Magnetized Dynamic Friction Force for Times Short Compared to Plasma Period

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COOL'2019 International Workshop

25 September 2019 – BINP, Novosibirsk, Russia

This work is supported by the US DOE, Office of Science, Office of Nuclear Physics, under Award # DE-SC0015212.

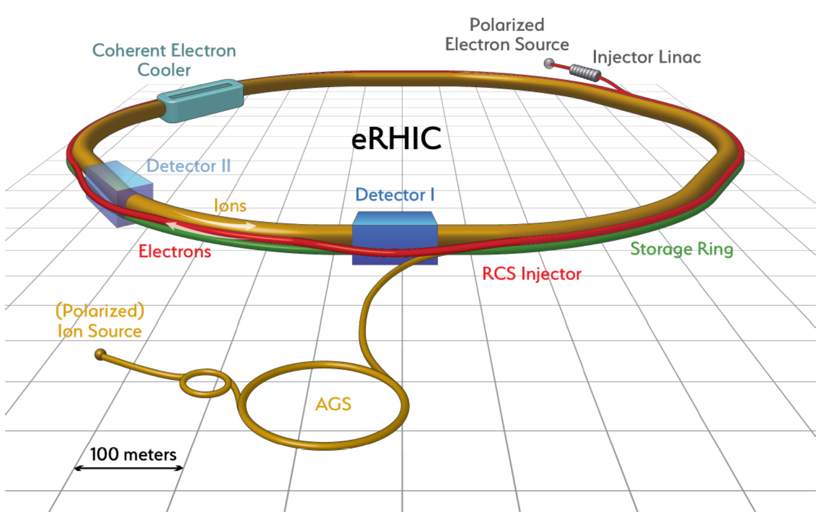
Outline

- Parameter regime for relativistic electron coolers
- Theoretical and semi-empirical models for the magnetized dynamical friction force
- Reduced binary interaction model and details of the calculation
- Preliminary results and a parametrized-fit model
- Comparison with other models
- Work in progress and future plans

Magnetized relativistic cooling is considered for the next-generation EIC designs

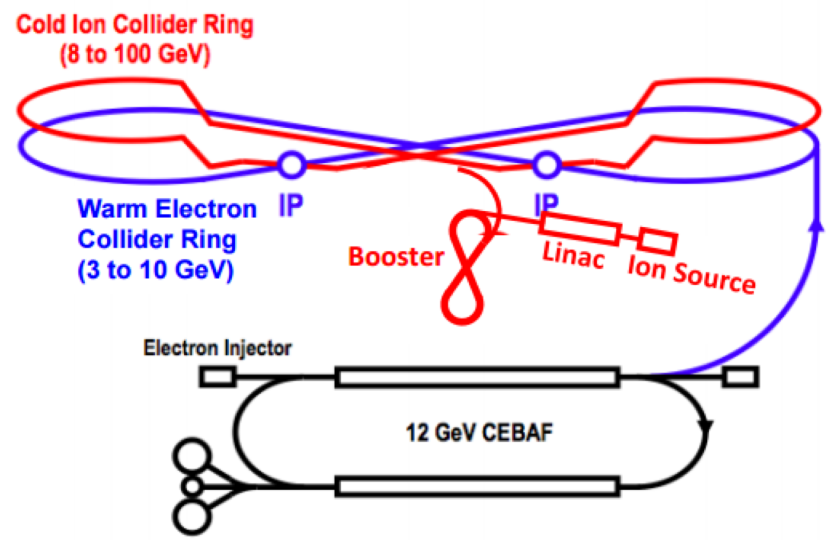
- Design of a polarized electron-ion collider (EIC) is a high priority for the nuclear physics community
- Relativistic, strongly-magnetized electron cooling
  - *may be essential for EIC, but never demonstrated experimentally*

eRHIC concept from BNL



C. Montag, "eRHIC Accelerator Design Overview," https://www.jlab.org/indico/event/281/session/0/contribution/3/material/slides/0.pptx

JLEIC concept from Jefferson Lab



S. Abeyrante et al., "MEIC Design Summary," https://arxiv.org/pdf/1504.07961.pdf

Relativistic cooling: short interaction time, new physics

- EIC requires cooling at high energy
  - *100 GeV/n → $\gamma \approx 107$ → 55 MeV bunched electrons, ~1 nC*

- Electron cooling at $\gamma \sim 100$ or higher requires different thinking
  - *friction force scales like $1/\gamma^2$ (Lorentz interaction-time contraction, electron density dilation in the beam frame)*
    - challenging to achieve the required dynamical friction force
    - not all of the processes that reduce the friction force have been quantified in this regime → significant technical risk
  - *normalized interaction time is reduced to order unity*
    - $\tau = t\omega_{pe} >> 1$ for nonrelativistic coolers
    - $\tau = t\omega_{pe} < 1$ (in the beam frame), for $\gamma \sim 100$
      - *violates the assumptions of introductory beam & plasma textbooks*
      - *breaks the intuition developed for non-relativistic coolers*
      - *as a result, the problem requires careful analysis*

Previous work: model of Derbenev and Skrinsky

- Model based on dielectric linear response of a plasma
  - *approximation of infinite magnetic field for electron motion ($v_{e\perp} = 0$)*
  - *actual values of B and transverse rms electron velocity enter through cut-off parameters in the Coulomb logarithm*

$$r_L = V_{rms,e,\perp} / \Omega_L \left(B_{\parallel}\right)$$

$$\rho_{max} = min\left\{max\left(\rho_{sh}, \left(\frac{3Z}{n_e}\right)^{1/3}\right), V_{ion}\tau\right\}$$

$$\rho_{sh} = \frac{\sqrt{V_{ion}^2 + V_{e,rms}}}{\omega_e}$$

$$L_M = \ln(\rho_{max}/\rho_L)$$

$$U = \sqrt{V_{\perp}^2 + (V_{\parallel} - v_e)^2}$$

$$\vec{V} = (V_{\perp}, V_{\parallel})$$

$$\vec{F} = -2\pi Z^2 n_e m_e (r_e c^2)^2 \frac{\partial}{\partial \vec{V}} \int \left[\frac{V_{\perp}^2}{U^3} L_M + \frac{1}{U}\right] f(v_e) dv_e$$

Ya. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

Ya. S. Derbenev and A.N. Skrinsky, "The Effect of an Accompanying Magnetic Field on Electron Cooling," Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, "Magnetization effects in electron cooling," Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

Previous work: asymptotics of the D-S model for cold, strongly magnetized electrons

Asymptotics for $V_{ion} >> \Delta_{e,\parallel}, \quad B \to \infty :$

$$\rho_{max} = min\left\{ max\left(\rho_{sh}, \left(\frac{3Z}{n_e}\right)^{1/3} \right), V_{ion}\tau \right\}$$

$$F_\parallel = -2\pi Z^2 n_e m_e \left(r_e c^2\right)^2 \left[3\left(\frac{V_\perp}{V_{ion}}\right)^2 \ln\left(\frac{\rho^A_{\max}}{\rho^A_{\min}}\right) + 1 \right] \frac{V_\parallel}{V^3_{ion}}$$

$$\rho_{min} = Z r_e c^2 \frac{1}{|\vec{V}_{ion} - \vec{v}_e|^2}$$

$$\rho_{sh} = \frac{\sqrt{V^2_{ion} + V_{e,rms}}}{\omega_e}$$

$$F_\perp = -2\pi Z^2 n_e m_e (r_e c^2)^2 \left[\frac{V^2_\perp - 2V^2_\parallel}{V^2_{ion}} \ln\left(\frac{\rho^A_{max}}{\rho^A_{min}}\right) \right] \frac{V_\perp}{V^3_{ion}}$$

$$r_L = V_{rms,e,\perp} / \Omega_L\left(B_\parallel\right)$$

$$\rho^A_{\min} = \max\left(r_L, \rho_{\min}\right)$$

Asymptotic result for *large V_{ion} parallel to* ***B***:

$$\rho^A_{\max} = \min\left(r_{beam}, \rho_{\max}\right)$$

$$F_\parallel\left(V_\perp = 0\right) = -2\pi Z^2 n_e m_e \left(r_e c^2\right)^2 \frac{1}{V^2_\parallel} \quad \text{(no dependence on } \tau\text{)}$$

$$V^2_{ion} = V^2_\parallel + V^2_\perp$$

Ya. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

Ya. S. Derbenev and A.N. Skrinsky, "The Effect of an Accompanying Magnetic Field on Electron Cooling," Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, "Magnetization effects in electron cooling," Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

Previous work: parametric model of Parkhomchuk for including finite B and thermal effects

$$\mathbf{F} = -4Z^2 n_e m_e \left(r_e c^2\right)^2 \ln\left(\frac{\rho_{\max} + \rho_{\min} + r_L}{\rho_{\min} + r_L}\right) \frac{\mathbf{V}_{ion}}{\left(V_{ion}^2 + V_{eff}^2\right)^{3/2}}$$

$$r_L = V_{rms,e,\perp} / \Omega_L\left(B_{\|}\right)$$

$$V_{eff}^2 = V_{e,rms,\|}^2 + \Delta V_{\perp e}^2$$

$$\rho_{max} = V_{ion}/(\omega_e + 1/\tau)$$

$$\rho_{\min} = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^2 = Z m_e r_e c^2 / V_{ion}^2$$ (as in the original paper)

$$\rho_{min}^B = Z m_e r_e c^2 / (V_{ion}^2 + V_{eff}^2)$$ (as implemented in BETACOOL)

V.V. Parkhomchuk, "New insights in the theory of electron cooling," Nucl. Instr. Meth. in Phys. Res. **A 441** (2000).

I. Meshkov, A. Sidorin, A. Smirnov, G. Trubnikov, R. Pivin, "BETACOOL Physics Guide," http://lepta.jinr.ru/betacool (2008).

Previous work: Asymptotics of Parkhomchuk's model for strong B, small V_{ion}

In the limit of $B \to \infty$:

$$\mathbf{F} = -4Z^2 n_e m_e (r_e c^2)^2 \ln\left(\frac{\rho_{max} + \rho_{min}}{\rho_{min}}\right) \frac{\mathbf{V}_{ion}}{(V_{ion}^2 + V_{eff}^2)^{3/2}}$$

$$V_{eff}^2 = V_{e,rms,\parallel}^2 + \Delta V_{\perp e}^2$$

$$\rho_{max} = V_{ion}/(\omega_e + 1/\tau)$$

$$\rho_{\min} = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^2 = Z m_e r_e c^2/V_{ion}^2$$ (as in the original paper)

$$\rho_{min}^B = Z m_e r_e c^2/(V_{ion}^2 + V_{eff}^2)$$ (as implemented in BETACOOL)

In the limit of strong B, cold e-beam, and small V_{ion}:

$$F_\parallel = -4Z n_e r_e c^2 \frac{V_{ion,\parallel}}{\omega_e + 1/\tau}$$ with plasma frequency $$\omega_e = \sqrt{4\pi n_e r_e c^2}$$

V.V. Parkhomchuk, "New insights in the theory of electron cooling," Nucl. Instr. Meth. in Phys. Res. **A 441** (2000).

I. Meshkov, A. Sidorin, A. Smirnov, G. Trubnikov, R. Pivin, "BETACOOL Physics Guide," http://lepta.jinr.ru/betacool (2008).

Previous work: Asymptotic representation by Meshkov

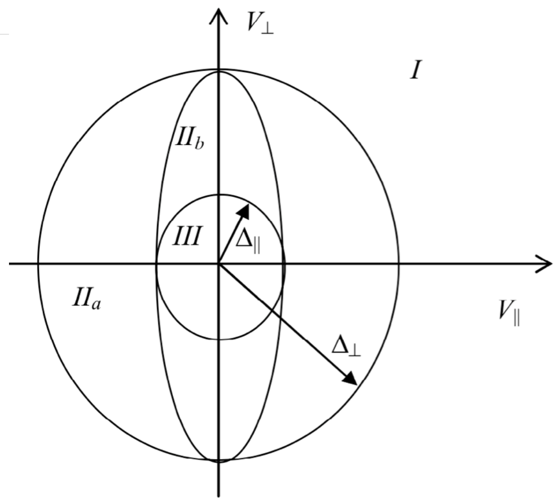
$$F_{\parallel} \approx -\frac{2\pi Z^2 e^4 n_e}{m} v_{\parallel} \begin{cases} \frac{1}{v^3}\left(2L_F + \frac{3V_{\perp}^2}{V^2} L_M + 2\right), \{I\} \\ \frac{2}{\Delta_{\perp}^2 V_{\parallel}}\left(L_F + N_{col} L_A\right) + \left(\frac{3V_{\perp}^2}{V^2} L_M + 2\right)\frac{1}{V^3}, \{II_a\} \\ \frac{2}{\Delta_{\perp}^2 \Delta_{\parallel}}\left(L_F + N_{col} L_A\right) + \frac{L_M}{\Delta_{\parallel}^3}, \{II_b, III\} \end{cases}$$

$$L_M = \ln\frac{R}{k\rho_{\perp}},\ L_A = \ln\frac{k\rho_{\perp}}{\rho_F},\ L_F = \ln\frac{\rho_F}{\rho_{\min}}.$$

$$\rho_{\min} = \frac{Ze^2}{m_e}\frac{1}{V^2 + \Delta_{\parallel}^2}$$

$$N_{coll} = 1 + \frac{\Delta_{\perp}}{\pi\sqrt{V^2 + \Delta_{\parallel}^2}}$$

$$F_{\perp} \approx -\frac{2\pi Z^2 e^4 n_e}{m} v_{\perp} \begin{cases} \frac{1}{v^3}\left(2L_F + \frac{V_{\perp}^2 - 2V_{\parallel}^2}{V^2} L_M\right), \{I\} \\ \frac{2}{\Delta_{\perp}^3}\left(L_F + N_{col} L_A\right) + \frac{V_{\perp}^2 - 2V_{\parallel}^2}{V^2}\frac{L_M}{V^3}, \{II\} \\ \frac{2}{\Delta_{\perp}^3}\left(L_F + N_{col} L_A\right) + \frac{L_M}{\Delta_{\parallel}^3}, \{III\} \end{cases}$$



I. Meshkov, "Electron Cooling; Status and Perspectives," Phys. Part. Nucl. **25** (1994), 631.

I. Meshkov, A. Sidorin, A. Smirnov, G. Trubnikov, R. Pivin, "BETACOOL Physics Guide," http://lepta.jinr.ru/betacool (2008).

Our approach is motivated by the work of *Ya. Derbenev*

Ya. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

THEORY OF ELECTRON COOLING

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- The E-fields associated with friction must be carefully identified
  - *these are the fields generated by the presence of the ion*

bulk fields | friction | statistical fluctuations

$$\vec{E}(\vec{r},\vec{v},t) = \langle \vec{E}^0 \rangle(\vec{r},t) + \langle \Delta\vec{E} \rangle(\vec{r},\vec{v},t) + \vec{E}^{fl}(\vec{r},\vec{v},t) \qquad (1.1)$$

- Friction force must be calculated along the ion trajectory:

$$\vec{F} = -ze\langle \Delta\vec{E} \rangle(\vec{r},\vec{v},t)\Big|_{\vec{r}=\vec{r}(t),\dot{\vec{r}}(t)=\vec{v}} \qquad (1.2)$$

  - *we do this numerically for each individual ion-electron interaction*
    - **total force obtained by summing over e^- distribution (i.e. no shielding)**
  - *bulk forces are removed by subtracting force from unperturbed e^-'s*

Our model: strongly magnetized, relativistic cooling regime → short interaction time, strong magnetic field

- Prototyping is done in the parameter regime of Fedotov *et al*. (2006)

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **9**, 074401 (2006)

Numerical study of the magnetized friction force

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(Received 14 November 2005; published 7 July 2006)

Proceedings of HB2006, Tsukuba, Japan

Analysis of the magnetized friction force *

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- For our test case, we considered the following beam frame parameters:
  - *e^- density, $n_e = 2x10^{15}$ m^{-3}*
  - *$V_{e,rms,||} = 0$ or 1.0×10^5 m/s and $V_{e,rms,\perp} = 4.2 \times 10^5$ m/s*
  - *ideal solenoid, B = 1T and 5T (theoretical models) and infinitely strong field (theoretical models and our simulations)*
  - *interaction time, $T_{int} = 4x10^{-10}$ s ~ 56 T_L ~ 0.16 T_{pl} (T_L for B = 5T)*
    - **16% of a plasma period → no shielding of the interaction**
  - *expectation value of distance to nearest e^-, r_1 ~ $4.9x10^{-6}$ m ~ 10 r_L*
    - **small Larmor radius → strong B-field assumption is reasonable**

Gyrokinetic averaging yields *1D e^-* oscillations

- Hamiltonian perturbation theory for single ion & e^-
  - *unperturbed motion: drifting ion and magnetized e^-*
  - *strong B assumption: D (impact parameter) >> r_L (Larmor radius)*
  - *longitudinal dynamics: $V_{ion,\perp} = 0$ (to be relaxed in future work)*
- choose ion to be stationary at the origin (convenient)
- to the leading order in perturbation theory, *e^-* gyrocenters stay on cylinder of constant radius *D* (different for different *e^-*'s)
  - *gyrocenters move in an effective nonlinear 1D potential:*

$$\ddot{z}(t) = -Zr_e c^2 \frac{z}{(D^2 + z^2)^{3/2}}$$

- a weakly nonlinear potential:
  - *larger amplitudes <=> longer oscillation periods;* $T_{min} = 2\pi\sqrt{D^3/Zr_e c^2}$
  - *both unbound and oscillatory e^- orbits, incl. trajectories with T > or >> T_{int}*
  - *net friction force is determined by contributions from different orbit types*
  - *1D numerical simulations are required to capture these effects*

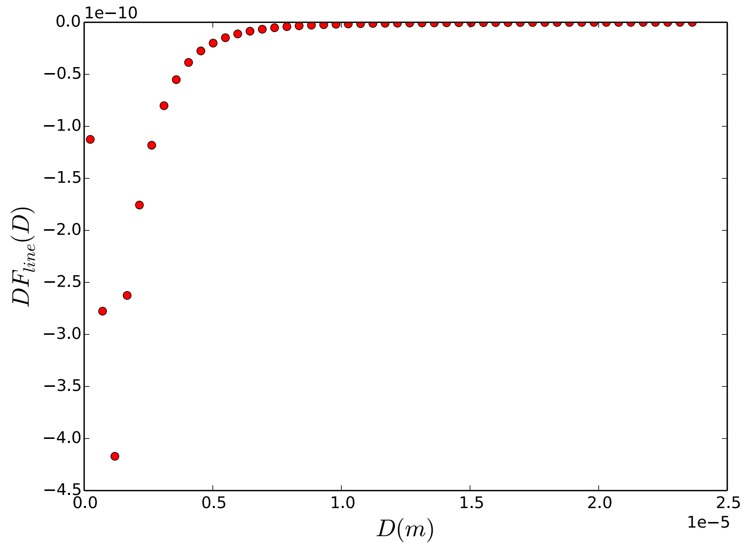
Key aspects of the numerical simulations

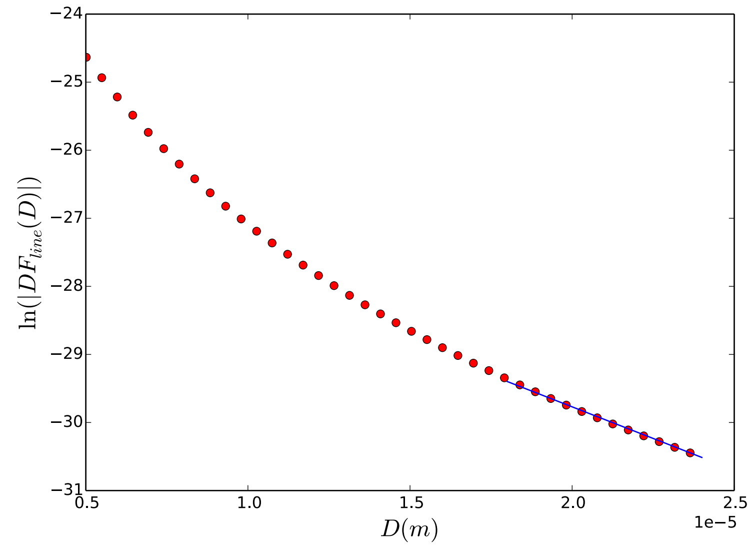
- Work in the system of reference where the ion is at rest
  - *assume ion velocity along the field lines of B (→ axial symmetry)*
  - *cold electrons → all have the same initial velocity w.r.t. ion*
  - *momentum kicks add up, averaged over T_{int}*
- Dynamical friction comes from ion-induced *density perturbation*
  - *add up the difference between force from e^-'s on perturbed & unperturbed paths*
    - **hence, we track pairs of electrons with identical initial conditions**
  - *this approach eliminates all bulk forces, both physical and numerical*
- Compute ensemble-average expectation value of friction
  - *we assume a locally-uniform electron density n_e*
  - *transversely, e^--s are uniformly distributed on lines of constant D*
    - **there is no logarithmic singularity for D→0, nor for D→∞**
  - *longitudinal distribution is uniform in initial z position, z_{ini}*
    - **finite range of z_{ini} values contributes non-negligibly to the friction force**
    - **range depends on: D (impact parameter), V_{ion}, Z (ion charge state)**
- Friction force for warm e^-'s is obtained via convolution

Finite friction for all ρ *(no logarithmic singularities)*

- First add up contributions to the friction force from initial conditions on lines of constant D, then integrate over the impact parameter:

$$F_{\|}(V_{\perp}=0) = 2\pi n_e \int_0^{\infty} dD D F_{line}(D) \equiv 2\pi n_e \int_0^{\infty} dD D \int_{-\infty}^{\infty} dz_{ini} F_{i-e}(z_{ini}, D)$$

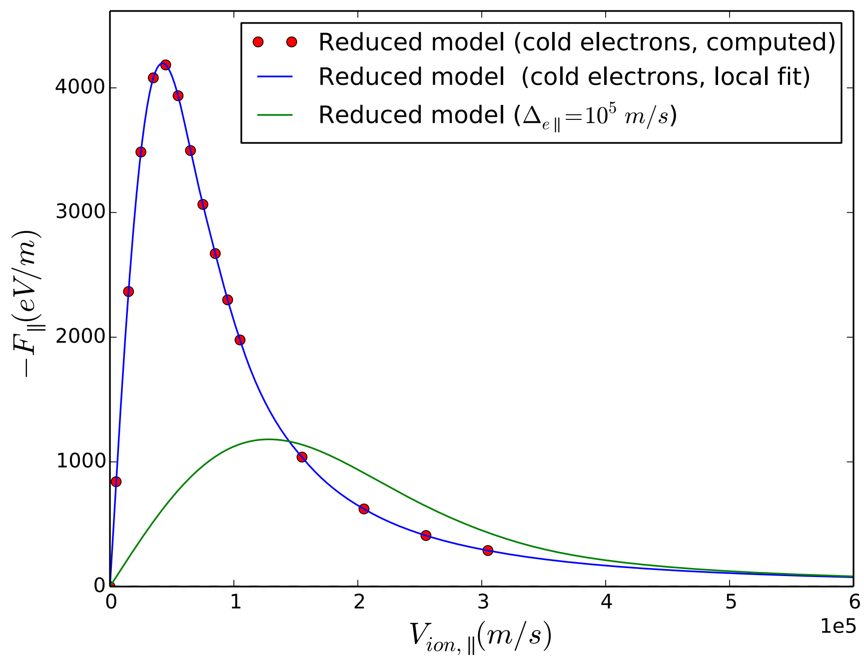




- Integrand is finite for small D & tails off exponentially => finite $F_{||}$
- Exponential fall-off for large D makes it possible to correct (analytically) for finite values of D_{max} in simulations
- Repeat for different values of $V_{ion,||}$ to compute $F_{||}$ $(V_{ion,||})$

Physically reasonable behavior of $F_{||}(V_{ion,||})$ seen for both small and large $V_{ion,||}$, cold and warm electrons

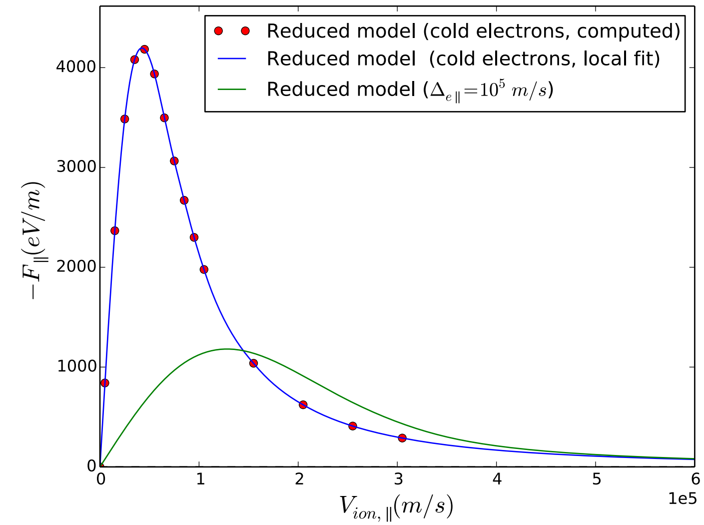
- For *cold* electrons and Au^{+79} ion, reasonable qualitative behavior of $F_{||}(V_{ion,||})$ seen for both small and large $V_{ion,||}$:
  - *linear in V for small V*
  - *$1/V^2$ for large V*
- For an arbitrary distribution $f(v_{e,||})$ of *warm* electrons, $F_{||}(V_{ion,||})$ is computed by convolution of $f(v_{e,||})$ with $F_{||}(V_{ion,||})$ for cold electrons



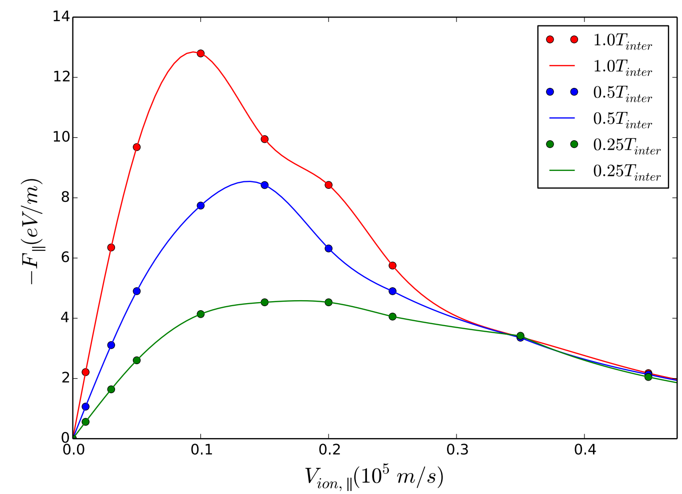
- Convolution with $f(v_{e,||})$ acts as a smoothing filter => peak of $F_{||}(V_{ion,||})$ for warm electrons is lower and shifted towards larger $V_{ion,||}$
- Just as for cold e⁻ gas, for warm electrons $F_{||}(V_{ion,||})$ is linear in $V_{ion,||}$ for small $V_{ion,||}$ and scales as $1/V^2$ in the large $V_{ion,||}$ region
- As expected, $F_{||}(V_{ion,||})$ for different electron temperatures converge as $V_{ion,||}$ gets larger

$F_{\|}(V_{ion,\|})$ for *cold* electrons: scaling in Z and T_{int}

- For cold electrons, looked at protons and Au^{+79} ion and different interaction times in the cooler (interaction-time-averaged force):
  - *for small $V_{ion,\|}$: $F_{\|}(V) \sim V$; slope $dF_{\|}(V)/dV \approx -2Z\, n_e\, m_e\, r_e\, c^2\, T_{int}$*
  - *large-V tail is well approximated by $F_{\|} \approx -2\pi Z^2 n_e m_e (r_e c^2)^2 / V^2$, with no dependence on T_{int}*
  - *for a given T_{int}, peak friction force scales approximately as $Z^{4/3}$*
- For $T_{int} < T_{pl}$ and small-to-moderate V_{ion}, $F_{\|}(V_{ion,\|})$ goes up with interaction time; large-V tail is T_{int}–independent
- $F_{\|}(V_{ion,\|})$ is linear in n_e by construction



Above: Gold ion, cold and warm e⁻'s

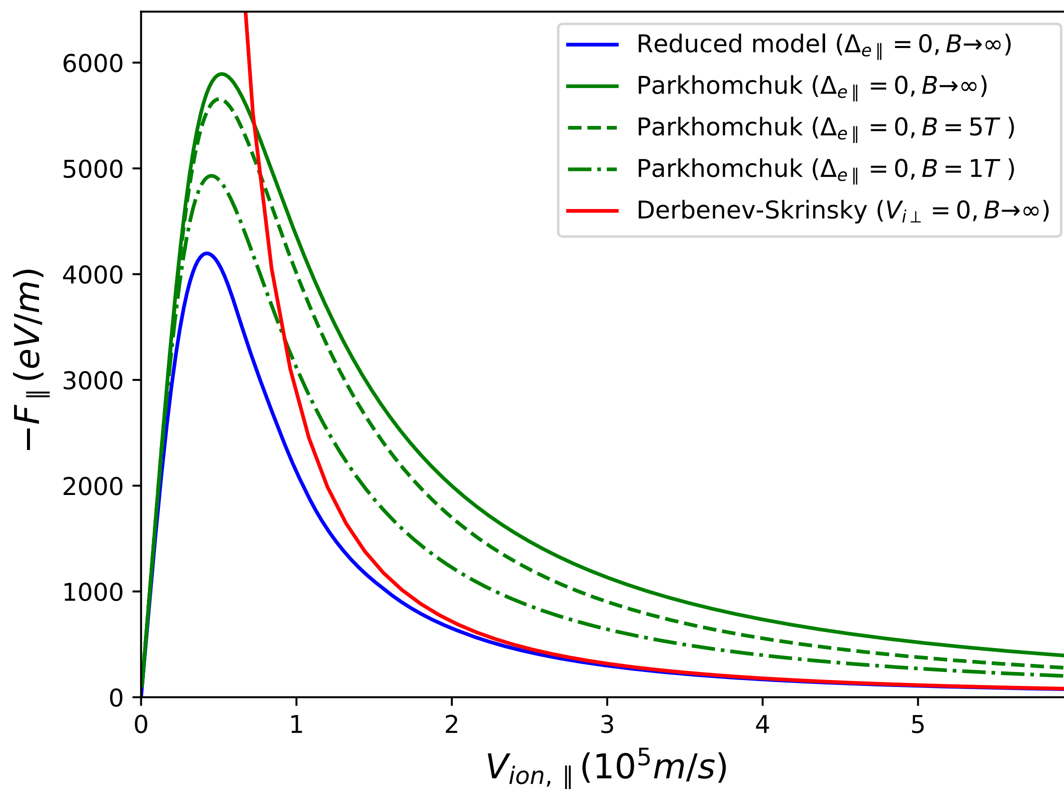


Above: Protons, cold e⁻'s, varying T_{int}

Compare with Derbenev-Skrinsky and Parkhomchuk (1)

- Comparison of new model for an Au^{+79} ion, with:
  - *Derbenev and Skrinsky (D&S) for $V_{ion,\perp} = 0$, cold e^-'s, strong B and large $V_{ion,\parallel}$*
  - *Parkhomchuk (P) with 0 effective longitudinal e^- temperature for $V_{ion,\perp} = 0$*

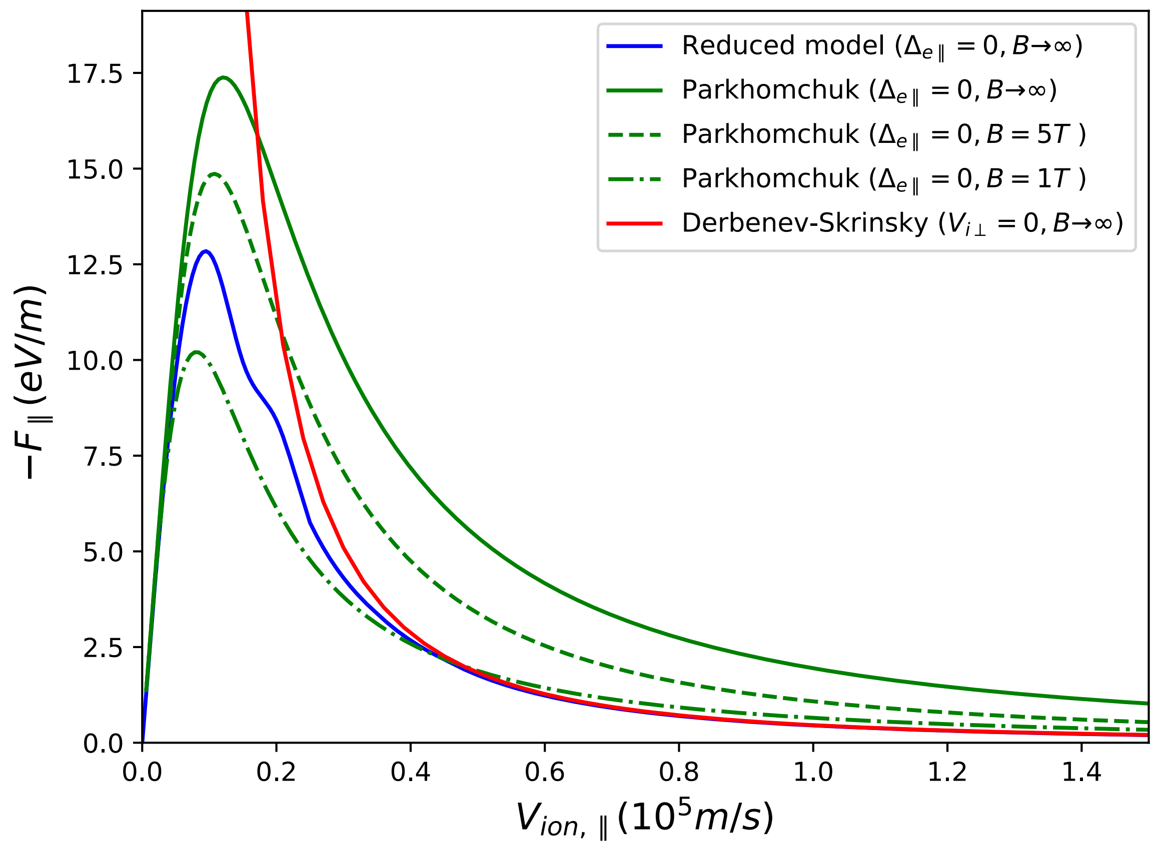
- All models $\sim V^{-2}$ for large V
  - *our simulation and semi-analytic model agree exactly with D&S*
  - *consistently lower force than Parkhomchuk in this limit*



Compare with Derbenev-Skrinsky and Parkhomchuk (2)

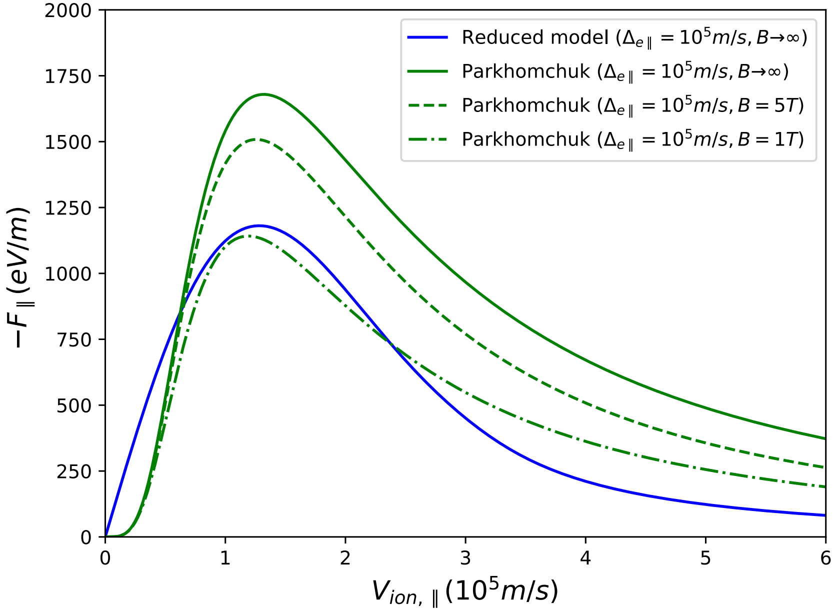
- Comparison of new model for protons, with:
  - *Derbenev and Skrinsky (D&S) for $V_{ion,\perp} = 0$, cold e^-'s, strong B and large $V_{ion,\parallel}$*
  - *Parkhomchuk (P) with 0 effective longitudinal e^- temperature for $V_{ion,\perp} = 0$*

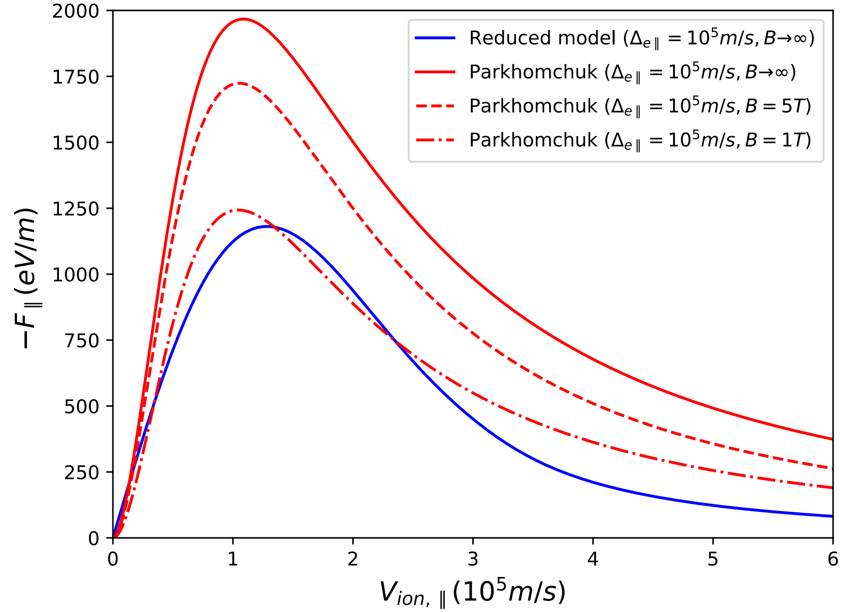
- For cold electrons:
  - *new model shows consistently lower force values (for a very strong B) than Parkhomchuk at **ALL** velocities*



Compare with Derbenev-Skrinsky and Parkhomchuk (3)

- Comparison of new model for Au^{+79} *(warm e$^-$'s)*, with Parkhomchuk:
  - *Parkhomchuk model with finite effective* longitudinal *e$^-$ temperature*
  - *ϱ_{min} as in the original paper and as implemented in BETACOOL* $\rho^B_{min} = Zm_e r_e c^2/(V^2_{ion} + V^2_{e,rms,\parallel})$

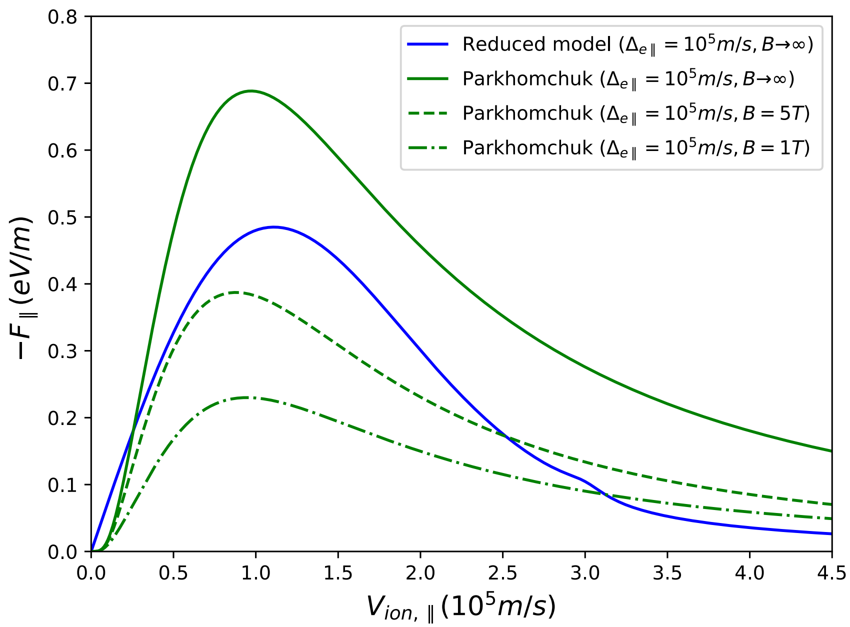


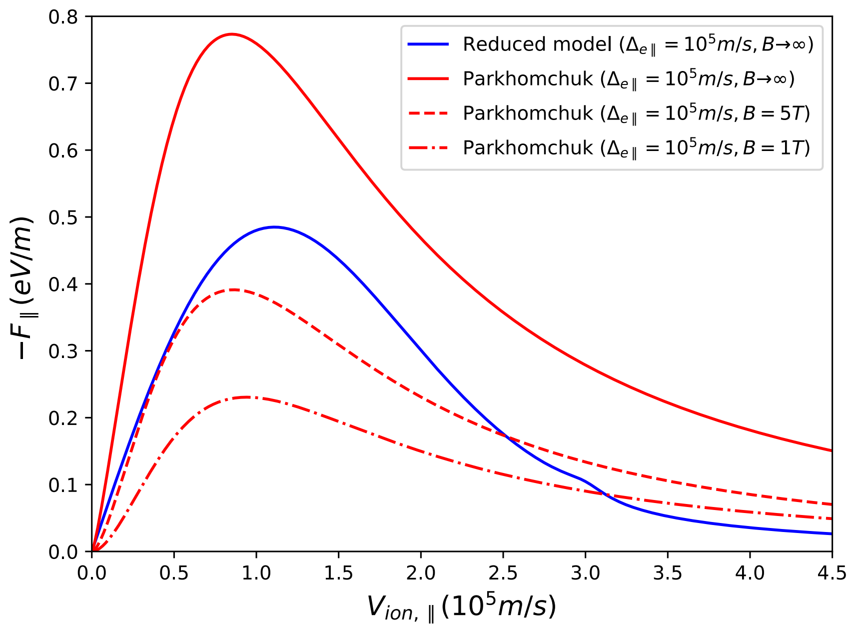


- For warm electrons, new model agrees approximately with Parkhomchuk (but details depend on Z, V_{ion})
  - *we are working to understand the details of how and in what sub-domain of parameter space this occurs*

Compare with Derbenev-Skrinsky and Parkhomchuk (4)

- Comparison of new model for protons *(warm e^-'s)*, with Parkhomchuk:
  - *Parkhomchuk model with finite effective* longitudinal *e^- temperature*
  - *ϱ_{min} as in the original paper and as implemented in BETACOOL* $\rho^B_{min} = Z m_e r_e c^2/(V^2_{ion} + V^2_{e,rms,\|})$





- For warm electrons, new model agrees approximately with Parkhomchuk (but details depend on Z, V_{ion})
  - *we are working to understand the details of how and in what sub-domain of parameter space this occurs*

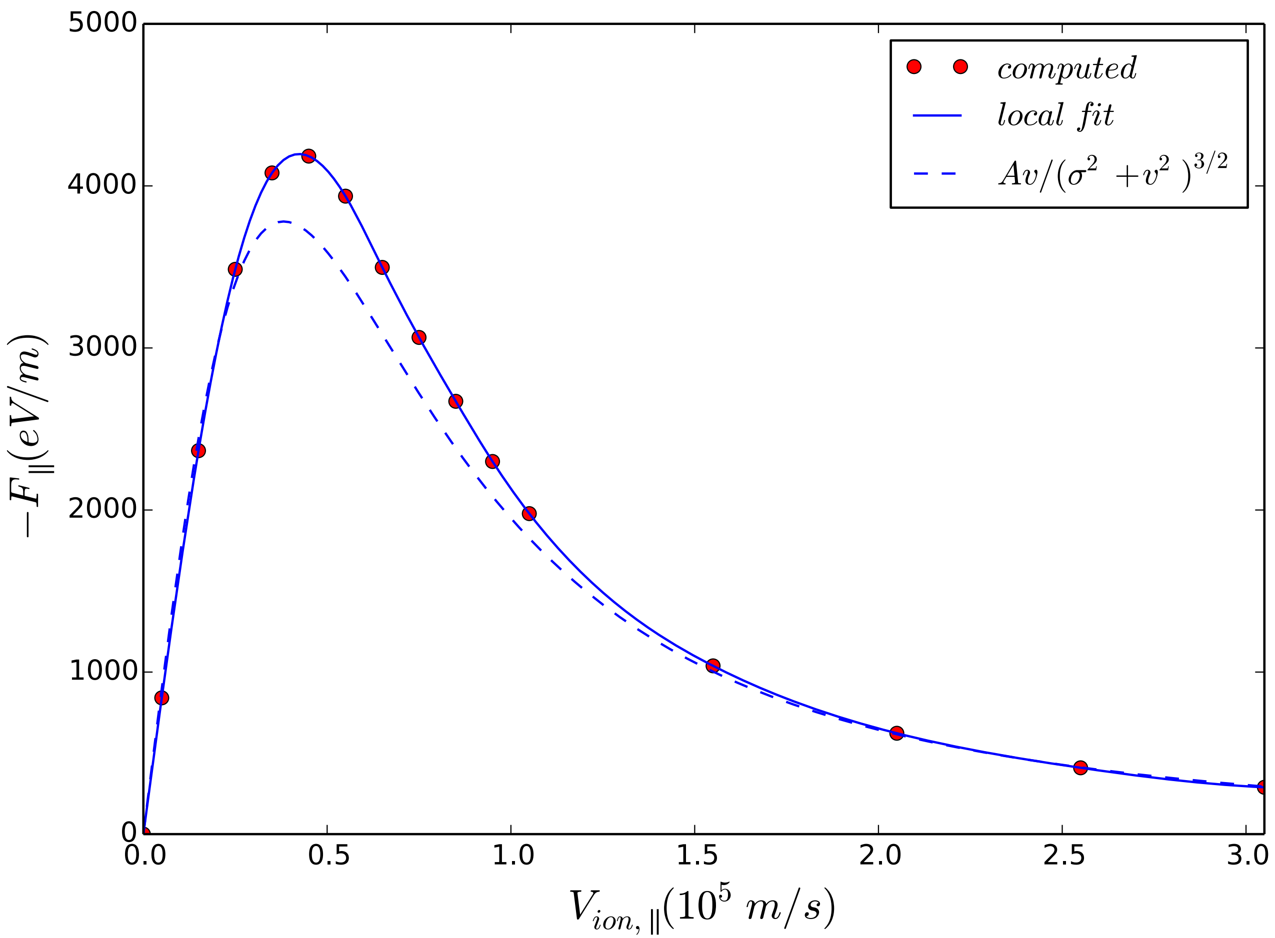
Simple, approximate 2-parameter model

$$F_{\|}(v) = -\frac{Av}{(\sigma^2 + v^2)^{3/2}}$$

$$A = 2\pi Z^2 n_e m_e (r_e c^2)^2$$

$$\sigma \approx (\pi Z r_e c^2 / T_{int})^{1/3}$$

- Large v:
  - $F_{\|} \sim A/v^2$
  - *A found via fit and dimensions and scaling analysis*
- Small v:
  - $dF/dv \sim A/\sigma^3$
  - *σ found via fit, dimensional and scaling analysis*
- Peak force is underestimated by ~10-15%
  - $F_{\|,max} \sim Z^{4/3}$

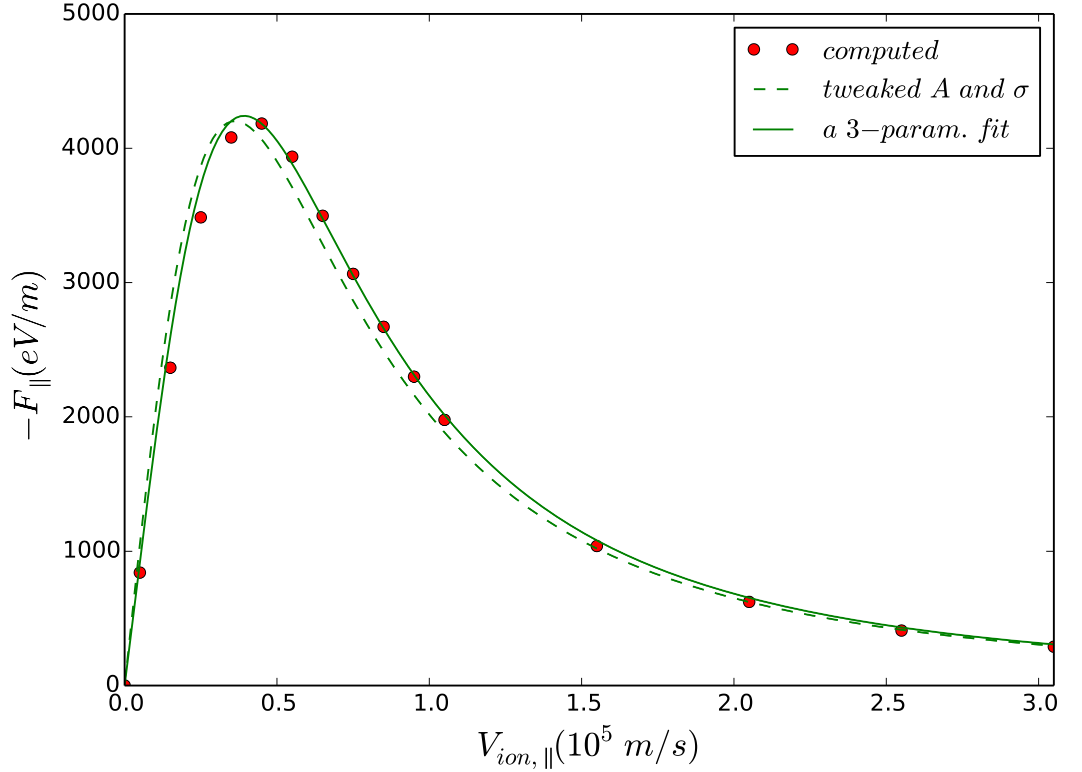


radiasoft

3-parameter model fits the calculations closely

- The physical system depends on 3 parameters:
  - n_e, Z, T_{int}
- Captured via perturbation of 2-parameter model:
  - *Adjust values of A and σ* or *add a small 3rd parameter*

- Improved parametric models are under development



Work in progress and future plans

- Improved parametrized models for cold electrons, and parametrized models for non-zero electron temperature
- Better understanding of the role of trapped (oscillatory) *vs* unbound electron orbits
- The case of finite B
- What happens to the magnitude of dynamic friction force as the interaction time approaches/exceeds T_{pl} ?
- Modeling transverse dynamic friction (have to work with non-zero electron temperature from the start)
- Statistical properties of $F(V)$: so far, only the expectation value was considered (in essence, the continuum limit)
- Adding new models to JSPEC as they become available, simulations in the EIC parameter regime
  - https://sirepo.com

Thank You!

Спасибо!

Comments or Questions?