EQUIVALENT CIRCUIT MODEL OF CYCLOTRON RF SYSTEM

M. Mohamadian, H. Afarideh, M. Salehi, S. Sabounchi,
Amirkabir University of Technology, Tehran, Iran
Mitra Ghergherehchi, SKKU University, Suwon, Korea

Abstract

Cyclotron cavity modelled via electromagnetic circuits in the desired frequency. The design performed according to resonator basis and also cyclotron acceleration requirements with ADS software and compared to simulations made by the CST microwave studio. The scattering parameters obtained for main resonators of the cyclotron and Dee parts as a diaphragm for each of cavity sections and also for the whole structure. All the characteristics modelled and calculated by the electromagnetic rules and theory of resonators from circuit model. Then it analysed with numerical methods for benchmarking. Finally, it shows that the circuit model able to modelled accurately the cyclotron cavity and especially it can estimate precisely the structure parameters without any time consuming numerical method simulations.

INTRODUCTION

A particle accelerator is a machine that uses electromagnetic fields to propel charged particles to nearly light speed and to contain them in well-defined beams [1]. One kind of accelerators is oscillating field accelerators, which use radio frequency electromagnetic fields to accelerate particles, and circumvent the breakdown problem. Also circular accelerators partition to several types such as; Cyclotrons, Synchrocyclotrons and isochronous cyclotrons, Betatrons and etc. [2].

Cyclotron which is our discussion about, are accelerators in which particles are propelled in spiral paths by the use of a constant magnetic field. This accelerator was invented for the first time by Ernest O. Lawrence in 1932 [3]. The cyclotron was one of the earliest types of particle accelerators, and is still used as the first stage of some large multi-stage particle accelerators. It makes use of the magnetic force on a moving charge to bend moving charges in a semicircular path between accelerations by an applied electric field. The applied electric field accelerates electrons between the "Dees" of the magnetic field region. The field is reversed at the cyclotron frequency to accelerate the electrons back across the gap [4]. One of the main parts of the cyclotron is the RF cavity. In this paper we have proposed the circuit model to consider the cyclotron RF cavity response to analyze its behavior in resonance frequency.

CYCLOTRON ACCELERATORS

The cyclotron principle involves using an electric field to accelerate charged particles across a gap between two "D-shaped" magnetic field regions. The magnetic field accelerates the particles in a semicircle, during which time the electric field is reversed in polarity to accelerate the charge particle again as it moves across the gap in the opposite direction. In this way a moderate electric field can accelerate charges to a higher energy [4]. Cyclotron includes some different sections that is illustrated in block diagram of Fig. 1 which the RF cavity is one of the main parts.

RF CAVITY STRUCTURE

With regard to the overall structure of the cyclotron accelerators been described above, its different parts are illustrated in Fig. 2. The main part of such a device is a resonator which provides resonance at considered frequency. Resonator structures are different. The oscillations in a resonator can be either electromagnetic or mechanical [5]. In the cyclotron accelerator, the cavity resonator is used. Due to the low resistance of their conductive walls, cavity resonators have very high quality factors; that is their bandwidth is very narrow. Thus, they can act as narrow band-pass filters [5]. The different cavity designs are depending on the specifications and geometric layout of a cyclotron [6].

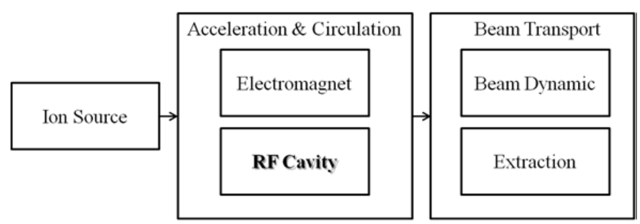

Figure 1: Brief block diagram of the cyclotron accelerator.

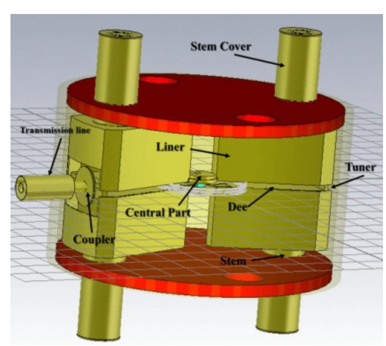

Figure 2: Overall structure of the RF cavity.

Therefore, a coaxial resonator in fixed frequency, double gap and superconducting can be suitable for this application. But as mentioned, the cyclotron, because of the conditions governing the charged particle rotation, requires a region in the center of the resonator to get energy to the particles through an electric field. As a result, in the center, it has to be a horizontal plate, called Dees, with a certain angle that generate the electric field, between the

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coaxial inner core and its outer shell which gives the required energy to the particle in each round of rotation. As it will express in past section, this part is in the form of the D-shape generally, for two accelerating gap design or in the form of triangular plates to have four accelerating gap. So to have the coaxial structure, the outer shell of this part of the transmission line is also triangular shape.

As a result, the appropriate resonator for such a system is composed of a cylindrical coaxial plus a roughly triangular shape coaxial which become short-circuit at the end (Fig. 3). Thus, in such state, in the resonance frequency, the impedance of coaxial TEM waveguide as resonator will be infinity because of creating the open-circuit on the other end, which physically will connected with Dees and so, the wave transmit completely. Also, in the other frequencies, the resonator attenuates the transmitted wave. Generally, it performs like a band-pass filter at the resonance frequency. Therefore, the resonator is modeled with two transmission lines that have different length, characteristic impedance and phase constant.

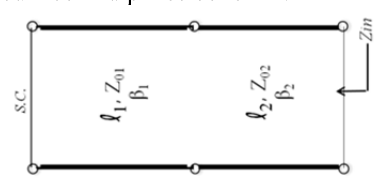


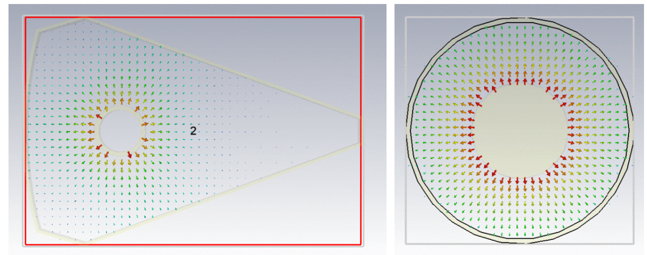
Figure 3: short circuit λ/4 transmission line, Equivalent Transmission Line.

EQUIVALENT CIRCUIT MODEL

The dimension of the resonator including, the quarter wavelength transmission line, calculate in some steps. At first, the phase constant of both cylindrical and triangular transmission line calculated separately. With simple calculation can be shown that the phase constant of both structures are the same as free space phase constant. So the overall structure length of these two parts will be equal to designed coaxial structure length. It means that, in designing the quarter wavelength transmission line, just the sum of the cylindrical and the triangular coaxial length will be important and not each of them alone. Therefore the height of each of these parts related to other considerations of manufacturing, including location of magnets and mechanical implement remarks.

Hence, in overall, resonator is a short circuit transmission line which its input impedance calculate by Zin=j Z0 tan(βl). As it can be shown at circuit model, there is two resonance points which is repeated frequently. These resonances occur in infinity and short circuit impedances which can be modeled with parallel and series LC circuit respectively. Since our resonance point is near the open circuit impedance. So we used parallel LC equivalent circuit. The part called Dees are attached to the inner conductor of coaxial. The Dees are triangular shape as it explained previously. The electric field patterns for a TEM wave are sketched in Fig. 4.

In addition, since we are going to design IranCYC10, our desired structure as discussed in [7] works on fourth harmonic and so the angle of the triangular part obtain approximately via $\theta_{dee} = \frac{180}{h} = \frac{180}{4} = 45$. Where θ_{dee} is the angle of Dee part and h is harmonic number [7].



a b

Figure 4: Electric field pattern of between diaphragm and Liner a) Triangular and b) cylindrical section.

For analysing the circuit model of the coaxial resonator, if the region between electrodes is a vacuum, TEM waves propagate with ω/k =c. Therefore, the line has a capacitance C and inductance L per unit length given by $\mathrm{C} = \frac{2\pi\varepsilon}{\mathrm{Ln}\left(\frac{R_0}{R_i}\right)}\left(\frac{\mathrm{F}}{\mathrm{m}}\right)$ and $\mathrm{L} = \left(\frac{\mu}{2\pi}\right)\mathrm{Ln}\left(\frac{R_0}{R_i}\right)\left(\frac{\mathrm{H}}{\mathrm{m}}\right)$ for cylindrical coaxial lines. But in the overall structure, these relations can't be applied.

Another case that distinguishes this designed resonator from conventional one is the necessity of the Dee part of this structure which discussed about it earlier. This part in resonator works such as a diaphragm in a waveguide. We considered the effects of the diaphragm structures in the triangular transmission line. According to electromagnetic consideration, TEM mode waves propagate inside the coaxial waveguide which at the diaphragm part has a capacitance effect between it and the outer conductor in both sides and inductance effect because of junction of the diaphragm with the inner conductor of coaxial line [8]. The magnetic fields are almost identical to those of the standard transmission line except for field exclusion from the Dees.

$$|S12|^2 = \frac{4}{4+\frac{Z_0^2}{(C\omega-\frac{1}{L\omega})^2}} = \frac{1}{2} \rightarrow \Delta\omega = BW. = \frac{2}{Z_0.C} \rightarrow \mathrm{C}=\frac{2}{Z_0*BW.},\ \mathrm{L}=\frac{1}{4\pi^2 {f_r}^2 C} \rightarrow \text{parallel LC} \quad (1)$$

$$|S12|^2 = \frac{4}{4+\frac{Z_0^2}{(L\omega-\frac{1}{C\omega})^2}} = \frac{1}{2} \rightarrow \Delta\omega = BW. = \frac{Z_0}{2L} \rightarrow \mathrm{L}=\frac{Z_0}{2*BW.},\ \mathrm{C}=\frac{1}{4\pi^2 {f_r}^2 L} \rightarrow \text{Series LC} \quad (2)$$

In contrast, radial electric fields cannot penetrate into the region between Dees. The electric fields are restricted to the region between the outer conductor and the Dees. In fact, in such a model, take the junction between to Dees in half section of overall structure into account.

Furthermore, the diaphragm modeled in CST and its scattering parameters versus frequency has demonstrated in Fig. 5.

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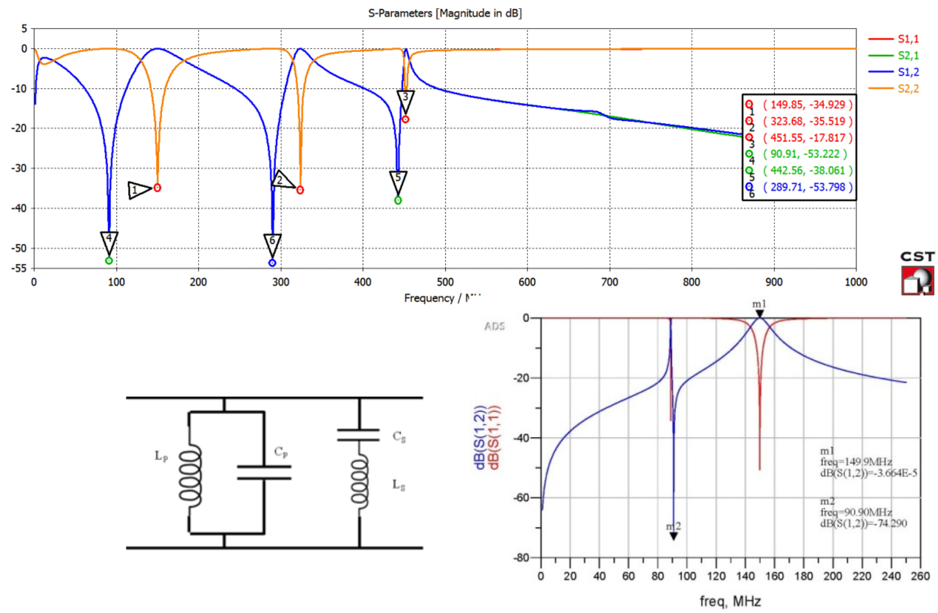

Figure 5: Scattering parameter of diaphragm consideration, a) Scattering parameters b) equivalent microwave circuit model.

It indicates several resonances which some of them are band-pass and another one is band-stop which show diaphragm behavior in this frequency interval as a parallel LC and series LC respectively. So the proper circuit model of this portion as examined with CST simulation will be an LC in the series and another LC in parallel which connected to each other.

RESULTS AND CONCLUSION

According to discussion previously, resonator itself works as a transmission line band pass filter and diaphragm acts as two LC circuit in series and parallel. So as it can be seen, diaphragm causes a notch in transmission power versus frequency in the higher than the resonance frequency of the resonator. So it should keep away from the resonance frequency sufficiently. Therefore, if relation of diaphragm's L and C parameters to be $L\omega<1/C\omega$, because of the inductive effect on the resonator, it decreases the resonance frequency of the total structure, and so if $L\omega>1/C\omega$ because of the capacitive effect on the resonator, it increases the resonance frequency. Ofcource, it also illustrates that the diaphragm makes better quality factor and decreases the resonance frequency. It could be tuned again via changing the resonator dimension. So in the final design includes the four similar resonators with a diaphragm which locate symmetrically in top and down of electron beam revolution path and in the left and right sides. In the equivalent circuit, these parts are parallel with each other. The circuit model demonstrates that the combination of all four sections makes the better quality factor.

According to previous section discussions, resonator, diaphragm and couplers are simulated in CST software. Here should be the structure's scattering matrix to take the effect of the resonator and the diaphragm to consideration. The phase of scattering parameter S11 of this short circuit line and its characteristic impedance in our desired resonance frequency in IranCYC10 which is at 71 MHz, has obtained. As a result of this simulation, the equivalent transmission line length or βl value will be -19.15 and -21.91 at the resonance frequency for cylindrical and triangular coaxial line respectively. So the equivalent capacitance and inductance will be calculated. Consequently, we found an accurate circuit model to design RF cavity which the comparison of the model and CST simulation indicates the acceptable engineering accuracy. So it could lead us to consider such structure with microwave circuit model to calculate its different parameter values to have appropriate cavity with high quality factor without any time consuming simulation.

REFERENCES

[1] C.R. Nave, Cyclotron, Hyperphysics. Dept. of Phy. and Ast., Georgia State University, 2014.

[2] M.S. Livingston, J. Blewett, Particle Accelerators, New York: McGraw-Hill, 1969.

[3] *Handbook of Accelerator Physics and Engineering (2nd Ed.)*, A.W. Chao, K.H. Mess, M. Tigner, and F. Zimmermann, Eds. World Scientific Press, May 2013.

[4] E.O. Lawrence, "Method and apparatus for the acceleration of ions", US patent 1948384, 1934.

[5] D.M. Pozar, *Microwave Engineering*, John Wiley & Sons, 2011.

[6] S.K. Jain, D. Sharma, V.K. Senecha, P.A. Naik and P.R. Hannurkar, "Study of microwave components for an electron cyclotron resonance source: Simulations and performance", Sadhana 39, Part 4, 2014, pp. 901–909.

[7] M. Mohamadian, H. Afarideh, M. Salehi, J.-S. Chai, M. Ghergherehchi, "Development of Optimized RF Cavity in 10 MeV Cyclotron", in *Proc. 7th Int. Particle Accelerator Conf.*, Busan, Korea, May 2016, pp. 1250-1252, doi:10.18429/JACoW-IPAC2016-TUPMR011

[8] T. Dong, L. Zhan, Y. Hong, M. Fan, "Cyclotron cavity analysis based on field circuit model", *Nucl. Inst. and Meth. in Phys. Research*, A 513, 2003, pp. 631–634.

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