TRACKING IN A CYCLOTRON WITH GEANT4

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Abstract

The tracking and simulation toolkit Geant4 [1] has been conceived and realised in a very general fashion, with careful attention given to the modeling of electric and magnetic fields and the accuracy of tracking charged particles through them. As evidenced by the G4Beamline application, Geant4 offers a unique simulation approach to beam lines and accelerators, in a 3D geometry and without some of the limitations posed by conventional optics and tracking codes. Its visualization tools allow detailed examination of trajectories as well as a particle's-eye view of the acceleration process. Here we apply G4Beamline to the TRIUMF cyclotron, describing the generation and input of the field data, accuracy of closed orbits and tunes, stability of multi-turn tracking, and tracking accelerated orbits.

INTRODUCTION

Originally conceived to meet the simulation needs of a new generation of high energy physics detectors at the LHC and elsewhere, the Geant4 software toolkit has become the mainstay simulation resource in a wide range of applications, from underground and underwater detectors to space experiments and medical physics. With its object-oriented architecture and C++ implementation, Geant4 has proved to be adaptible to many types of problems. One of the fundamentals is its precise tracking of charged particles in electric and magnetic fields, with user control of tracking error limits and a flexible interface for defining fields.

Geant4 can be useful in accelerator studies simply as a versatile and accurate ray-tracer with 3D geometry and fields, but it offers much more: the wide range of physics models can be used to simulate decays, foil scattering, production targets, collimators, ionization cooling, and so on. The tracking can include polarization, multiple particle species, and precise detection of particle losses.

A notable related development is the G4Beamline[2] application, which puts together the Geant4 components with a powerful scripting and analysis interface that is designed for the needs of accelerator physicists. It allows accelerators and beam lines to be defined, layed out, populated with beams, and instrumented with virtual detectors. Data output options in a variety of formats are provided.

In the course of using G4Beamline for TRIUMF applications, we found it to be easy to adapt an OPERA field map for a dipole magnet to G4Beamline and use it to do accurate ray-tracing for analysis of the aberrations. This raised the question: could G4Beamline handle a really large and complex field map, e.g. for the TRIUMF 500 MeV cyclotron? If so, could it track accurately enough to exhibit repeatable equilibrium orbits? If we added a time-dependent dee-gap field, could it accelerate to the maximum energy? This prompted the present study, where the cyclotron model is a good vehicle to test both the spatial accuracy of Geant4 tracking as well as the time-of-flight accuracy needed to produce isochronism and successful acceleration.

FIELD MAP ADAPTATION

The reference field map of the TRIUMF cyclotron has been in use for over 30 years. It is expressed in Fourier harmonics as a function of radius, derived from the original survey data at 3-inch and 1-degree intervals, together with the trim coil contributions.

In addition to predefined beam line elements, G4Beamline offers several options for specifying electric and magnetic fields, which can be placed at arbitrary locations in the simulation "world". Overlapping and superimposed fields are automatically combined, to first order in the contributing fields. Here we have utilized the `fieldmap` element which reads in a self-describing ascii file containing the mesh information (2D cylindrical or 3D cartesian) and componentwise field data. For converting the TRIUMF data to this format we re-used some existing C++ code to process the Fourier data and evaluate the field components on a 3D cartesian grid of 0.5" spacing.

Geant4 by design handles arbitrary ions but due to limitations in the G4Beamline "external beam" input options, we have conducted all our tests using proton beams instead of H$^-$. To preserve isochronism it is sufficient to scale the magnetic field globally by the ratio of masses m_p/m_{H^-} as will be verified in the next section.

In this study we have used the default tracking settings provided by G4Beamline, including the most general and safe 4th-order Runge-Kutta integrator (one of several offered by Geant4) and relatively stringent error controls. G4Beamline currently implements only an 8-point linear interpolation method for field evaluation, and the limitations of this are discussed below.

EQUILIBRIUM ORBITS AND ISOCHRONISM

A first test of Geant4 tracking is to track known equilibrium orbits (EOs) and see if they close with sufficient accuracy, and to measure their time-of-flight to see if the particle velocity and path-length are also accurately accounted for.

We tracked 95 EOs from CYCLOPS[3] H$^-$ orbit data based on the same field, from 0.1 to 520 MeV, using a small converter program to simply assign the same coordinates and energy to the protons and write out a "BLTrack-

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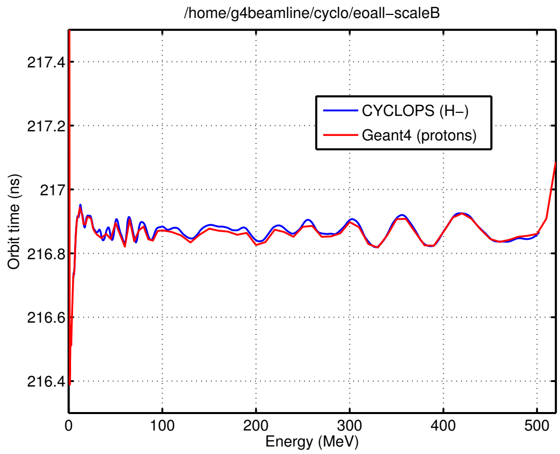

Figure 1: Isochronism of CYCLOPS and Geant4 orbits.

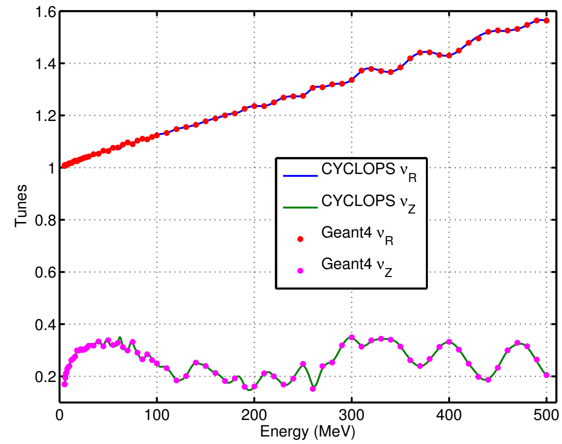

Figure 2: Tunes from CYCLOPS and Geant4 orbits.

File" input file for G4Beamline. The orbit times of the two codes agree very closely and without any systematic deviation, indicating equivalent time-of-flight accuracy in tracking and isochronism of the field scaled for protons.

STABILITY OF ORBITS

To determine the closure and repeatability of the EOs in G4Beamline, tracking tests were performed at 9 energies from 5 to 500 MeV and analyzed to find the true EOs, which differed from the CYCLOPS orbits by about 1–3 mm depending on the energy. Tracking again on the true orbits, we observed systematic but very small orbit drifts, of at most 0.5 mm after 10000 turns and at most 2 mm after 50000 turns. This seems remarkable, but tracking of non-equilibrium particles at a small offset (5 mm) from the EO revealed the limitations of the non-symplectic tracking and linear interpolation, with rapid emittance growth or loss of around $\pm$ 50% within 1000 turns. This could be reduced to more acceptable levels ($\pm$ 7%) by reducing the field mesh size from 0.5" (used in all other tracking results presented here) to 0.25", but the resulting field map is huge (>1 GB) and reduces tracking performance. For multi-particle multi-turn applications a more efficient and accurate approach would be to generalize G4Beamline's 2D cylindrical mesh to 3D, utilize the original field data, and adopt a higher order interpolation method.

TUNE MEASUREMENTS

Betatron tunes were measured at the same equilibrium orbit energies as the isochronism test, by tracking test particles at a small displacement (5 mm in each plane) from the EO, accumulating 1000 turns at each energy, sampling the particle position once per turn. Results based on FFTs of this tracking data are shown in Figure 2. At all energies from 4.9 to 502 MeV the observed tunes in Geant4 show detailed agreement with those calculated by CYCLOPS.

ACCELERATION

The complexity of the TRIUMF cyclotron central region is not treated in this simple model, so we have tested acceleration by starting protons on the 5 MeV and 10 MeV equilibrium orbits with a starting RF phase of $-7°$, comparing with equivalent runs using the GOBLIN[4] tracking code. In GOBLIN the accelerating gap is treated as an energy kick at the center of the gap, together with correction terms for the finite gap size. In G4Beamline the RF field extends over the physical gap region and is sampled in detail (space and time) during tracking of the particle through the gap, so no additional corrections are needed.

The dee gap field proves a good example of the ease and flexibility of G4Beamline input:

```
param tau=0.2*216.8719431
fieldexpr RFGAP Ex=0.190/(9*0.0254) \
     time=cos(2*$pi*t/$tau) length=660*$INCH \
     width=9*25.4 height=200 period=$tau
place RFGAP z=0 rotation=Y-90
```

where Ex is the peak field in MV/m, the peak RF voltage is 190 kV, the effective gap width (electric field flat-top region) is 9 inches[5], and the parameter tau is the RF (5th harmonic) period based on the GOBLIN RF frequency of 23.05508 MHz. The simplification of modeling only the flat-field region of the gap results in a slight underestimation of the energy gain, which is compensated by increasing the RF voltage by ~1% from its nominal value of 188 kV. As in the real cyclotron, we observed that the acceleration behavior in Geant4, was extremely sensitive to the RF frequency, and found that a slight adjustment of about 1 part in 10^5 gave better agreement with GOBLIN. Some tuning of the Geant4 starting phase at 5 MeV was also required, due to the finite-gap effect and slightly different starting azimuths in the two codes. With these adjustments the phase excursions agree very well. There remains a small systematic timing difference of 2-3 degrees, but this has very little effect on the acceleration to 500 MeV, as seen in Figure 3.

PHASE TRANSMISSION

To assure that successful acceleration to 500 MeV in Geant4 was not just a matter of luck in choosing initial conditions, we conducted a phase scan to see if a range of rf phases can be transmitted, in accord with GOBLIN results.

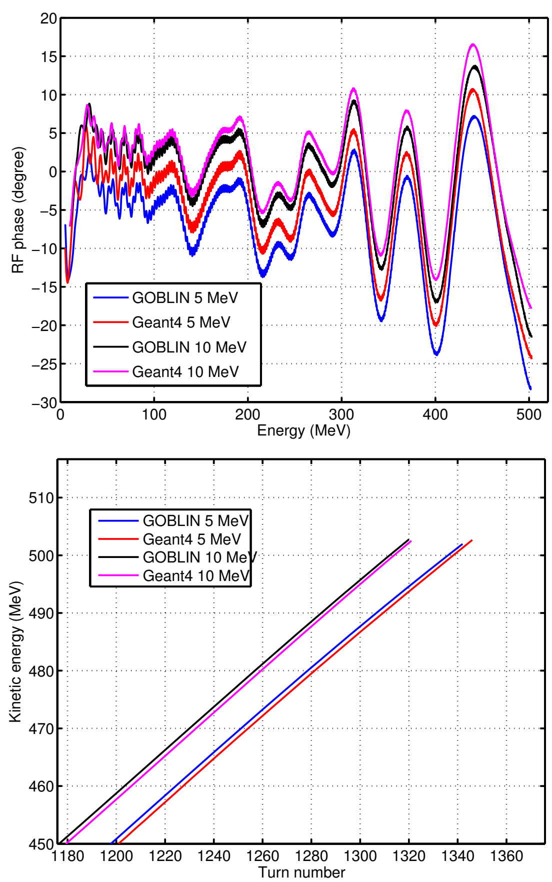

Figure 3: RF phase (top) and energy (bottom) of accelerated protons, starting at 5 and 10 MeV.

In both codes, a series of 10 MeV protons were launched at 1-degree RF phase intervals and tracked for 1320 turns. Figure 4 shows the final energy achieved as a function of starting phase. As in the phase histories, there is a shift of about 3° between the two codes, but otherwise the acceleration behavior matches very closely.

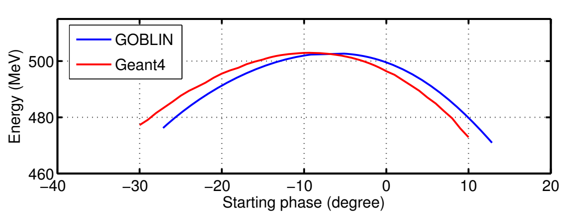

Figure 4: Scans showing RF phase transmission.

VISUALIZATION

Simulation with Geant4 involves placing objects, fields, and particles into a 3D world and then following the trajectories of the particles, their interactions in matter, and secondary particles created by decays or interactions. Interactive 3D visualization is an indispensable part of this process and at TRIUMF we have developed a new "navigating viewer" for Geant4 with features useful for beam lines and other extended structures. It provides fast navigation to any beam line element and follows particle trajectories through series of elements, with rotations, zooms, and bookmarking of views, as well as readouts of trajectory and element data. This viewer is now available in Geant4 and is expected to be available in the next release of G4Beamline.

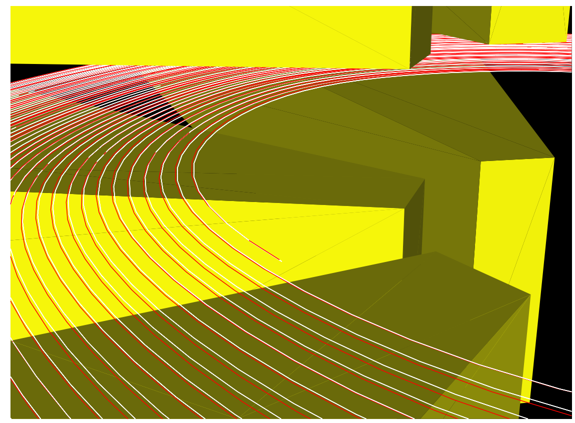

Figure 5: Accelerated proton tracks starting at 5 MeV (magnet sectors are not realistic).

CONCLUSIONS

Geant4 was not designed with cyclotrons in mind, but it is sufficiently flexible and accurate to implement a cyclotron tracking model producing results that compare well with the CYCLOPS and GOBLIN codes.

Geant4's object-oriented architecture, 3D geometry modeling, and flexible treatment of electric and magnetic fields, offer interesting possibilities for simulations of cyclotrons and FFAGs, particularly where interactions with matter (*e.g.* extraction foils, probes, particle losses) play a role. The G4Beamline application, together with Geant4's built-in 3D visualization, allows one to easily construct different scenarios and interactively observe the beam behavior. Within this framework, however, a higher-order interpolation method, as well as tuning of integrator type and accuracy parameters, will be required to support longer-term (∼1000 turns) multiparticle tracking.

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