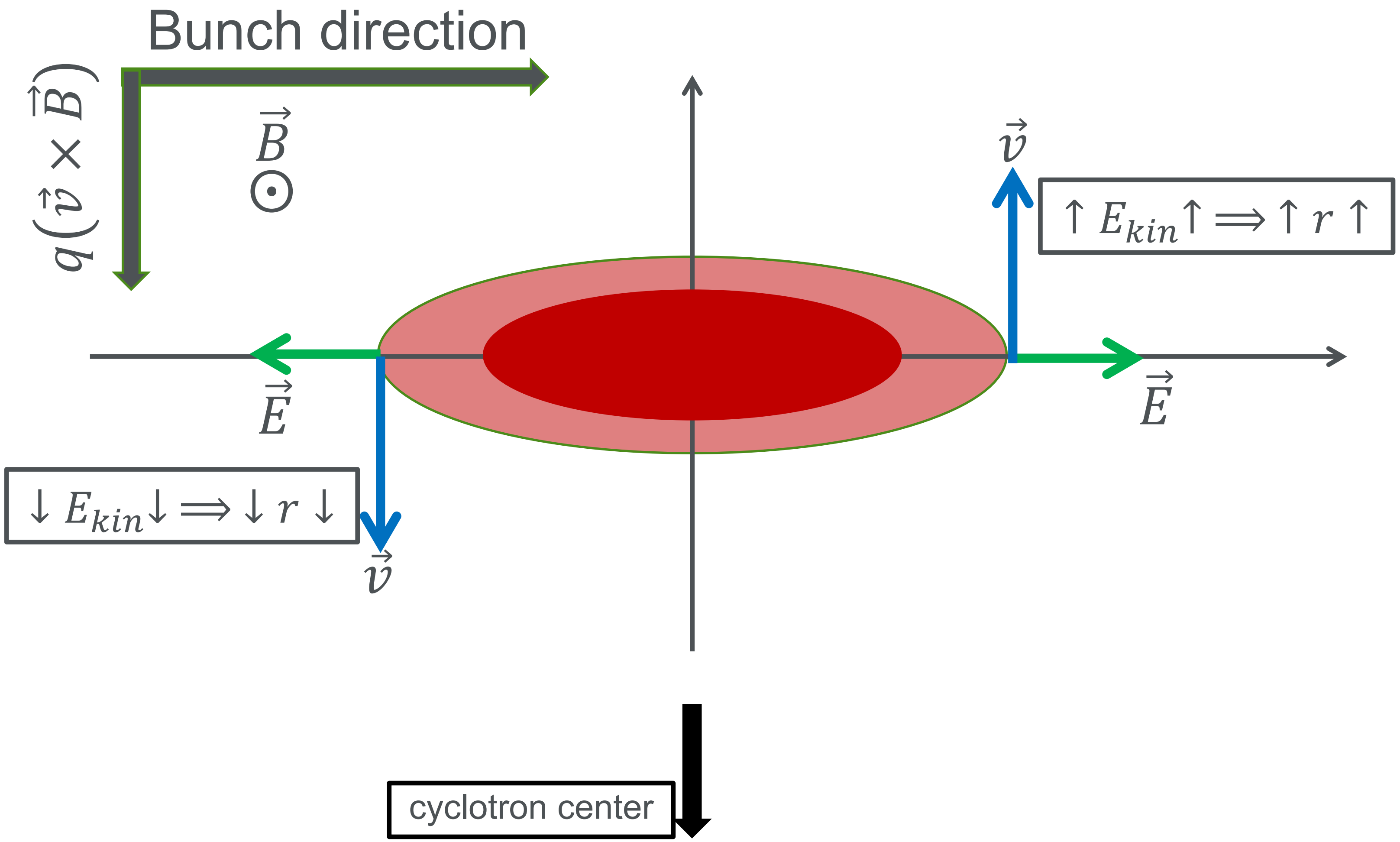

- Due to the outward directed space charge force, the leading particles gain energy
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- Trailing particles loose energy and move to lower radius

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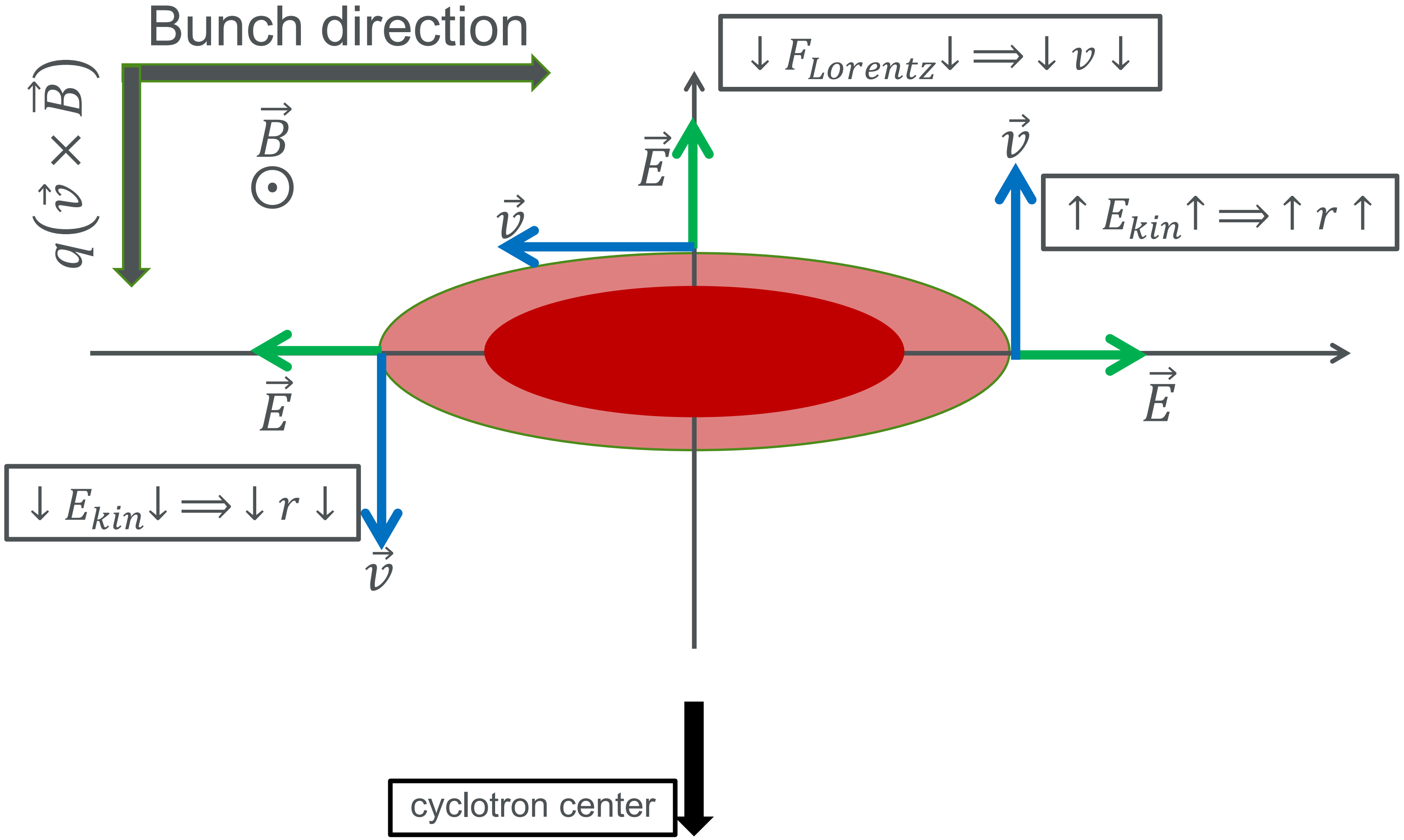

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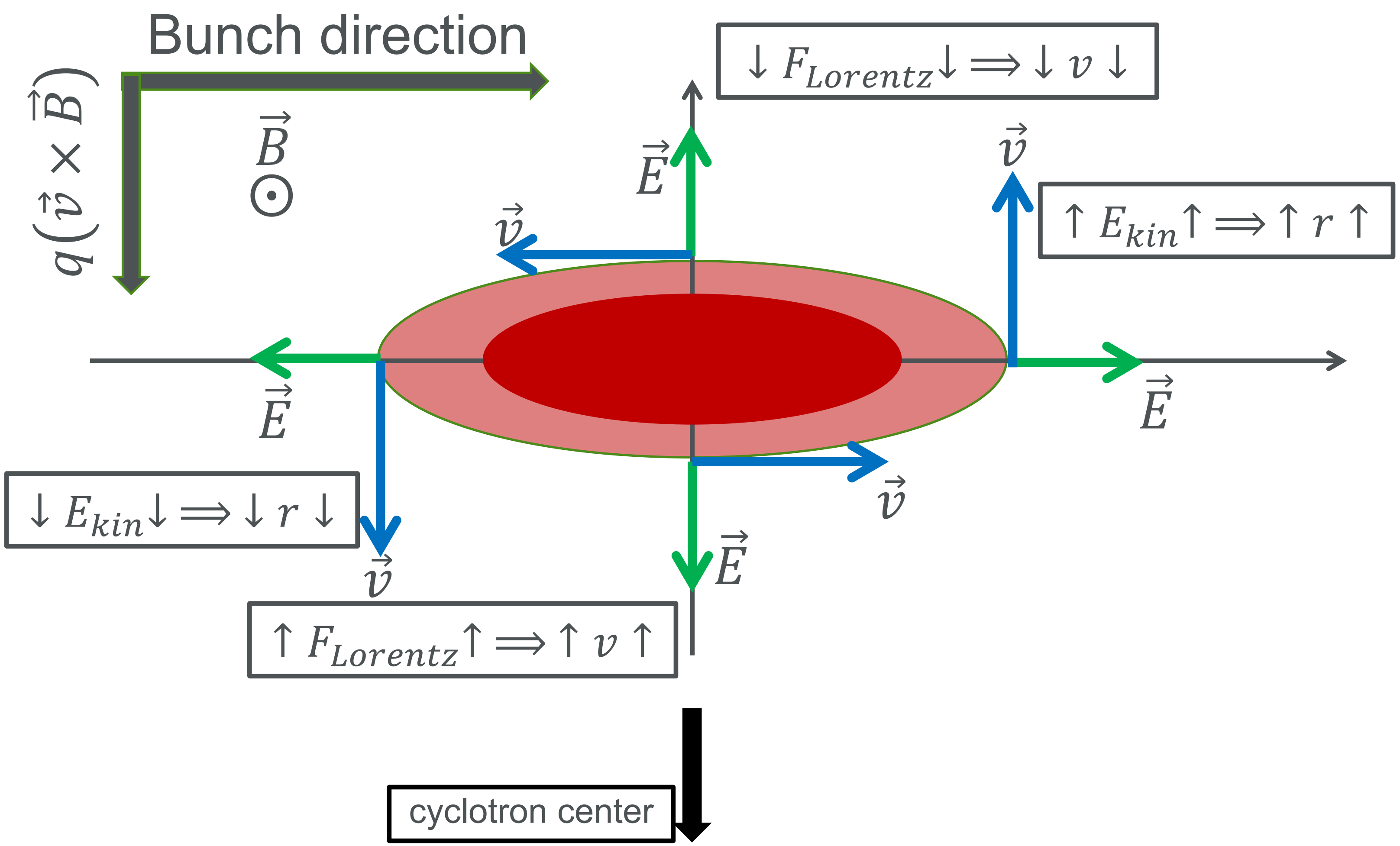

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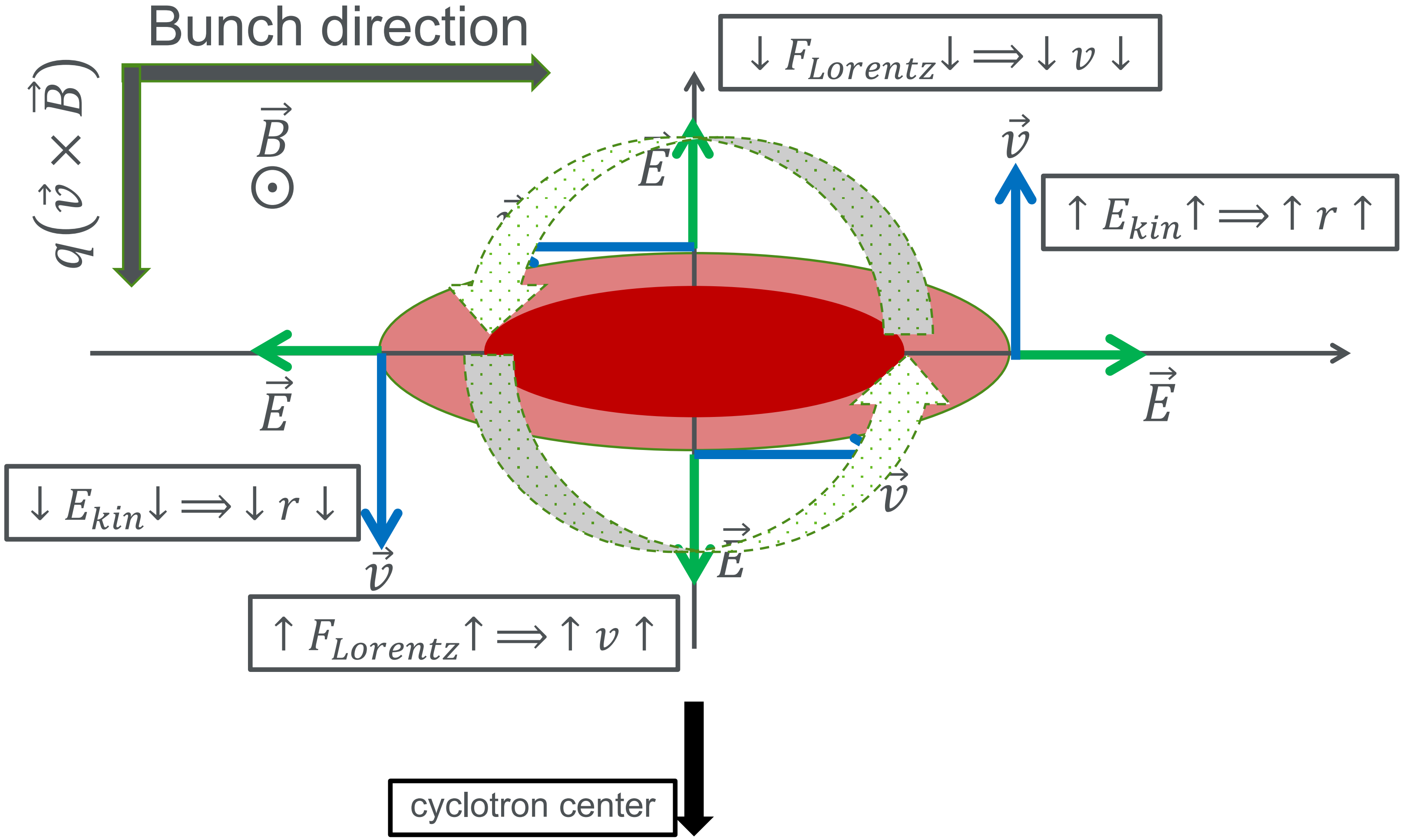

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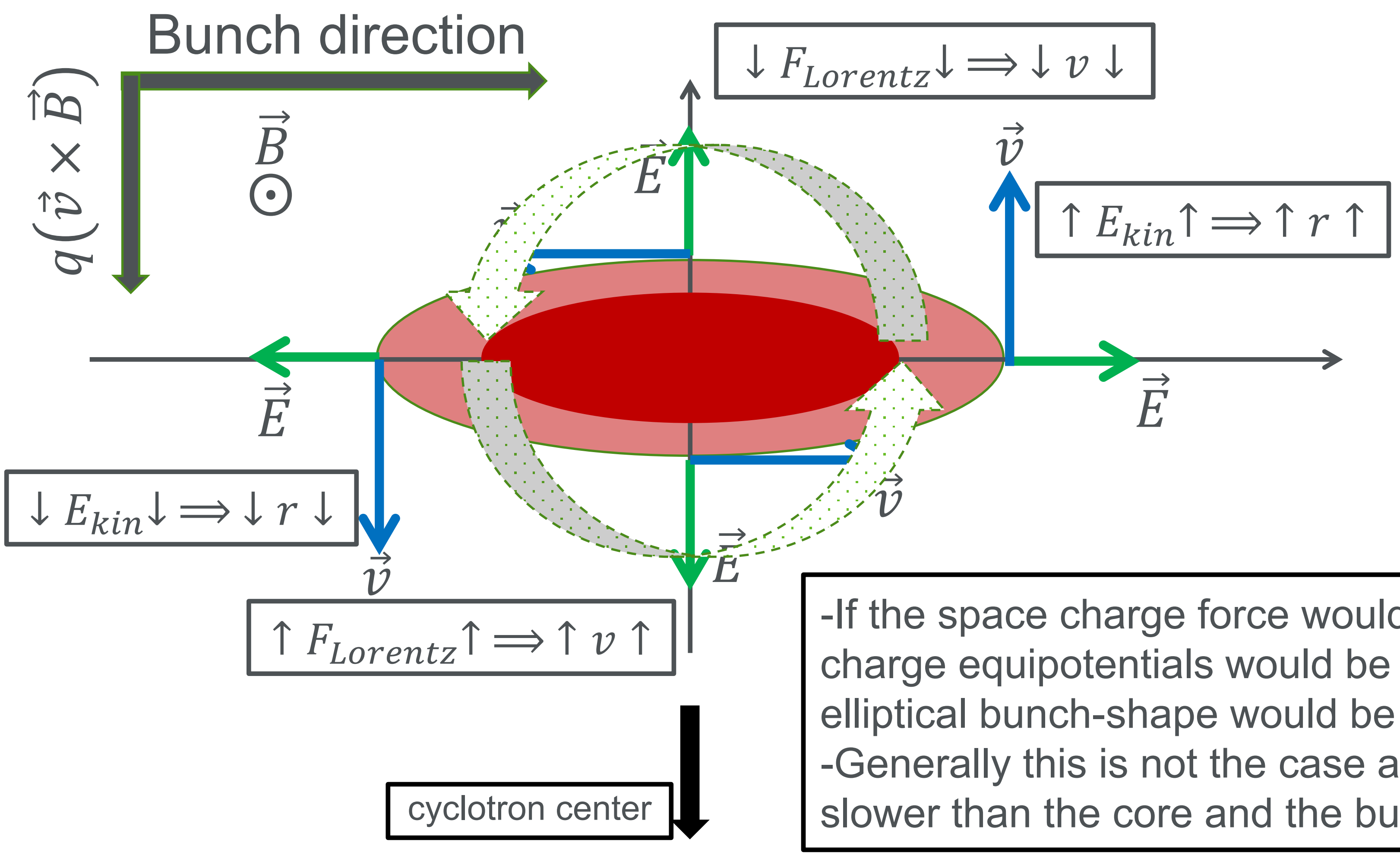

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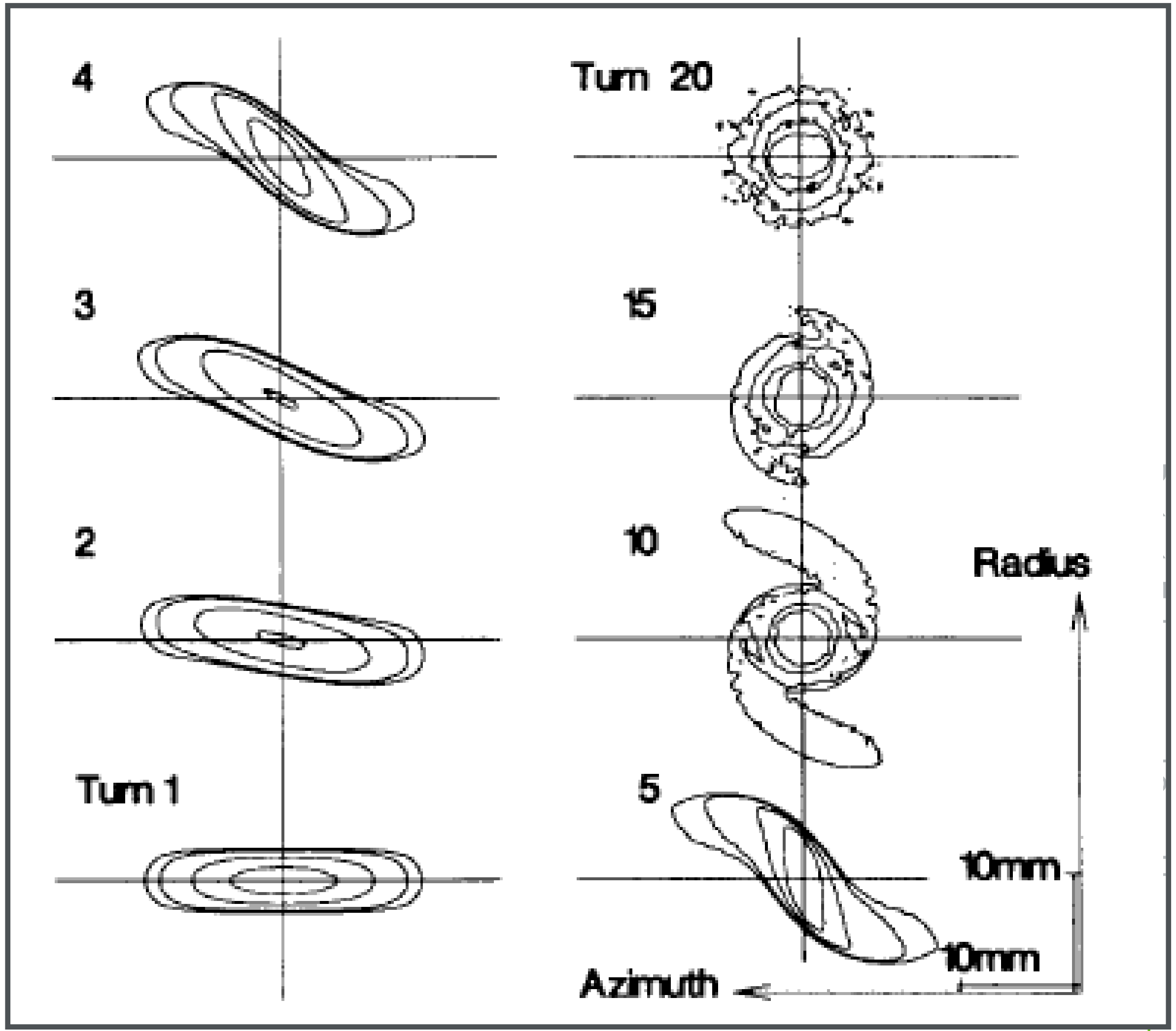
-The opposite happens for the interior particles

-The bunch density shows an effect of macroscopic rotation (vortex).

-If the space charge force would be perfectly linear and the space charge equipotentials would be similar to that of the ellipse, then the elliptical bunch-shape would be conserved.
-Generally this is not the case and the outer part of the bunch will rotate slower than the core and the bunch starts to deform (spiralize)

Discovery of the vortex motion at PSI

- Stefan Adam from PSI first started with efforts to simulate the longitudinal space charge effects in their injector and ring cyclotron[4].
- The right figure shows a later (1995) simulation[5] of a 1 mA bunch accelerated in the PSI injector II, with an initial phase width of 15°, during 20 turns. It is seen that the core of the bunch rotates faster than the envelope, resulting in an initial deformation of the bunch.
- After 10 to 15 turns a round beam emerges with an intense core, surrounded by halo.
- Halo reduces with better initial matching

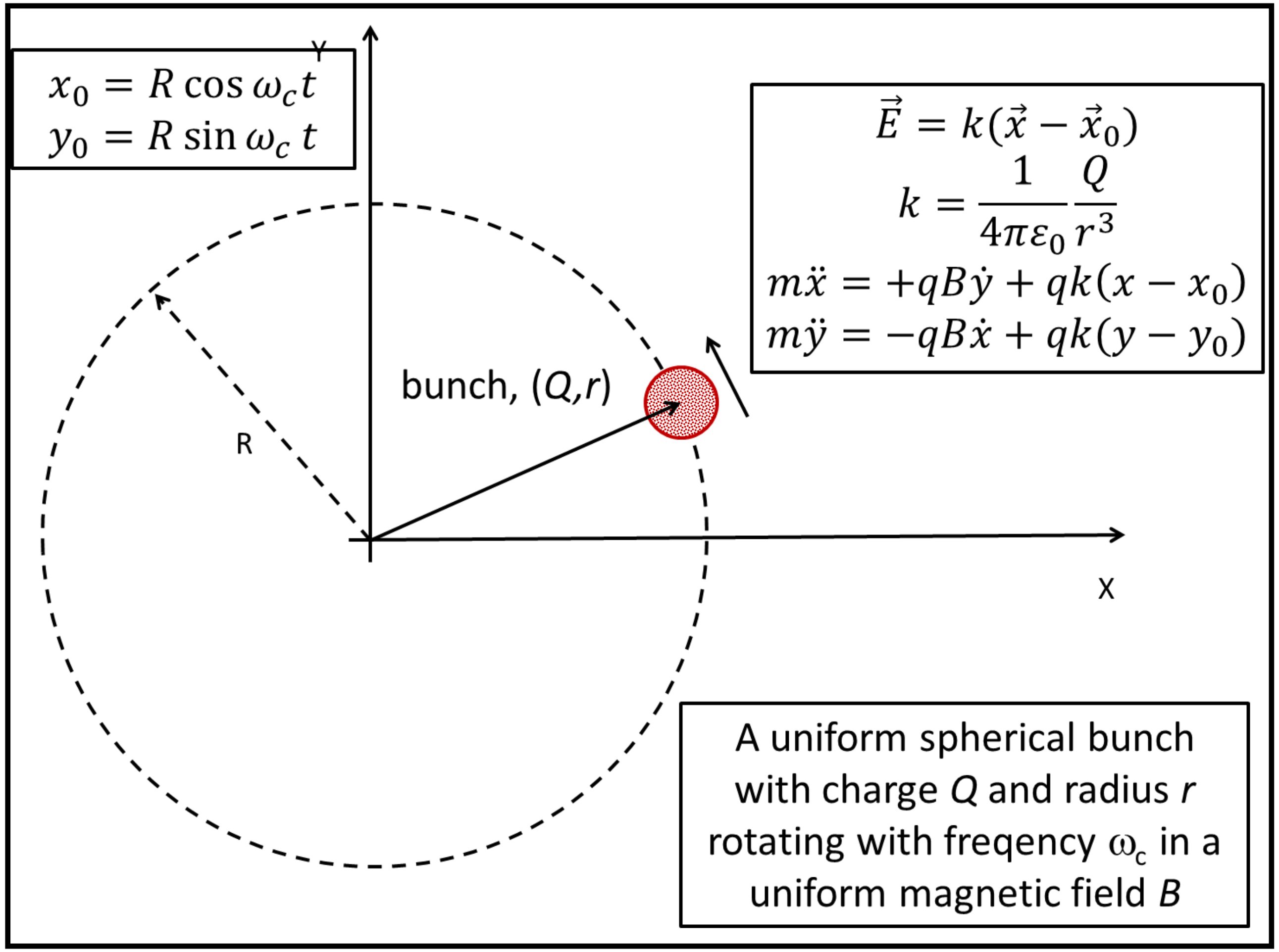


Vortex effect, some literature on analytical approach(1)

- The vortex effect was first recognized in 1969 by Mort Gordon[7] who explained that, due to the Coriolis forces in a rotating frame, the particles in the bunch execute a steady state velocity pattern which is directed along the equipotential curves of the space charge potential. However, at the same time he found that this effect could be neglected in the then known practical case where the bunches were much longer than wide.
- In 1981 Werner Joho[8] elaborted furher on Gordon's idea, using a model of multiple turns with constant azimuthal length (sector- or pie-model) to calculate the space charge induced energy-spread and its effect on the turn-separation. This resulted in a formula showing an intensity limit proportional to the cube of the energy gain per turn (or RF-voltage; so a $1/n^3$ dependence). This formula is still found to be correct although the sector model has been invalidated later with the numerical confirmation of the vortex motion at PSI.
- In 1988 I myself [9] applied the 3D beam-envelope approach of Sacherer[10] to derive differential equations for the full set of second moments of a space charge bunch in an Azimuthally Varying Field (AVF) cyclotron. One of the outcomes was a proof of existence of round bunches. Another was that such bunches follow envelope equations that are similar to the Kapchinsky-Vladimirsky (KV) equations.

Vortex effect, some literature on analytical approach(2)

- In 2001 Bertrand and Ricaud[11] propose an elegant and simple model of a spherical non-relativistic bunch in a homogeneous magnetic field. The resulting linear equations of motion of particles in the bunch can be easily solved analytically.



This motion is found to be stable if for a given bunch radius (*r*) the total bunch charge (*Q*) is smaller than a threshold Q_{max}

$$Q < Q_{max} = \pi\left(\frac{V_m}{cZ_0}\right)\frac{\boldsymbol{r^3}}{R_\infty{}^2}$$

$V_m = mc^2/q$ and $R_\infty = c/\omega_c$

This can be written as a local condition on the plasma-frequency ω_c

$$\Rightarrow \omega_p < \frac{1}{2}\sqrt{3}\omega_c$$

$$\omega_p = \sqrt{q^2N/\varepsilon_0 m}$$

Vortex effect, some literature on analytical approach(3)

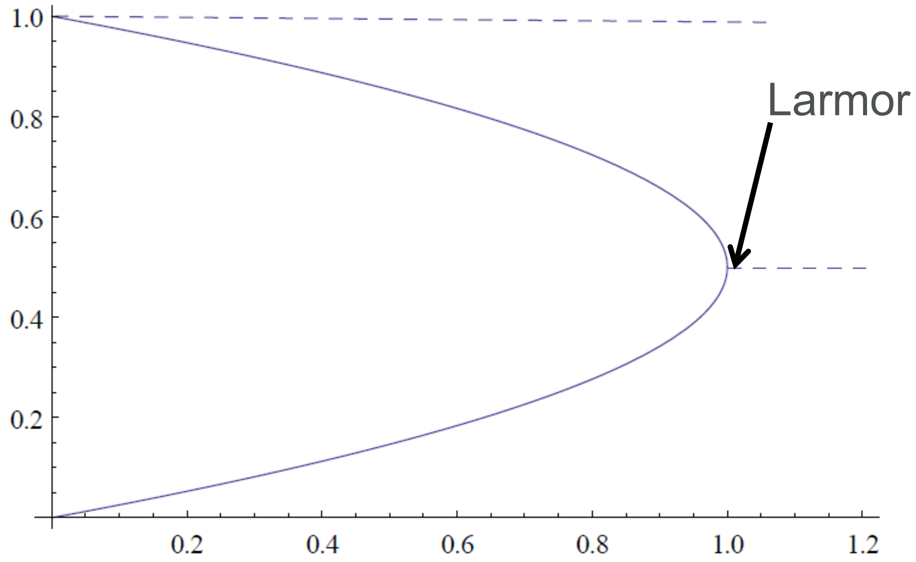
- In his 2013 paper Baartman[6] further explores the models myself and that of Bertrand/Ricaud and also provides a deeper insight into the physics of the vortex motion.

The tunes of two oscillations are obtained and interpreted as modes of betatron (r,P_r) motion and dispersion (E,Φ) motion. For $Q \ll Q_{max}$ the betatron oscillations are fast and the energy oscillations slow. For $Q = Q_{max}$ the acceptance approaches zero and both frequencies are equal to 1/2. This is a beam with zero emittance and laminar flow.

Two modes of coupled motion

$$\nu_{r\pm} = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{Q}{Q_{\max}}}\right)$$

Mode tunes $\nu_{r\pm}$ vs. $Q/Q_{\max}$



Vortex effect, some literature on analytical approach(4)

A formula is derived for the intensity limit of separated turn cyclotrons which applies if the injected bunch is sufficiently short such that the vortex motion causes the bunch to curl up into a single droplet

Acceleration effects are considered and a qualitative threshold is found for the vortex motion to take place: below the threshold the bunches maintain their phase length (thus bunch length increasing like RΔθ), but above it the bunch length remains constant and thus decreases in phase length.

- Intensity limit
  - 3rd power on RF-voltage
  - Scaling laws with respect to particle type, energy and tune and also bunch aspect ratio.

$$I_{\max} = \frac{h}{2 g_r \xi^3 \beta^3 \gamma \nu_x{}^4} \frac{V_{rf}{}^3}{V_m{}^2 Z_0}$$

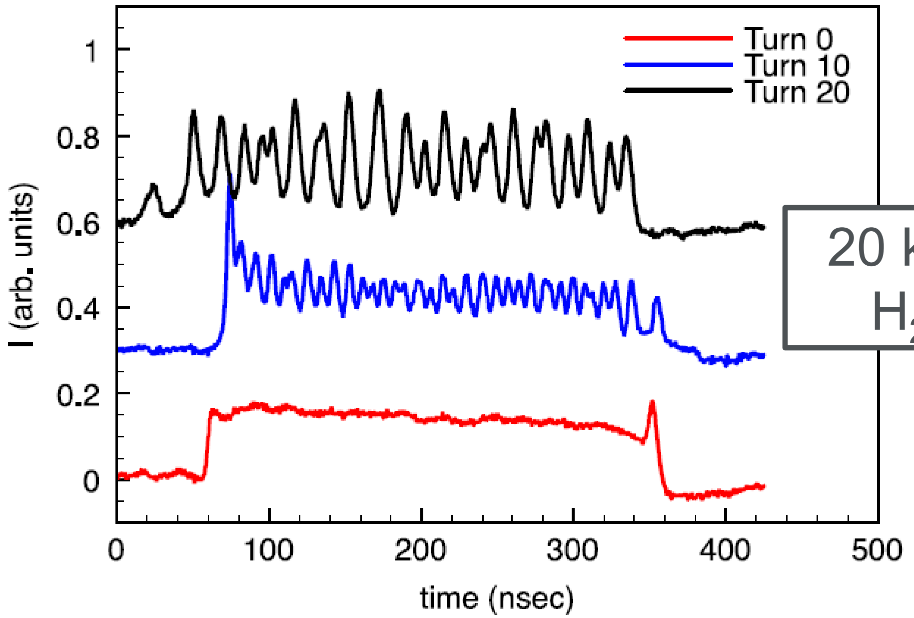
- Vortex threshold:

  The space charge induced tune shift must be (considerably) larger than the relative velocity increase per turn due to acceleration

$$2\pi\Delta\nu_r \gg \frac{\delta\beta}{\beta}$$

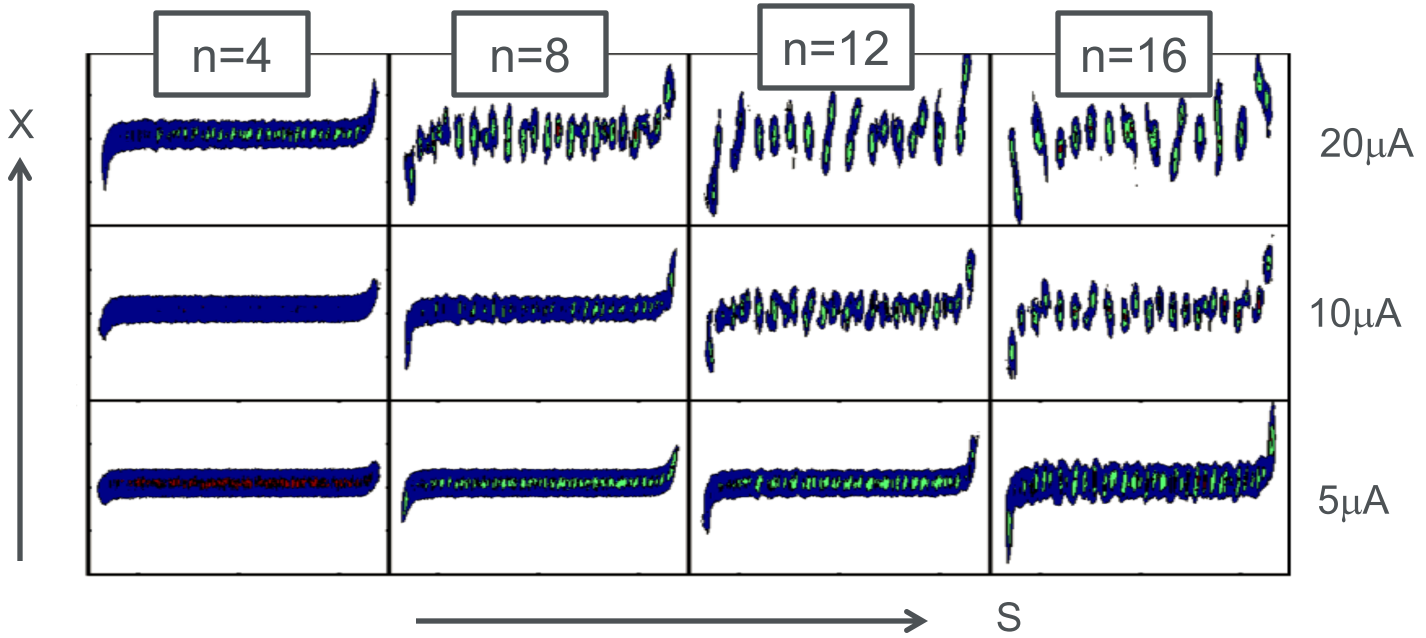
Break-up of a bunch into droplets

- The vortex effect makes that bunches with a high length/width ratio break up into small approximately circular droplets. This was shown both experimentally and by simulations in the 2003 thesis study of Pozdeyev[12] on the Small Isochronous Ring (SIR) at MSU.



20 keV H_2^+

Longitudinal time structure measured on a fast Faraday cup at 0, 10 and 20 turns after injection in the SIR at MSU for a bunch peak current of 10 μA (from Pozdeyev[13])



Pozdeyev[13] beam simulations in the SIR with the PIC tracking code CYCO. Each frame contains a median plane projection of the bunch and has a size of 5 cm × 45 cm (x × s). Droplets start to appear quickly, depending on the peak current

Fluid dynamics approach to space charge vortex effects (1)

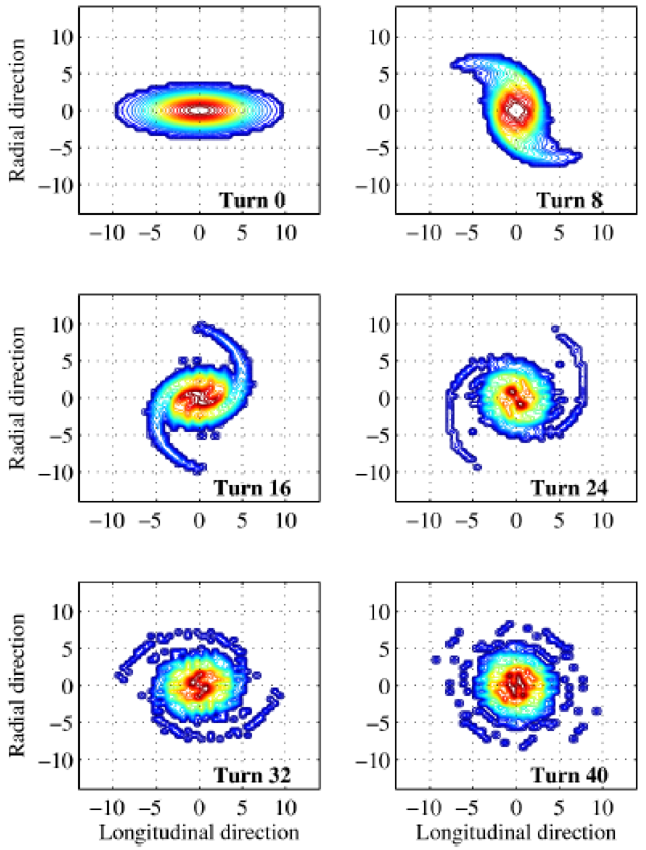
- Recently Antoine Cerfon has introduced a new approach in which he directly obtains an approximate solution of the collisionless Vlasov equation [14-16]
- I am not expert in this field of fluid dynamics, but I think this approach looks so promising that is must be mentioned
- The main simplifications and assumptions made are the following:
  - The space charge is not too strong such that the incoherent tune shift is small ($\omega_p < \omega_c$)
  - The beam size is mainly determined by dispersion and little by emittance (close to laminar flow)
  - A non-relativistic, 2-dimensional coasting beam in uniform B-field
- The 1st assumption makes that the time-scale associated with betatron oscillations is much faster than the time scale associated with space charge
- This allows to apply an averaging procedure to the Vlasov equation
- Together with the Poisson equation this gives two simple coupled 2D partial differential equations for the bunch-density

Fluid dynamics approach to space charge vortex effects (2)

The vortex motion can be intuitively understood as the nonlinear advection of the bunch by the **E X B** velocity field (similar to Gordon's idea).

an example

Formation of a round beam core surrounded by a low density halo with the fluid-dynamic approach by Cerfon[15].
δ^2=0.2



E × B

$$\frac{\partial n}{\partial t} + \delta^2 \nabla\phi \times \mathbf{e}_z \cdot \nabla n = 0$$

$$\nabla^2 \phi = -n$$

$$\delta^2 = \left({}^{\omega_p}/_{\omega_c}\right)^2$$

n=charge density; ϕ is SC potential

ω_p , ω_c plasma and cyclotron frequency

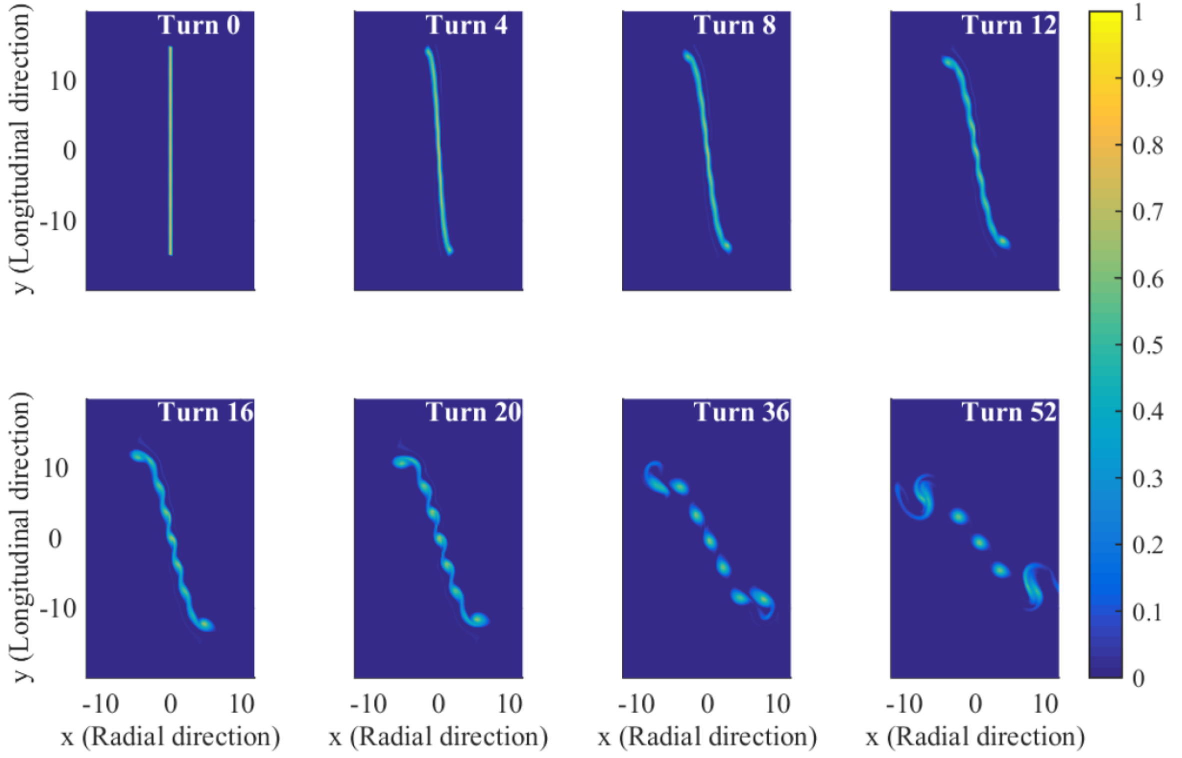
δ^2 is a measure of SC, proportional to I_{peak}

Note that δ^2 can just be removed by a scaling of time => the nature of the vortex motion does not depend on intensity but only the time scale.

Fluid dynamics approach to space charge vortex effects (3)

- The model could serve as an interpretation of complicated PIC simulation results and identifies the basic contributing mechanisms
- As compared to PIC methods, the solution of the equations does not require large supercomputers and long computing times.
- Of course the approach does not give the quantative precision as PIC codes such as for example OPAL. Such precision is needed for actual designs. Both approaches are complementary

another example



A simulation of the beam break-up with the fluid-dynamic approach by Cerfon[16]

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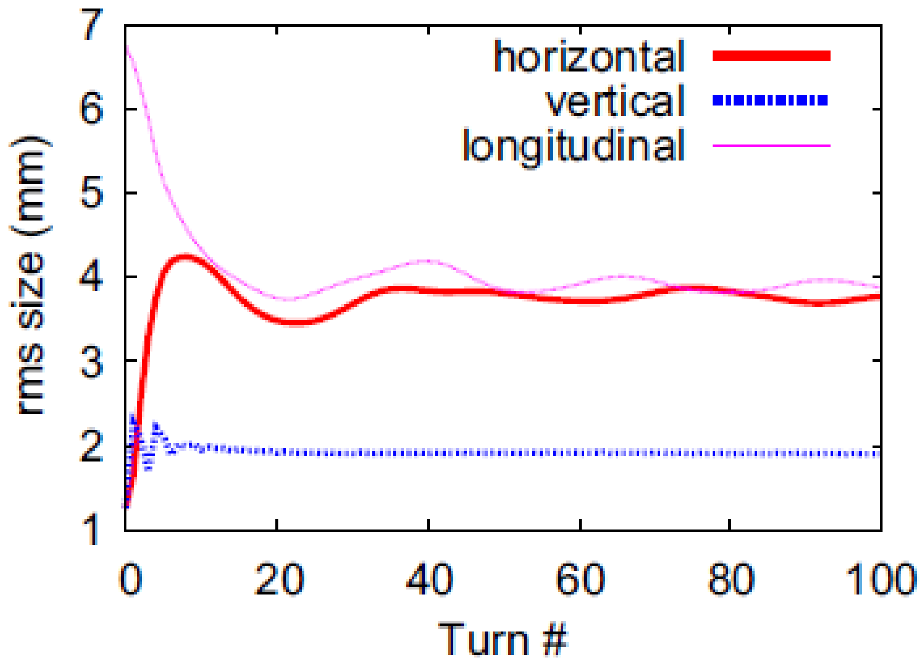
Importance of the vortex effect

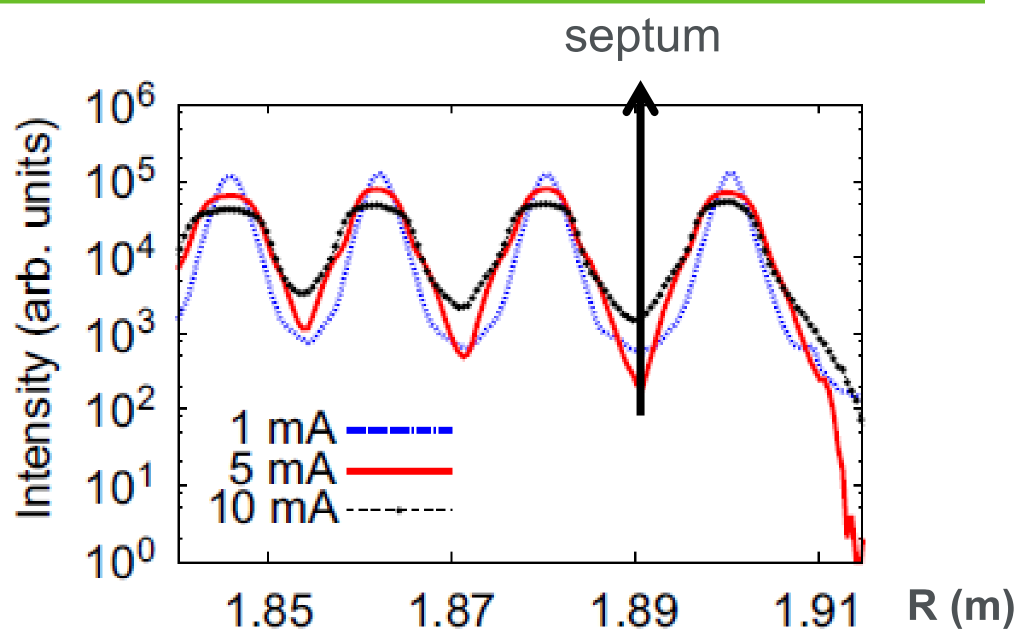
- For cyclotrons using stripping extraction (such as H^- at Triumf or H_2^+ in the proposed Daedalus ring-cyclotron[17]) turn-separation at extraction is not needed and the energy spread induced by space charge has no importance. The relation between energy and radius remains unique (except for the smal contribution of emittance) and the quality of extracted beam is not really affected. These cyclotrons can accept a large phase width at injection.
- For high-intensity cyclotrons with an ESD, turn-separation at extraction is crucial (avoid septum losses) and a good matching is needed at injection such that the vortex-effect occurs quickly, resulting in circular bunches. For too long bunches their sizes increase and a large halo develops, resulting in high extraction losses.
- For the PSI injector II cyclotron the vortex motion is so strong that very short bunches are obtained such that the flattop system is no longer needed[18] (and is actually used to accelerate).
- In the PSI ring cyclotron the space charge effect is not strong enough to produce the circular bunches. Here the relative phase of the flattop cavities is detuned such that the energy gain in the tail of the bunch is larger than in the head, thereby counteracting the longidutinal space charge [19].

OPAL: Object oriented Parallel Accelerator Library

- OPAL is a PIC space charge tracking code, for large accelerator structures and is developed mainly at PSI[20]. It extensively relies on parallel processing and is able to simulate large numbers of accelerated particles (order10^6) in cyclotrons resulting in very precise beam density and profile predictions
- It has been (and still is) used extensively in simulations of the PSI injector II cyclotron as reported in the talk of Anna Kolana[21]
- It also has been applied for other cyclotrons such as for example the PSI ring cyclotron[19,22], the CIEA 100 MeV H^- cyclotron[23] and the proposed Daedalus cyclotrons[17]. But also for other machines such as FFAG's[24]
- The code remains under further development. It can simulate the space charge effect of neighboring bunches[22]. Besides space charge it has various other simulation applications such as wake-fields, multipacting and particle-matter interaction
- An update on OPAL will be given by Andreas Adelmann later this morning[25]. Therefore I limit myself here to a few examples.

Examples of OPAL simulations of the IsoDAR cyclotron





An initially mis-matched long Gaussian bunch (aspect ratio 5/1, $\Delta\Phi$=40°) of 5 mA H_2^+ is coasting at 3 MeV and followed during 100 turns. After 10 to 15 turns, the beam has obtained the circular match due to the vortex effect. This suggest that, as in the PSI injector II, flattop cavities are not needed and 4 accelerating cavities can be placed in the valleys

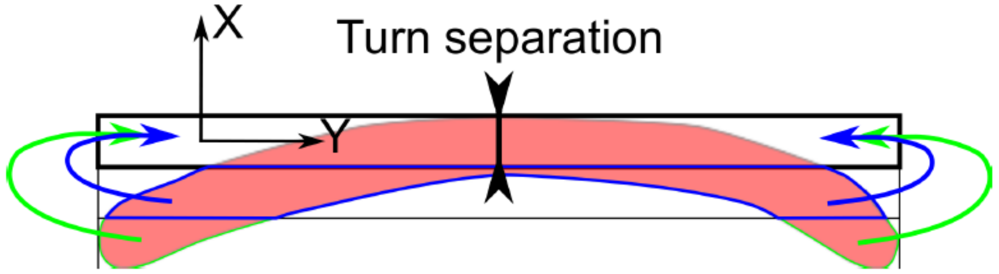
Using a large number of macroparticles (10^6) OPAL allows to predict with high dynamic range (10^4) the density profile in between the last two turns. This allows to estimate the losses on the septum (0.5 mm) as function of the beam current and also to optimize collimators placed closer to the center. These collimators partly remove beam-halo. Based on such simulations it is claimed that IsoDAR can extract 5 mA of H2+.

Simulation of SC for cyclotrons with small turn-separation

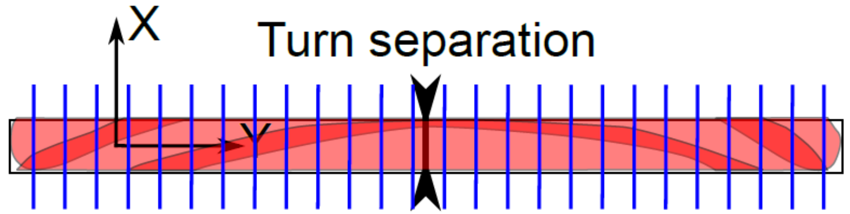
- H^- cyclotrons require no large turn-separation ΔR because of stripping extraction.
- These machines have a large phase acceptance (up to 60°).
- Bunches are long and also have a large radial extend due to large energy spread.
- At large radii, turns overlap and the effects of neighboring turns become essential
- The neighboring bunch feature is available in OPAL
- For the Triumf case, with extreme long bunches, OPAL would require a very large number of particles and a large grid.
- Thomas Planche [3,26] from Triumf made a code (TRICYCLE) which uses periodic boundary conditions in the radial direction to solve the Poisson equation.
- This considerably reduces computation time, because only one bunch needs to be followed during the simulation.

The use of radial periodicity: simulation code TRICYCLE

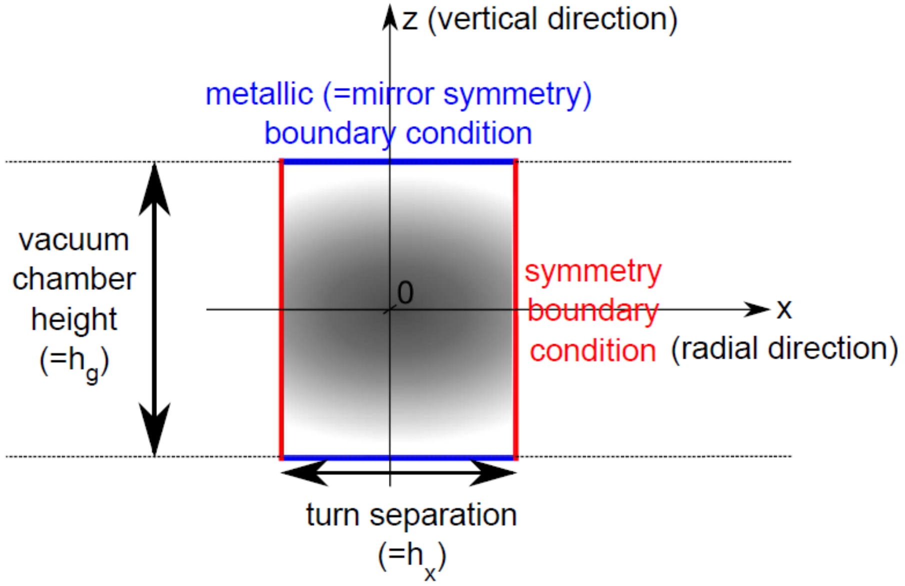
Bunch shape evolves slowly turn by turn and one may assume periodic boundary conditions in the r-direction. Bunches are sliced radially by a box with a width of the turn separation ΔR. Parts of the bunch that are outside of the box, are returned into it by a radial shift of ΔR



The box is sliced longitudinally to create a number of 2D surface-charge density distribution (in X-Z plane)



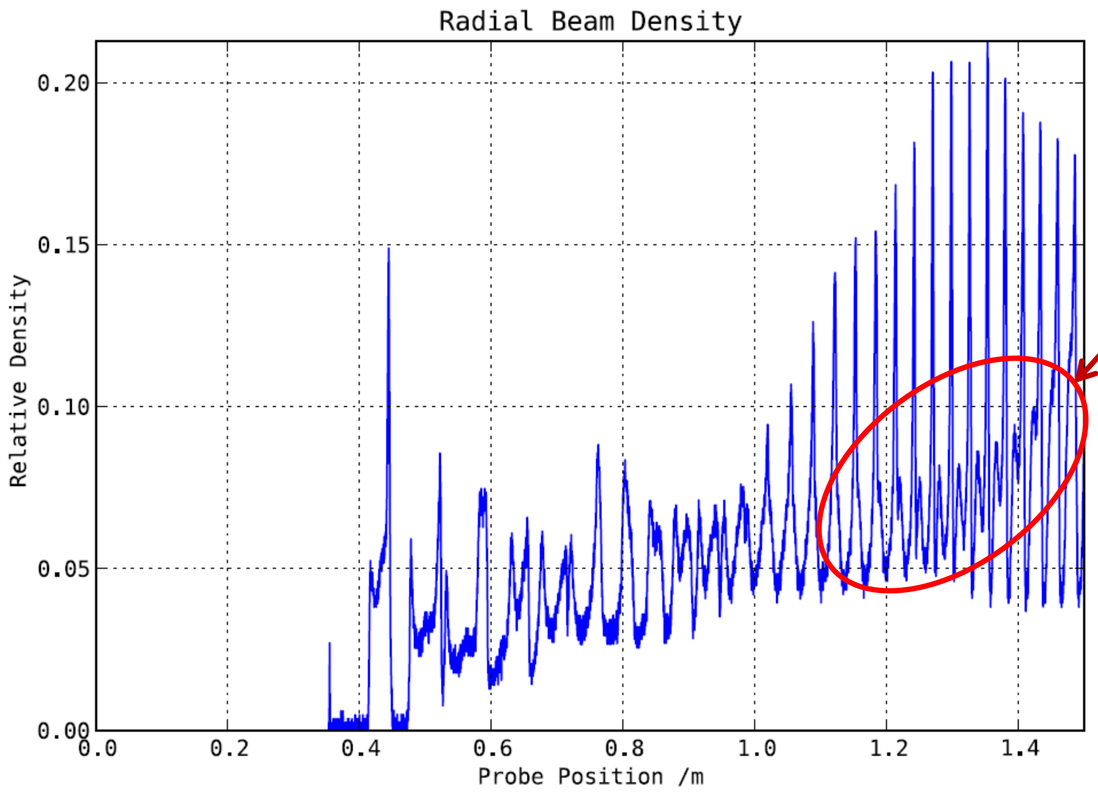
For each slice the 3D Poisson equation is solved by FFT with proper boundary condition (x ⇒ periodic, y ⇒ open, z ⇒ metallic)



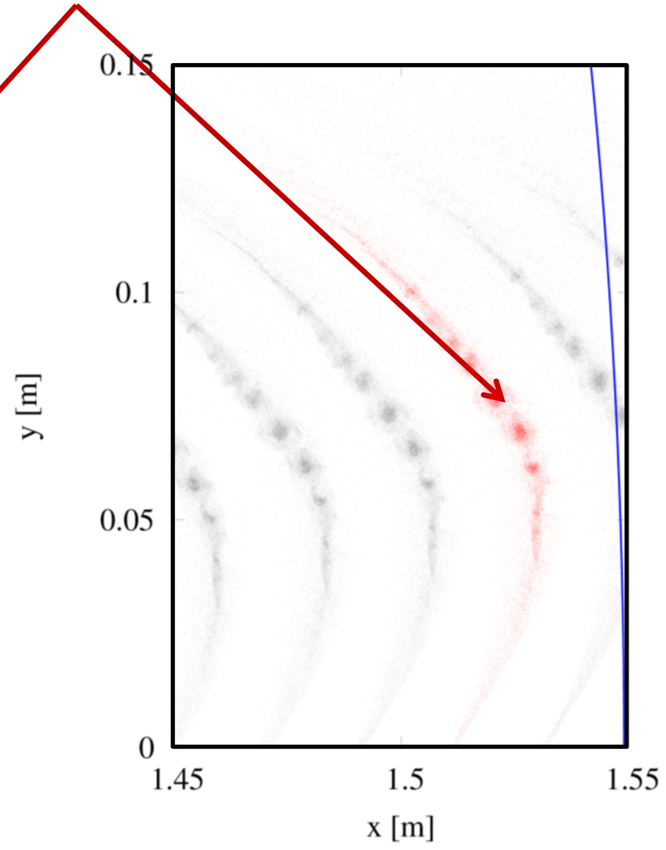
Simulation of Triumf central region with TRIYCLE

Simulation of high space charge bunched beam (410 μA) during the first 30 turns in the Triumf cyclotron central region [26].

Measured radial probe track



Simulation at turn 30, showing the breakup of the bunch in droplets



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Some recent progress at IBA

During the last few years IBA made quite some efforts in order to precisely simulate orbits and beams in all our different types of accelerators

1. The program `phase-motion` integrates the coupled motion of orbit-center and longitudinal phase space over multiple pulses (several10^5 RF periods) in our proton-therapy superconducting synchro-cyclotron S2C2. This will be reported by Jarno Van de Walle tomorrow
   - Note that our inspiration for this code relates directly back to work of Henk Hagedoorn
2. The Advanced Orbit Code (`AOC`) [38] facilitates design studies of critical systems and processes in medical and industrial accelerators. The space charge module will be discussed in more detail and two examples are shown

Some applications of AOC

1. Different accelerators (Cyclotron, Synchrocyclotron, FFAG, Rhodotron, Gantry)
2. Cyclotron Injection (with spiral inflector or internal source) and extraction (ESD, stripping, regenerative)
3. Beam capture and longitudinal dynamics studies in synchrocyclotrons
4. Beam simulation from the ion source to the extraction
5. Calculation of Twiss- functions
6. Space charge effects

Space charge in AOC

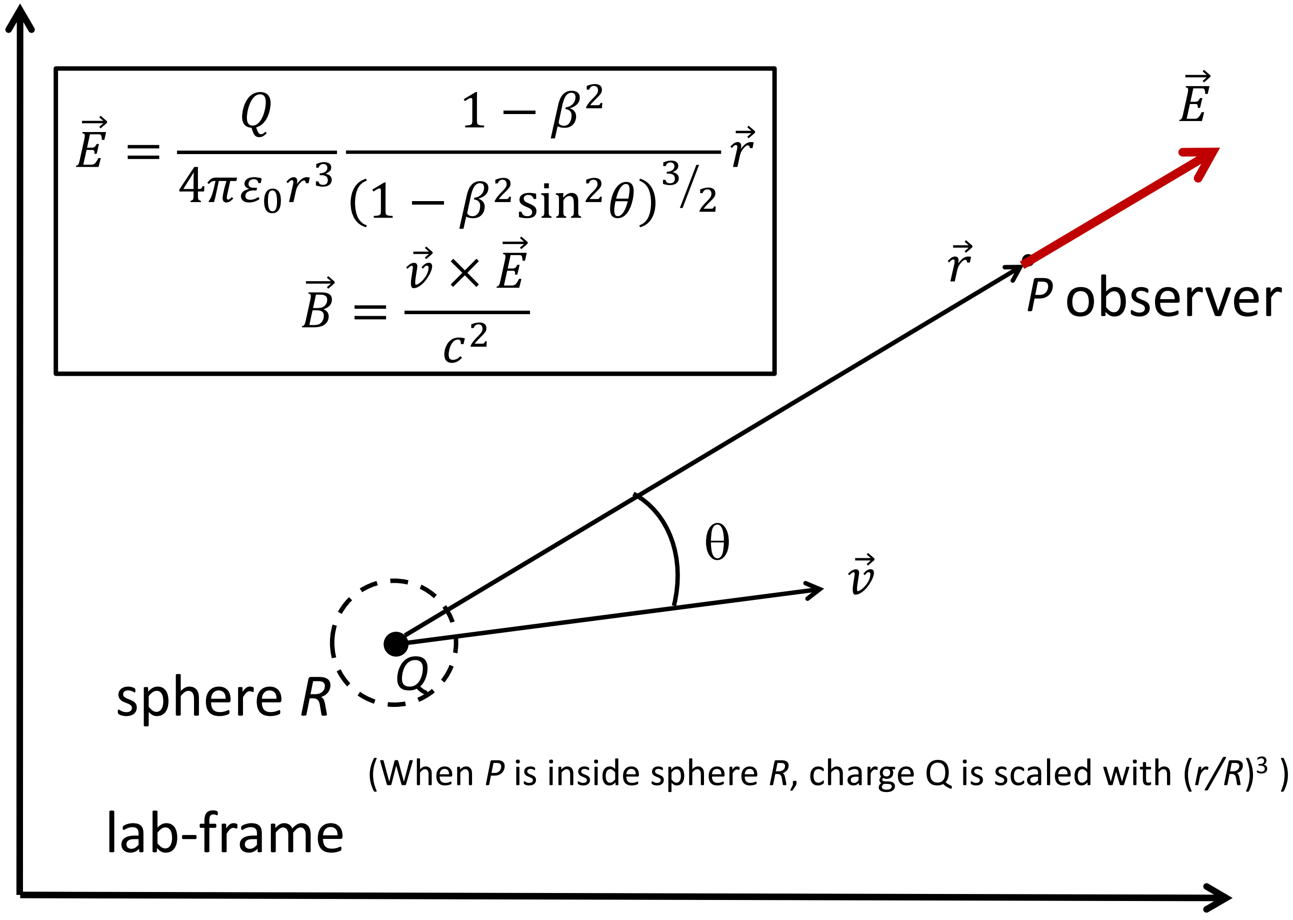
- A particle-to-particle solution was chosen (self-field acting on one particle is obtained as the sum of contributions of all other particles.
- **Disadvantage**: becomes slow for large N because computing time scales as N^2. But… the cyclotron injection and central region problem (and also the Rhodotron) does not require very large number of particles: 10000 can be enough.
- **Advantage**: we can immediately integrate the SC-option with the fully 3D features of the (**E**,**B**)-fields and the complex 3D shape of a reference orbit (inflector) available in AOC.
- We want to also simulate the IBA industrial electron accelerator Rhodotron,
  - where electrons go from low energy (30 keV) to fully relativistic (10 MeV),
  - and one bunch may be very far from mono-energetic,
  - the direction of particle velocities in the bunch may differ over 180° in the re-circulating dipole magnets,
- Therefore, we have chosen for fully relativistic approach (No Lorentz frame moving with the bunch)

AOC self-field calculation

- Relativistic ***E***-field from a moving point charge is radially directed but not isotropic
- Magnetic field is perpendicular to ***E*** and ***v***
- In order to avoid singularities a virtual sphere *R* is placed around the charge Q. If the observer is inside this sphere, then Q is scaled with $(r/R)^3$
- *R* is estimated from the rms volume of the bunch and the number of particles in the bunch

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^3}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\vec{r}$$

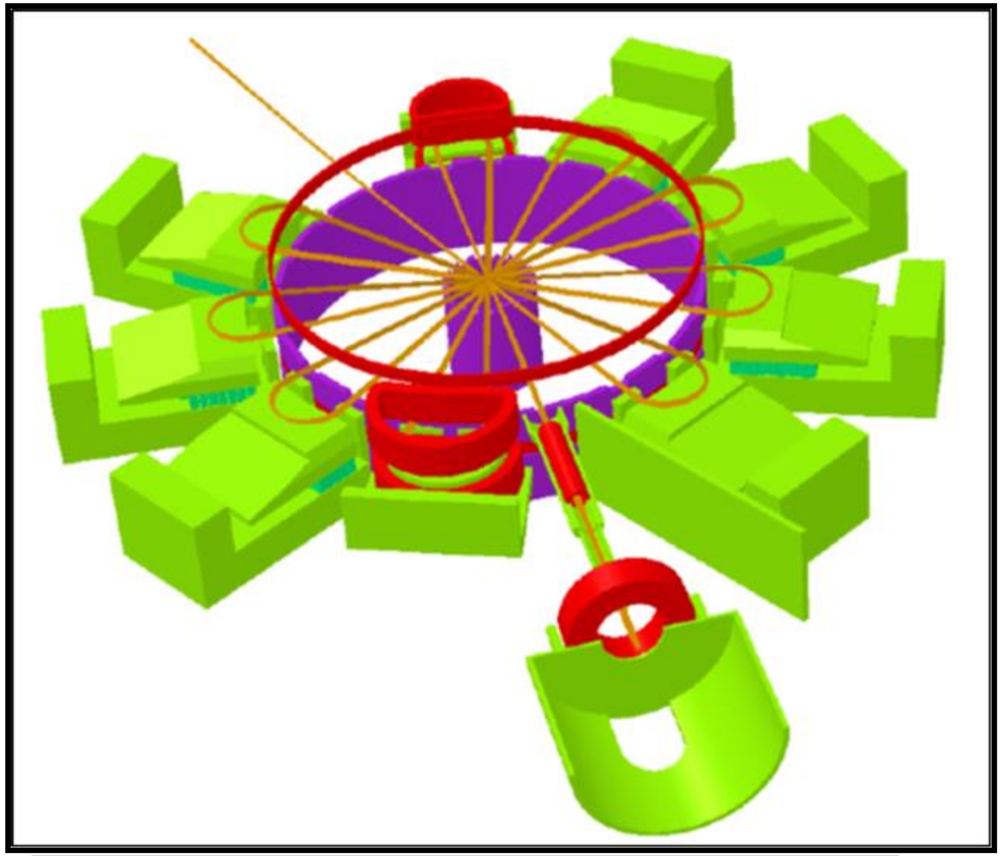
$$\vec{B} = \frac{\vec{v}\times\vec{E}}{c^2}$$



(When *P* is inside sphere *R*, charge Q is scaled with $(r/R)^3$)

Example 1: simulation of the Rhodotron

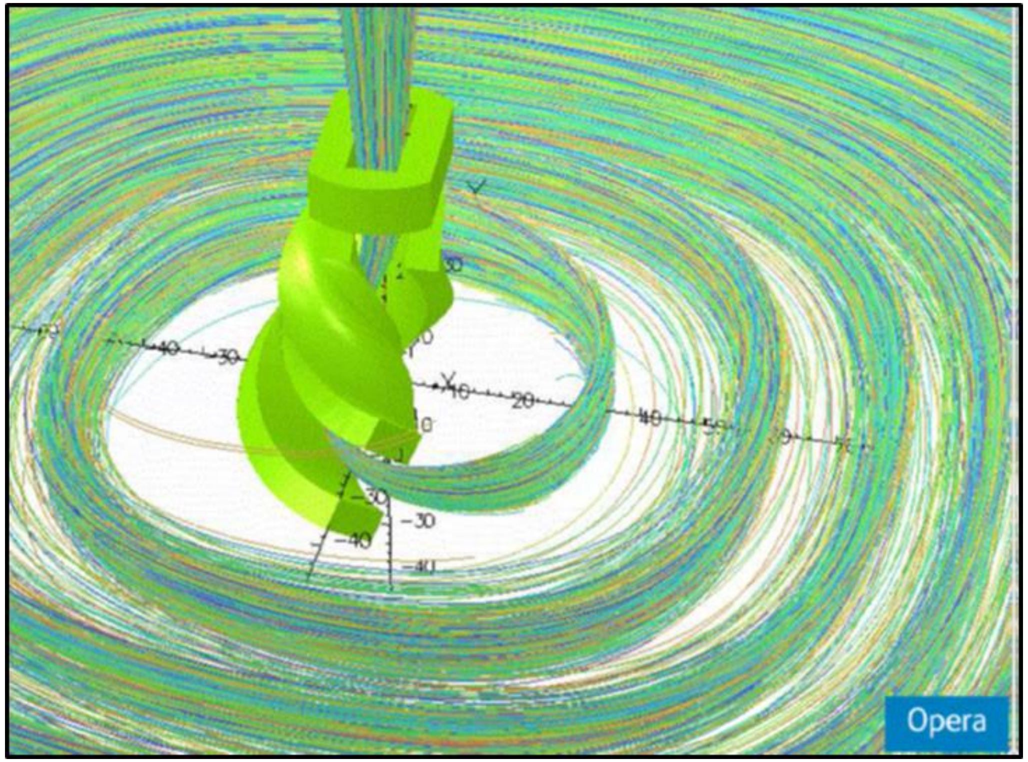
- The Rhodotron is an industrial CW electron accelerator in the range from 5 to 10 MeV and high average beam power in the range of 20 kW to 700 kW
- There is some similarity to cyclotrons: the beam is re-circulating several (order 10) times into one coaxial accelerating-cavity running with an RF similar to cyclotrons (100-200 MHz) and one orbit period is equal to an RF period
- Space charge effects are important especially at the lower energies (up to 1 MeV) and at the high beam powers.
- During the first pass, there is a large spread in particle velocity and direction: the Lorentz frame traveling with the bunch as commonly used in existing PIC space charge codes was therefore not applied in AOC



Simulation of a 10 mA (average current) SC beam in the TT50

Example 2: calculation of an inflector with space charge

- A bunch with 5000 particles was simulated through the C70 cyclotron axial bore, spiral inflector and central region during about 5 turns
- The particle-to-particle solution programmed in AOC immediately allows the full 3D complexity of the inflector static E-fields, the RF E-fields and the magnet B-fields
- The structure of the program is such that an RF buncher and injection line focusing elements could be included in a rather straightfort manner



Simulation of 2mA (average current) SC beam injected into the C70 cyclotron. For clarity, the dee-structure and magnet structure are not shown.

Some examples of recent progress of beam-dynamics simulations in cyclotrons

Thank you !

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