# SPACE-CHARGE SIMULATION OF TRIUMF 500 MeV CYCLOTRON*

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## *Abstract*

We present a method to improve computation efficiency of space charge simulations in cyclotrons. This method is particularly efficient for simulating long bunches where length is large compared to both transverse size and turn separation. We show results of application to space charge effects in the TRIUMF 500 MeV cyclotron.


## INTRODUCTION

The TRIUMF 500 MeV cyclotron accelerates $H^-$ ions, and uses charge exchange extraction. No turn separation is required for extraction, which allows a very large phase acceptance of this machine (about 60°) [1]. Bunches are very long, and have a very large energy spread between the head and the tail (see for instance Fig. 8). Each bunch therefore occupies a large and slim volume in real space. Solving Poisson equation in a PIC code over such a large volume would require a significant computation time.

In addition, at high energy the turn separation is several times smaller than the radial beam size even for an infinitesimal phase slice. It is therefore essential to take into account the effect of many overlapping neighbouring turns. The multibunch calculation used in `OPAL` [2], is most appropriate when bunch length and width are comparable, but in the TRIUMF case, the bunch length can be 400 times its width.

To overcome these difficulties, we use periodic boundary conditions in the radial direction. This trick, originally proposed by Pozdeyev as a possible way to improve his code `CYCO` [3, 4], is presented in Ref. [5]. The radial dimension of the box inside which Poisson's equation is solved is equal to the turn separation. Particles of the bunch that fall out of this box are returned to the box assuming radial periodicity (see Fig. 1). In fact, these particles appear to belong to the neighbouring turns. The charge density in this 3D box is divided onto slices cut along the $y$ direction (see Fig. 1). To take into account the image charge, we use "metallic" boundary conditions in the vertical direction; to simulate the effect of neighbouring turns, we use periodic boundary conditions in the radial direction.

We have implemented such a 3-D Poisson's equation solver into a piece of code that we call `tricycle`

### *Poisson Solver Test*

To test our Poisson's equation solver we compute the electric potential from a static sphere of charge constituted of $10^6$ randomly distributed macro-particles; results are shown in Fig. 2.

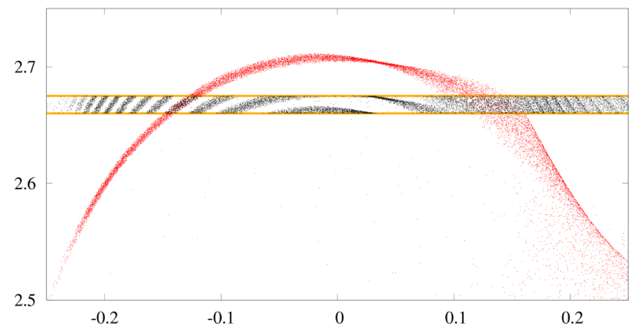


Figure 1: Beam folding applied to accelerated turn #100 of TRIUMF. The original particle distribution is shown in red (in top view). The folded beam in show in black. The yellow line materializes the edge of the box inside which Poisson's equation is solved; perodic boundary condition is applied along those two sides. Coordinates are in metres.

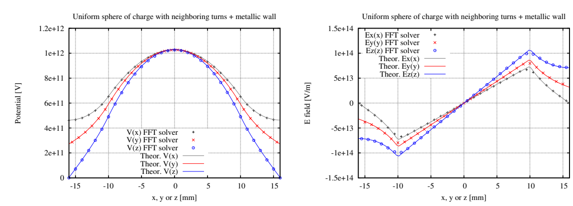


Figure 2: Electrostatic potential (left) and electric field (right) from a uniform sphere of charge; the dots presents results from `tricycle` with a $32 \times 32 \times 32$ size PIC grid; solid lines are from theory. Note the three different boundary conditions in x, y, and z: periodic (neighbouring turns), open, and metallic (potential=0), respectively.

The theoretical electrostatic potential from a uniform sphere of charge with such boundary conditions writes:

$$V(x,y,z) = \sum_{i_x=-\infty}^{+\infty} \sum_{i_z=-\infty}^{+\infty} (-1)^{i_z} f(x - i_x \Delta x, y, z - i_z \Delta z) \quad (1)$$

where $\Delta z$ is the vertical gap of the vacuum chamber, $\Delta x$ the turn separation ($x$ here is along the radial direction), and $f(x,y,z)$ is derived from Gauss's law:

$$f(x,y,z) = \begin{cases} \dfrac{Q}{\epsilon_0 4\pi r} & r \geq R\,, \\[2ex] \dfrac{Q}{\epsilon_0 8\pi R}\left(3 - \dfrac{r^2}{R^2}\right) & r < R\,, \end{cases} \quad (2)$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $R$ is the radius of the sphere, and $\epsilon_0$ is the vacuum permitivity.

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## SINGLE PARTICLE TRACKING

### *Closed Orbits and Tune*

An equilibrium orbit search algorithm similar to the one proposed by Gordon [6] was implemented; linear optics parameters were calculated based on Ref. [6]; results agree well with `CYCLOPS` (see Fig. 3).

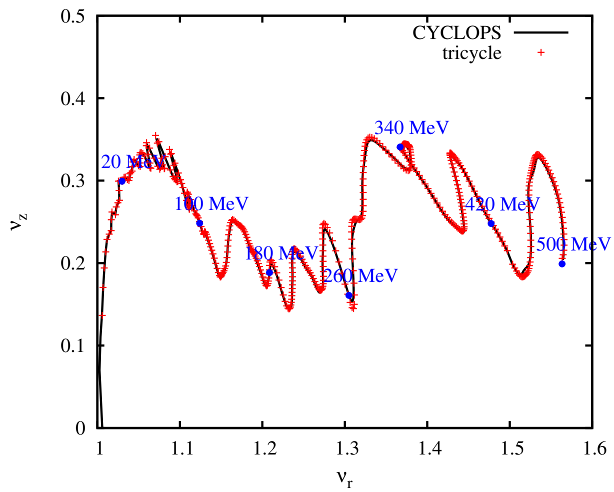


Figure 3: Tune diagram obtained by `CYCLOPS` and `tricycle` using TRIUMF cyclotron standard field map "policyinita6.dat".

### *RF Acceleration*

Next step was to implement acceleration. To simplify the tracking through the rf gap, and to save on computation time, we chose to model the gap crossing as a thin rf kick using:

$$\begin{aligned} \delta E_k &= qV_{\rm rf} \sin\phi \\ \delta z' &= K\,\frac{1-a}{2} z \\ \delta r' &= K\,\frac{1+a}{2}(r-r_0) \end{aligned} \qquad (3)$$

where $\delta E_k$ is the energy kick; $q$ the charge of the particle; $V_{\rm rf}$ the rf field amplitude; $\phi$ the rf phase; $\delta z'$ and $\delta r'$ are the vertical and radial focusing kicks; $a$ is a geometrical factor describing the geometry of the rf gap: $a = -1$ is for an horizontal rf gap (focusing only vertically), $a = 0$ is for a round (or square) gap, etc; $r_0$ is the radial position of the gap center; the focal power $K$ is given by Reiser's formula [7]:

$$K = \frac{V_{\rm rf}}{V_c}\frac{\pi}{\beta\lambda_{\rm rf}}\cos(\phi) + \frac{F}{2b\pi}\left(\frac{V_{\rm rf}}{V_c}\right)^2 \sin^2(\phi) \qquad (4)$$

with $\beta$ the ratio of the particle to the speed of light; $\lambda_{\rm rf}$ the rf wave length; $V_c$ the average of the particle 'kinetic' energy across the rf gap; $F$ a form factor that we took equal to 1. $b$ is of the order of (half) the extend of the electric field: it changes with $R$ from 0.5 to 2 inches following the geometry of our rf gap.

Single particle tracking with acceleration was compared with results from our reference code `CYCLONE` for the first few turns in the cyclotron where vertical focusing is mostly electric. `CYCLONE` uses a 3D electric field map to model rf acceleration. As shown in Figs. 4,5 our thin kick model agrees resonably well with `CYCLONE`.

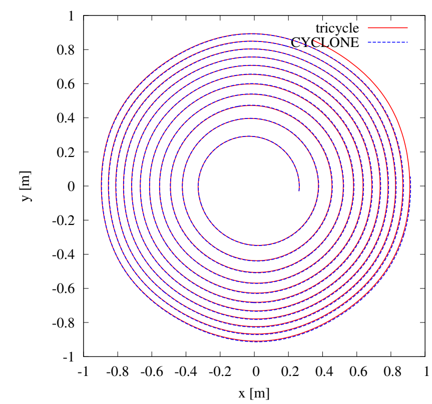


Figure 4: Acceleated orbit obtained from `CYCLONE` and `tricycle`

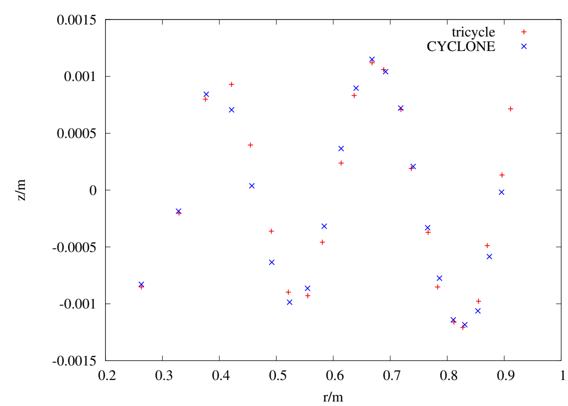


Figure 5: Vertical motion of a single particle (seen at each rf gap crossing) obtained from `CYCLONE` and `tricycle`

## MULTIPARTICLE TRACKING

### *Without Space Charge*

The $6 \times 6$ beam $\sigma$-matrix calculated by `TRANSOPTR` at the exit of the spiral inflector/deflector (see [8]) was used to generate a 6D fully correlated initial particle distribution, including the coupling effects of the axial cyclotron field, longitudinal motion, and the coupling in all 3 planes due to the spiral inflector. A Gaussian distribution is used in every direction. Simulation results are compared to actual measurements in Fig. 6 and Fig. 7.

### *With Space Charge*

We use here the initial particle distribution as in the previous section, with space charge of 22 pC per bunch (*i.e.* $\sim 500\mu$A at 100% duty cycle). The beam breakup is clearly

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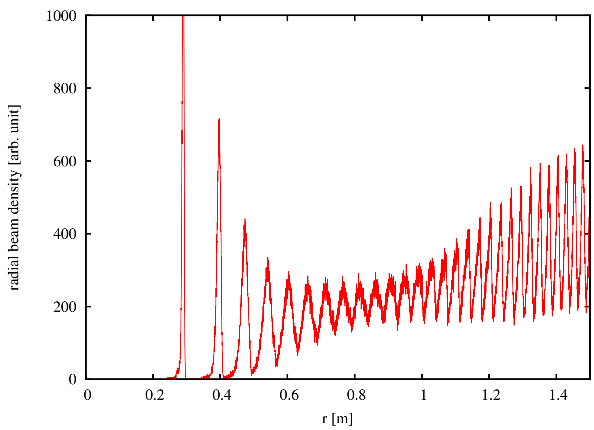


Figure 6: Simulation result from tracking a bunched beam without space charge, plotted as one of our radially-scanning low energy probes would see it.

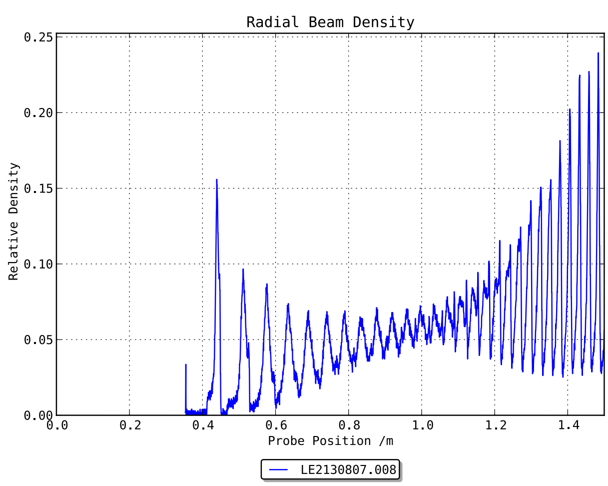


Figure 7: Measurement results from a radial low energy probe scan, when injected a low space-charge beam (using a ‘pepper-pot’ to reduce intensity by a factor 100) bunched with only the first harmonic buncher, and injected ~ 20° off crest (on the falling side of the rf wave). Note that our low energy probe cannot quite reach the first turn.

visible at turn#30 (see Fig. 11), and becomes very convoluted by turn #300 (see Fig. 12). The beam breakup causes fine structure to appear on the radial density plot from about turn#20.

The fine structure measured experimentally (see Fig. 10) from the very first turns is not due to vortex motion in the cyclotron but rather comes from the complex on-linear space-charge effects during the bunching process (see Fig. 13). Only the smoothed Gaussian 6D hyperellipsoid from TRANSOPTR is used to start our multi-particle simulation. We plan to expand `tricycle` to include the injection line.

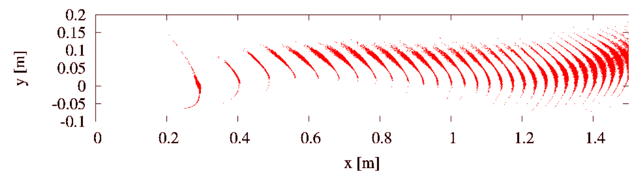


Figure 8: Simulation result from tracking bunched beam without space charge for 35 turns; this top view shows “snapshots” taken every 5 (=harmonic number) rf periods.

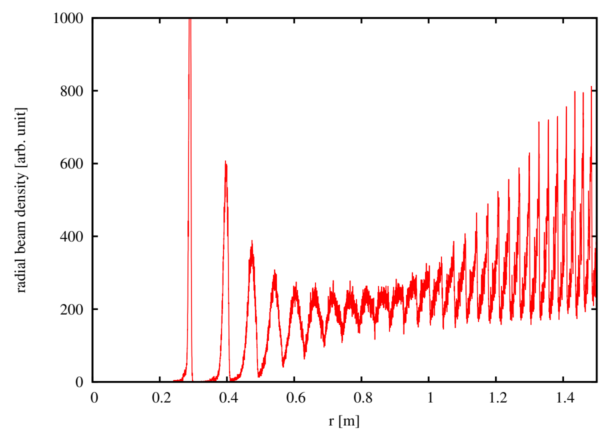


Figure 9: Simulation result from tracking a 500 $\mu$A bunched beam with space charge, plotted as one of our radially-scanning low energy probes would see it.

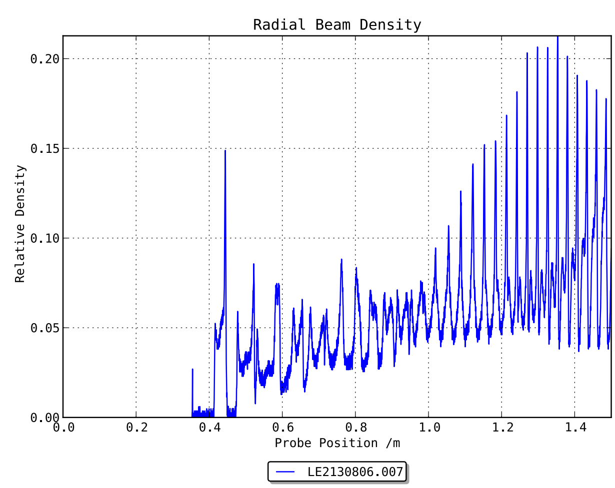


Figure 10: Measurement results from a radial low energy probe scan, when injected a high space charge beam when injecting ~ 410 $\mu$A equivalent (100% duty cycle) beam.

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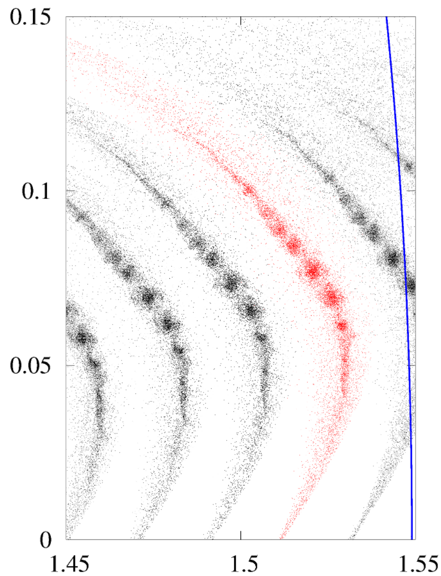


Figure 11: Turn #30 shown together with neighbouring turns. The Blue lines is an arc of circle centered on the machine center. Plot is Cartesian; the $x$-axis is mainly radial and $y$ is azimuthal; both in metres.

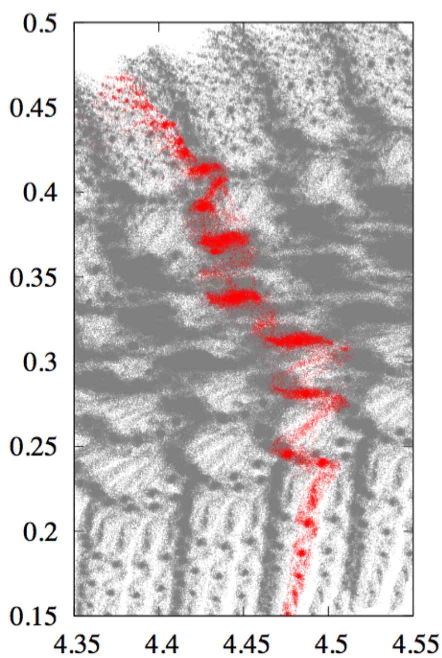


Figure 12: Simulated TRIUMF turn #300 (red) shown together with neighbouring turns. Plot is Cartesian; the $x$-axis is mainly radial and $y$ is azimuthal; both in metres. Colour has been darkened to make fine structure more visible.

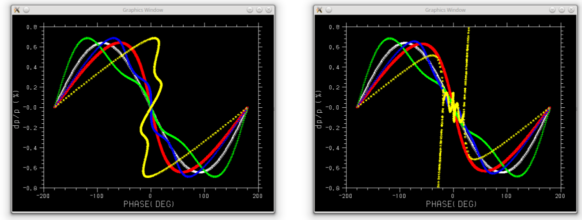

Figure 13: Longitudinal phase space simulation of bunching with (right) and without (left) space charge through our injection line using SPUNCH [9]; each colour corresponds to different locations along the beamline. Our bunching system is constituted of two separate bunchers, operating respectively on the first and second harmonics of the main rf frequency.

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