LONGITUDINAL PARTICLE DYNAMICS AND COOLING IN NICA COLLIDER

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Abstract
A feature of the NICA acceleration complex is high luminosity of colliding beams. Three types of RF stations will be used in the NICA Collider to reach the necessary beam parameters. The first one is for accumulation of particles in the longitudinal phase space with the moving barrier buckets under action of stochastic and/or electron cooling systems. The second and third RF stations are for formation of the final bunch size in the colliding regime. This report presents brief description of constructed in BINP three types of RF station and numerical simulations of longitudinal beam dynamics which take into account the longitudinal space charge effect, cooling and IBS during the accumulation and bunching procedures.

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Content:

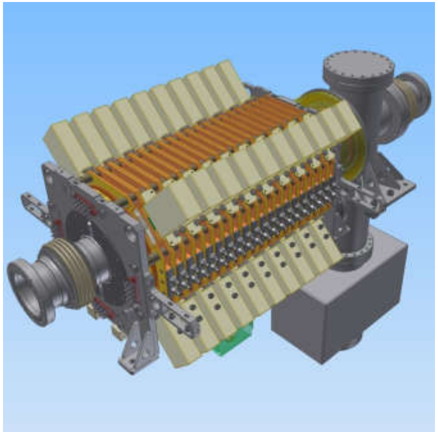
Introduction

The goal of the NICA facility in the heavy ion collision mode is to reach the luminosity level of 10^{27} $cm^{-2}s^{-1}$ in the energy range from 1 GeV/n to 4.5 GeV/n.

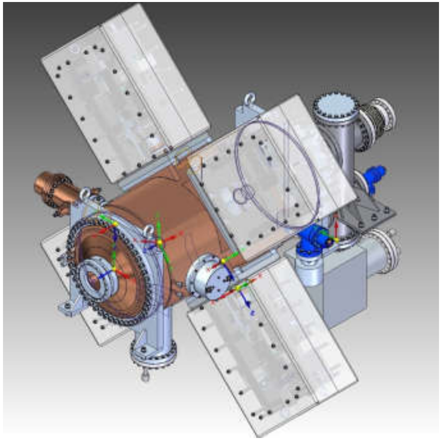
Collider RF systems have to provide accumulation of required numbers of ions in the energy range 1-3.9 GeV/n, accumulation at some optimum energy and acceleration to the energy of the experiment in the range of 1-4.5 GeV/n, formation of 22 ion bunches, and achievement of the required bunch parameters.

This can be done with the help of three RF systems, one of the broad-band type and two narrow-bands ones. The first one accumulates particles in longitudinal phase space with application of RF barrier bucket technique. The maximal voltage of the barrier is 5 kV, it has rectangular shape with phase length $\pi/12$. By applying additional voltage of 300 V, one can also use the meander between the barriers for inductive acceleration. The second RF system works on the 22th harmonic of the revolution frequency and is used for formation of the proper number of bunches. The maximal RF2 voltage corresponds to 100 kV. The RF2 can also be used for beam acceleration or deceleration. The third RF system works on the 66th harmonic and is used for the final bunch formation and maintenance of the bunch parameters during the collision mode. The maximal RF3 voltage is 1 MV. The RF3 system is also used for ion beam acceleration or deceleration. All stages of the bunch formation as well as the collision mode are accompanied by a cooling process, either stochastic or electron.

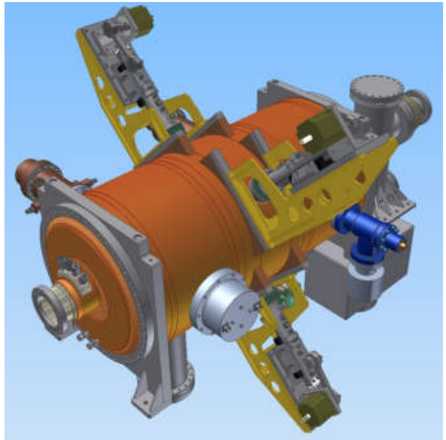
Construction of RF stations (3D models)



RF1 station



The cavity of RF2 station



The cavity of RF3 station

RF1: a sequence of voltage impulses $\pm 5kV$ with duration 80 nsec or $\pi / 12$.

	RF2	RF3
harmonic	22	66
Frequency, MHz	11.484÷12.914	34.452÷38.742
Rsh,Ohms	$3.12 \cdot 10^5$	$2.68 \cdot 10^6$
Q	3900	6700
RF voltage, kV	25	125
Number of cavities per ring	4	8

Previous calculations modelling longitudinal beam dynamics were fulfilled in approach neglecting change of transverse emittance and cooling time during accumulation or bunching. Now we take into account **dependence of IBS and electron cooling force on transverse emittance** which also changes in accordance with these effects in RMS model.

Accumulation of ions (RF1, 1-st harmonic)

Moving barrier buckets.

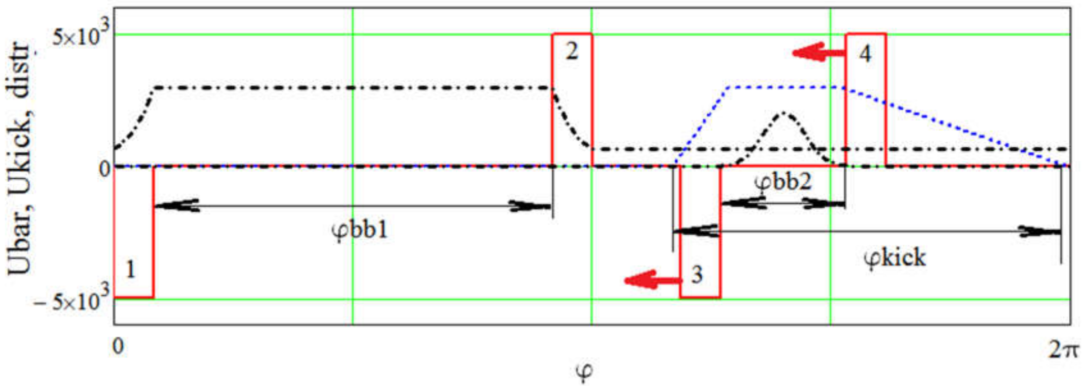


Fig. Barrier voltage (red line), density of stored and newly injected beam (black dash-dot line), impulse of kicker(blue dashes).

Accumulation is fulfilles with separated regions of injection and storage. Two pairs of voltage impulses form 2 separatrices, the 1st one for injection, the 2nd - for storage of ions (stack). After injection the impulses of injection sepahatrix move close to the stack, then impulses separating injected bunch from stack decrease, and separatrices join. If the length of combined separatrix exceeds half of the ring perimeter, it will be compressed.

Calculation model.

At the calculation all the effects are separated (movement of barrier buckets, cooling, IBS, loss of ions at injection). All movements are slow, with conserved longitudinal emittance.

Electron cooling force

$$\vec{F}_{e1}(\vec{v}_1)/m_i = \frac{d\vec{v}_1}{dt_1} = -\xi_1 \frac{\vec{v}_1}{\left(1 + v_1{}^2 / v_{eff1}{}^2\right)}, \quad \text{(BS)}, \qquad \text{(V.Parkhomchuk formula)}$$

Parameters: $I_e = 1A$, $r_e = 1\,cm$, $T_{et} = 5\,V$, $T_{el} = 5\,mV$

Gaussian distribution functions $f_{g\|,\perp}(v_{1\|,\perp}, \sigma_{1\|,\perp})$ (BS)

We use in calculacion the longitudinal component of cooling force averaged over transverse velocities and averaged over all 3 velocities distributions values of longitudinal and transverse decrements:

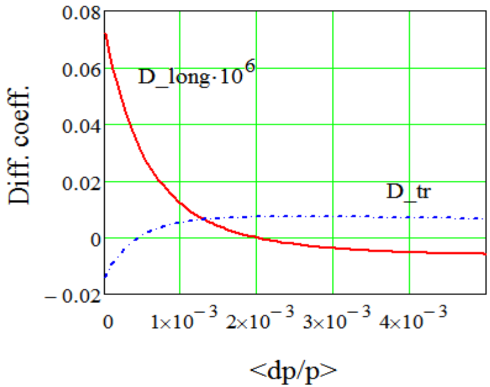
$$F_{e1\|av}(v_{1\|}, \sigma_{1\perp}, \xi_1) = \int F_{e1\|}(v_{1\|}, v_{1\perp}, \xi_1) f_{g\perp}(v_{1\perp})^2\, 2\pi v_{1\perp} dv_{1\perp}, \qquad (BS)$$

$$\xi_{av\|,\perp}(LS) = \frac{1}{\gamma_s} \frac{1}{\sigma_{v1\|,\perp}{}^2} \left\langle v_{1\|,\perp} \frac{dv_{1\|,\perp}}{dt_1} \right\rangle = \frac{1}{\gamma_s m_i} \int v_{1\|,\perp} F_{e1\|,\perp}(v_{1\|}, v_{1\perp}, \xi_1) f_{g\|}(v_{1\|}) f_{g\perp}(v_{1\perp})^2\, 2\pi v_{1\perp} dv_{1\perp} dv_{1\|},$$

BS, LS - beam and laboratory reference systems.

IBS diffusion coefficient

(S.Nagaitsev model, NICA magnetic structure 2018)



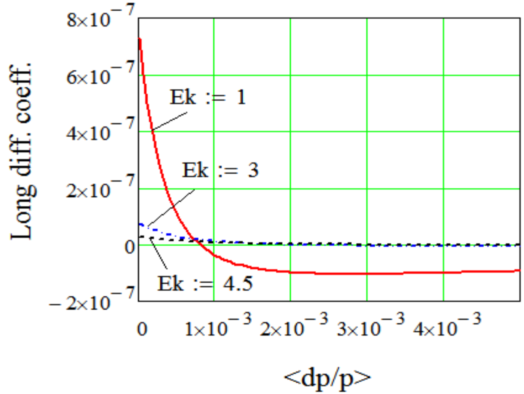


Fig. Left: IBS longitudinal and transverse diffusion coefficient $D_0(\sigma_p)$ at $N_0 = 6\cdot 10^9$, $\sigma_{s0} = 0.6м$, $E_k = 3GeV/n$, $\varepsilon_{x0} = 1\,\pi\cdot mm\cdot mrad$. Right: longitudinal diffusion coefficient for energies $E_k = 1, 3, 4.5\ GeV/n$.

At others values $N, \varepsilon_x, \sigma_s$: $D(N,\sigma_p,\sigma_s,\varepsilon_x) = \frac{N}{N_0}\frac{\sigma_{s0}}{\sigma_s}\left(\frac{\varepsilon_{x0}}{\varepsilon_x}\right)^{1.4}\cdot D_0(\sigma_p\sqrt{\varepsilon_{x0}/\varepsilon_x})$.

Cooling between injections.

After uniting 2 separatrices till a new injection rms beam parameters change with account of averaged decrements and stationary values as

$$\sigma_p(\Delta t)^2 = \sigma_{pst}{}^2 + \left(\sigma_p(0)^2 - \sigma_{pst}{}^2\right)\exp(-2\xi_{\|av}\Delta t),$$

$$\varepsilon_\perp(\Delta t) = \varepsilon_{\perp st} + \left(\varepsilon_\perp(0) - \varepsilon_{\perp st}\right)\exp(-2\xi_{\perp av}\Delta t),$$

where stationary values of impulse spread and transverse emittance are defined as a solution of the system of equations:

$$\begin{cases} \sigma_{pst}{}^2 = 0.5\xi_{\|av}{}^{-1}D_\|\left(\sigma_{pst}, N, \varepsilon_{\perp st}\right), \\ \varepsilon_{\perp st} = 0.5\xi_{\perp av}{}^{-1}D_\perp\left(\sigma_{pst}, N, \varepsilon_{\perp st}\right). \end{cases}$$

The main goal of these calculations was an attempt to take into account nonlinearity of the cooling force and it's influence on the distribution function and hence on the losses at injection.

One-dimentional models of distribution over longitudinal impulses.

2 models:

1)**gaussian** distribution with current value of σ_p (for **linear** cooling force)

2) stationary distribution for current values of diffusion coefficient and **nonlinear** averaged over transverse distribution longitudinal **cooling force** (**nonlinear distribution**):

This distribution we try to get from the 1-dimentional Fokker-Plank equation (for long. uniform beam)

$$\frac{\partial f(\delta_p)}{\partial t}+\frac{\partial}{\partial(\delta_p)}\left(p^{-1}F_{e1\|av}(\delta_p\cdot\beta c)\cdot f(\delta_p)\right)-\frac{\partial^2}{\partial(\delta_p)^2}\left(D\cdot f(\delta_p)\right)=0\,...,$$

$$\delta_p=\Delta p/p=v_1/(\beta c)\,.$$

Boundary conditions $f(0)'=0,\quad f(\delta_{p\,\max})=0$; time dependence $f(t,\delta_p)=f(\delta_p)e^{-t/\tau}$.

In stationary case (d/dt=0) there are 2 solutions:

$$f_0(\delta_p)=\exp\left(\int_0^{\delta_p}\frac{F_{e1\|av}(\delta_p\beta c)}{pD}d\delta_p\right),\qquad f_1(\delta_p)=f_0(\delta_p)\cdot\left(1-\frac{\int_0^{\delta p}f_0(\delta_p)^{-1}d(\delta_p)}{\int_0^{\delta p_{\max}}f_0(\delta_p)^{-1}d(\delta_p)}\right).$$

$f_0'(\delta_{p\,\max})=0$ $\qquad\qquad$ ($f_1(\delta_{p\,\max})=0$)

Unstationary case - iterations :

$$f_k(\delta_p) = C \cdot f_0(\delta_p) \cdot \left(1 - \frac{\int_0^{\delta p} f_{k-1}(\delta_p)^{-1} d(\delta_p)}{\int_0^{\delta p_{\max}} f_{k-1}(\delta_p)^{-1} d(\delta_p)} \right)$$

(satisfy both boundary conditions)

The first approach - $f_1(\delta_p)$ - is used as the model stationary 1-dimentional solution.

$\Delta t_{inj} >> \tau_{av} \quad \Rightarrow$ the losses are calculated with $f_1(\delta_p, D)$: $D = D(\sigma_{p\,st})$, $\sigma_{p\,st} = \sigma_p\left(f_1(D(\sigma_{p\,st})\right)$ instead of gaussian solution.

$\Delta t_{inj} << \tau_{av} \quad \Rightarrow \quad \sigma_p = \sigma_p(t)$ the losses are calculated with $f_1(\delta_p, D_1)$ for current $\sigma_p = \sigma_p(t)$: $D_1 = D(\sigma_{p1} \neq \sigma_p)$, $\sigma_p = \sigma_p\left(f_1(D_1)\right)$, analogously to the gaussian with changing $\sigma_p(t)$.

Application of 1-dimentional distribution to calcutation of ion losses at injection.

Above - 1dimentional distribution. But the stored beam occupies only a half of the ring and the losses are determined by the the distribution function in another half of the ring.
Outside of voltage barrier buckets the distribution is 1-dimentional:

$$f(\delta_p, s) = \begin{cases} f_1(\delta_{p1}) & inside\ \ the\ \ stack, \\ f_2(\delta_{p2}) & outside\ \ the\ \ stack. \end{cases}$$

$f_1(\delta_{p1})$ is proposed to be the same as for longitudinally uniform beam with the same density.
These functions are coupled with equation of motion along the phase trajectory and equation of continuity:

$$\begin{cases} f_1(\delta_p)\delta_p d(\delta_p) = f_2(\delta_{p2})\delta_{p2} d(\delta_{p2}), \\ (\delta_p)^2 / 2 = (\delta_{p2})^2 / 2 + (\delta_{psep})^2 / 2, \end{cases} \quad \Rightarrow \ f_1(\delta_p) = f_2(\delta_{p2}),$$

$$(\delta_{psep})^2 = kU_0\varphi_b$$

Thus, the number of lost ions can be defined as

$$\Delta N_{loss} = \frac{\varphi_{kick}}{\varphi_{bb}} N_0 \int_0^{\delta_{p\max}} f_2(\delta_{p2}) d\delta_{p2} = \frac{\varphi_{kick}}{\varphi_{bb}} N_0 \int_0^{\sqrt{\delta_{p\max}{}^2 - \delta_{psep}{}^2}} f_1(\sqrt{\delta_p{}^2 + \delta_{psep}{}^2}) d\delta_p$$

During injection the kicker injecting new portion of ions into stationary orbit simultaneously removes all ions of previously stored beam placed in the region of kicker impulse. The number of ions reaches its maximum when the numbers of injected and lost ions equal.

Effect of space charge field in the tube of vacuum chamber.

Density distribution of **hollow tube beam** with radius r_b (=0.5cm) in vacuum chamber with radius R (=5cm):

$$\rho(r,z) = dQ(r_b)\cdot\lambda(z)\cdot\frac{\delta(r-r_b)}{2\pi r_b},\quad \int\lambda(z)dz = 1,$$

Its longitudinal electric field

$$dE_z(r,z,r_b) = -\frac{1}{4\pi\varepsilon_0}\frac{2dQ(r_b)}{\gamma^2}\left(d\lambda/dz\right)\cdot\begin{cases}\ln(R/\mathrm{r}_b), & r<r_b,\\ \ln(R/\mathrm{r}), & r\geq r_b.\end{cases}\qquad \left|d\lambda/dz\right|\frac{R}{\gamma} << |\lambda|$$

Integration over transverse beam density distribution gives potential defined by ion beam space charge effects

$$W_{sc}(\varphi) = k_0 Ze\int_{\varphi_s}^{\varphi}U_{sc}(\varphi,\sigma_b)d\varphi = \frac{k_0 Ze}{\varepsilon_0}F(\sigma_b)\cdot\frac{Q}{\gamma^2}\left(\lambda(-\varphi R_{ring}) - \lambda(-\varphi_s R_{ring})\right),$$

Estimation:

$$\left|\Delta W_{sc}\right|_{\max} = \frac{k_0 Ze}{4\pi\varepsilon_0}F(\sigma_b)\cdot 2\pi\frac{2Q}{\gamma^2}\lambda_{\max},\quad \lambda_{\max}\approx 2/C_{ring},\quad F(\sigma_b)\approx 2,$$

$$\left|\Delta W_{sc}\right|_{\max}/\left|\Delta W_{barrier}\right|_{\max} = \frac{F(\sigma_b)}{\varphi_b}\cdot\frac{2}{\gamma^2\beta}\frac{I_0 Z_0}{U_0}\sim\frac{1.2 I_0}{\gamma^2\beta}\approx 0.14$$

For $E_k = 1\,GeV/n,\quad I_0 = 0.4A,\quad \varphi_b = \pi/12,\quad U_0 = 5000V$.

For calculation of losses space charge effect can be taken into account by decrease of barrier voltage by the factor $\left|\Delta W_{sc}\right|_{\max} / \left|\Delta W_{barrier}\right|_{\max}$.

For $E_k = 3\,GeV/n$ or for $E_k = 1\,GeV/n, \quad I_0 = 0.04A$ space charge can be neglected.

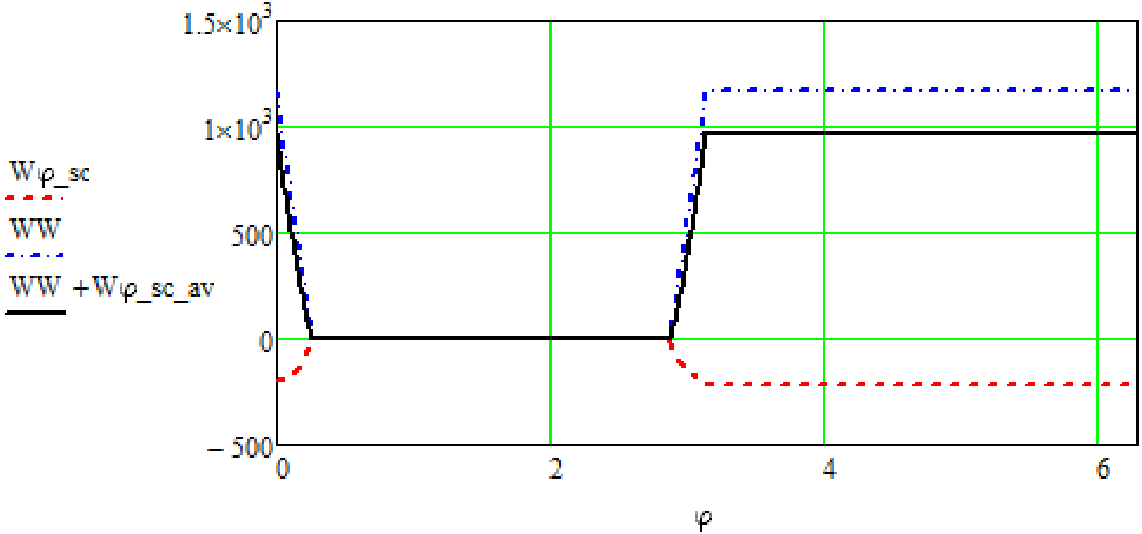


Fig. Potential defined by space charge effects(red dashes), RF barrier voltage (blue dash-dotes) and sum of them (black line).

Comparison of stationary results with Betacool program

a)

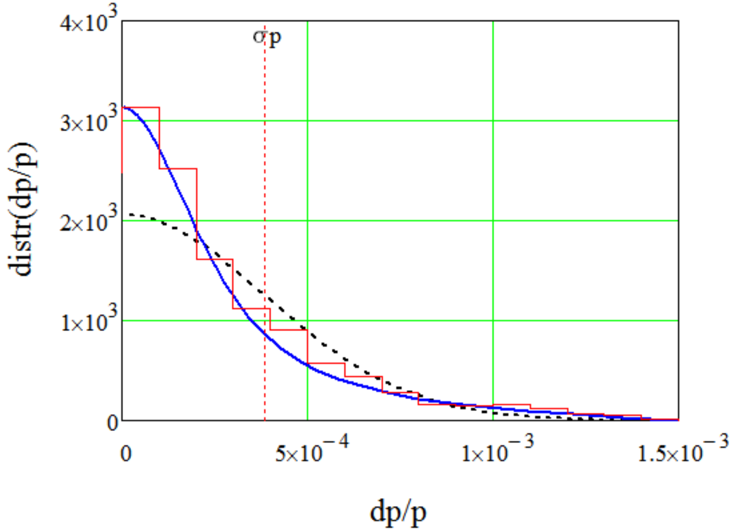
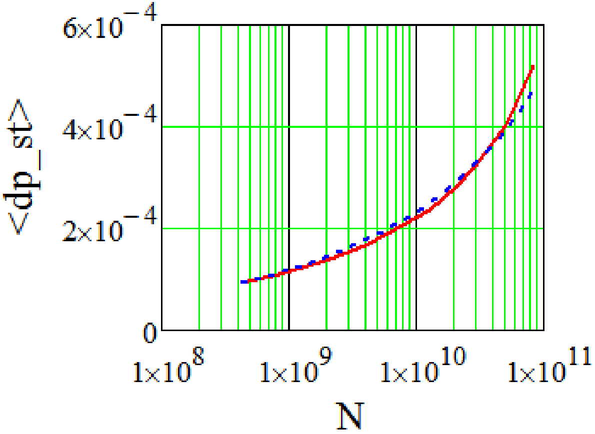
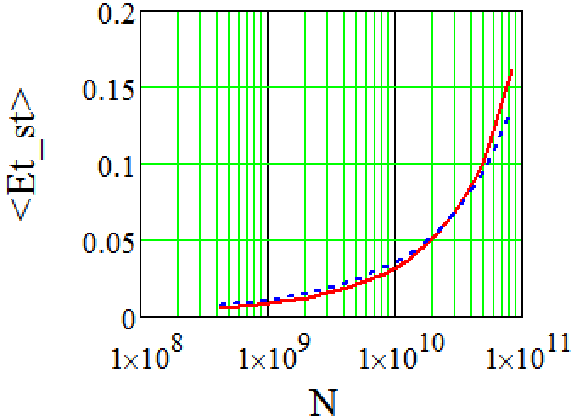


Fig. Stationary distribution solution obtained with Betacool program (red steps), proposed nonlinear distribution (blue line) and gaussian distribution for the same impulse spread (black dashes), for $I_e = 1\ A$, $\varphi_{kick} = 0.35 \cdot 2\pi,\quad \varphi_{bb} = 0.42 \cdot 2\pi,\quad \delta_{p\ \max} = 1.5 \cdot 10^{-3}$.

Calculacion were made for $\delta_{p\ \max} = 1.5 \cdot 10^{-3}$ instead of $\delta_{p\ \max} = 0.01$ in order to decrease calculation time and number of model particles for getting Betacool stationary solution.

b)Stationary impulse spread and transverse emittance ($\pi \cdot mm \cdot mrad$) in dependence on number of ions. Red line - Betacool; blue dashes - nonlinear model:





Stationary solutions are in good accordance with Betacool rms results.

Other stationary results.

a)

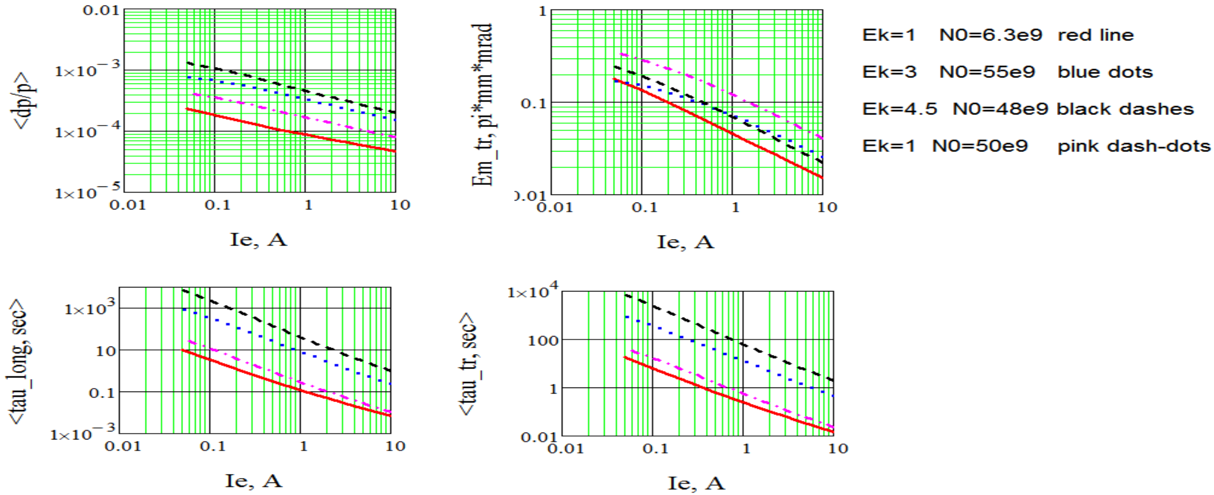


Fig. **Stationary impulse spread and transverse emittance** for ion energies E_k=1, 3, 4.5 *GeV/n* , number of ions N0, in dependence on electron beam current. Averaged cooling time for these parameters.

At the energy E_k=3 *GeV/n* and impulse spread $\sigma_p = 0.37 \cdot 10^{-3} = \delta p_{sep} / 3$ necessary number of ions $N_0 = 55 \cdot 10^9$ can be accumulated in stationary regime ($\Delta t_{inj} >> \tau_{long}$ at the electron beam currents $I_e > 0.7\ A$. For these parameters $\varepsilon_x \approx 0.1$, $\tau_{long} \sim 10\,\sec$.

b)

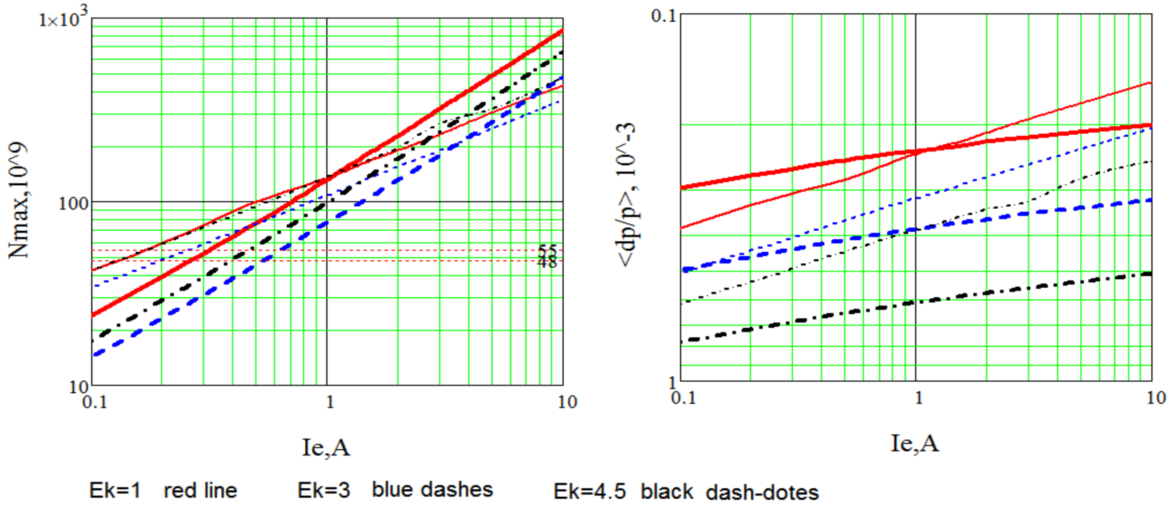


Fig. **Maximal stored number of ions** (at which $N_{loss} = N_{inj1}$) in stationary regime for ion energies E_k=1, 3, 4.5 *GeV/n* (red line, blue dashes, black dash-dotes), in dependence on electron beam current. Gaussian (thick lines) and nonlinear distribution (thin line) models.

At strong cooling maximal stored ion number is bigger for the gaussian model because the tails of the distribution which determine losses contain lesser number of ions than that for the nonlinear model. At weaker cooling the naximal ion number for gaussian model is less because it's impulse spread is greater and hence the losses are greater than for nonlinear model.

Necessary minimal electron beam current for accumulation of necessary number of ions at different energies (stationary regime).

E_k, *GeV/n*	*N0*,10^9	*Ie, A*		$\tau_{long\ av}$, sec	
		Nonlinear distr.	Gaussian distr.	Nonlinear distr.	Gaussian distr.
1	7	0.01	0.17		
	50	0.15	0.28		
3	55	0.25	0.65	80	22
4.5	48	0.14	0.4	1000	300

Thus, in stationary regime at energies 3 and 4.5 *GeV/n* one can accumulate necessary number of ions at the electron current *Ie*=0.65 *A*. with cooling time several tens of seconds at 3 *GeV/n* and several hundreds of seconds at 4.5 *GeV/n*. At 1 *GeV/n* the cooling time is small, and the regime of accumulation is stationary.

Accumulation of ions (for short time between injections)

Model of accumulation of ions for small time per 1 injection Δt_{inj} in comparison with cooling time (E_k=3 *GeV/n*)

One should take into account change of RMS parameters at each injection, at uniting separatrices at cooling between 2 consequental injections and at stack compression down till half of the storage ring perimeter with conserving longitudinal emittance:

Averaging : $\sigma_{p2}{}^2 = \dfrac{\sigma_{p1}{}^2 (N_1 - N_{loss}) + \sigma_{pi}{}^2 N_i}{N_1 - N_{loss} + N_i}$ (ε_x - similarly)

Cooling : $\sigma_{p2}{}^2 = \sigma_{p\,st}{}^2 + (\sigma_{p1}{}^2 - \sigma_{p\,st}{}^2) e^{-2\Delta t_{inj}/\tau_{av}}$ (ε_x - similarly)

Compression: $\sigma_{p2} = \sigma_{p1} \dfrac{L_1}{L_2}$ (ε_x does not change)

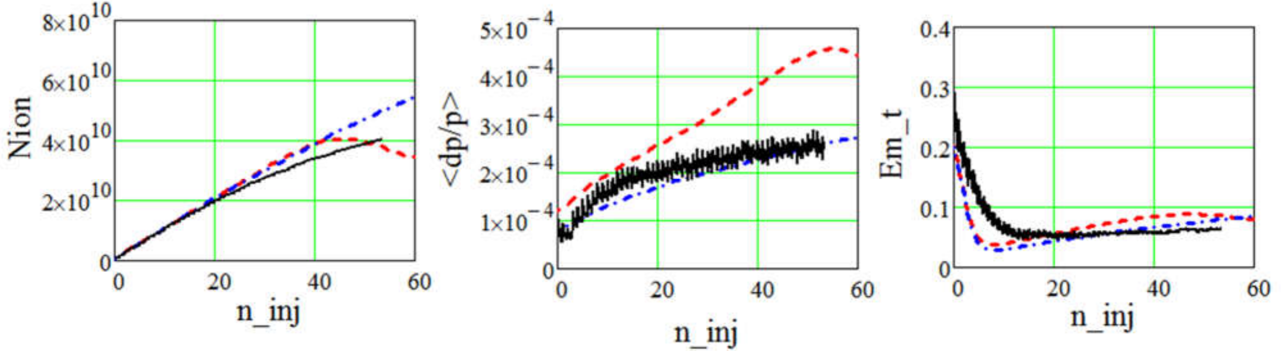
Losses: $\Delta N_{loss} = \dfrac{\varphi_{kick}}{\varphi_{bb}} N_0 \displaystyle\int_0^{\sqrt{\delta_{p\max}{}^2 - \delta_{psep}{}^2}} f_1\left(\sqrt{\delta_p{}^2 + \delta_{psep}{}^2}\right) d\delta_p$,

f_1 - for current value of σ_p .

Parameters of injected bunch: $\sigma_{si} = 10\ m$, $\sigma_{pi} = 1.2 \cdot 10^{-4}$, $N_i = 10^9$, $\Delta t_{inj} = 8 \sec$, $\varphi_{bb2} = 6\sigma_{si}$ - length of injection separatrix.

Results of calculation of accumulation of ions in dependence of number of injections

Energy E_k=3 *GeV/n,* electron current *Ie*= 1 *A*. Comparison with Betacool.



Black line - Betacool; red dashes - gaussian model; blue dash-dots - nonlinear model. *Ie=1A*:

50 injections => $4 \cdot 10^{10}$ (Betacool) / $4 \cdot 10^{10}$ (gaussian model) / $5 \cdot 10^{10}$ (nonlinear model) ions.

$5.5 \cdot 10^{10}$ **ions => ~80 inj.(Betacool)** / not reached (gaussian model) / **55 inj. (nonlinear model**).

Gaussian model, *Ie=1.25A*, 60 inj. => $5.5 \cdot 10^{10}$ **ions**.

Impulse spread for gaussian model sufficiontly greater than for nonlinear model and for Betacool.

The nonlinear model of 1-dimensional code is more close to Betacool solution than the gaussian model, but with overestimated cooling(~30%) or underestimated losses.

There remains a question if the stationary form of distribution has time to be formed or one should use this model with moving boundary of impulse distribution (under action of IBS and cooling) instead of a constant boundary defined by longitudinal acceptance.

Adiabatic capture and bunchung of ions (RF2, RF3)

Preparation of beams for ion-ion collision occurs in two stages. Firstly 22 bunches are produced using adiabatic capture technique at slowly increasing RF voltage.
When RF2 voltage reaches the maximum of 100 kV, the electron cooling is switched on for some time. When the bunch length becomes short enough due to cooling and RF2 maximal voltage, the voltage of the RF3 system working on the 66th harmonic (after a time of cooling) starts adiabatically increasing from 22 kV.

The maximal RF2 voltage together with cooling should provide conditions when the final longitudinal bunch length at interception into RF3 system voltage must be equal to the length completely fitting into the bucket of the RF3 system. However, a small number of ions at interception can be captured in the parasitic side separatrix of 66th harmonic. This leads to parasitic collisions in the Collider. The ratio of the number of captured ions in the side parasitic separatrix to the total number of bunch ions strongly depends on the rms bunch length after the RF2 bunching and cooling. Further adiabatic increase in the RF3 voltage together with cooling provides formation of an ion bunch with the length of 60 cm and momentum spread of 10^{-3} required for colliding experiments.

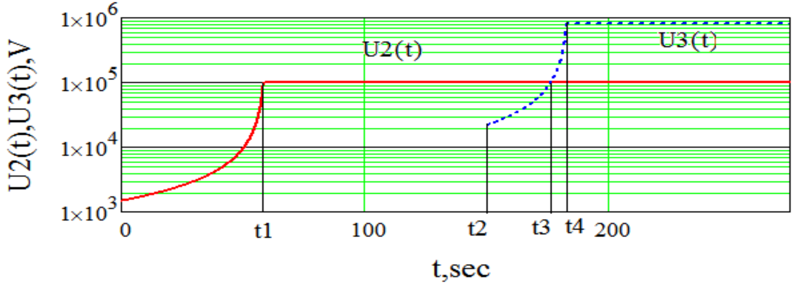


Fig. Amplitudes of voltages RF2, RF3 versus time. [t1,t2] - region of additional cooling after increassing voltage RF2 for 100% capture of ions into separatrix and further interseption into central separatrix of 66-th harmonic. t>t4 - region of additional cooling after increase of voltage RF3 for reaching final rms parameters of the beam.

Choice of starting voltages RF2, RF3.

Neglecting induced voltage, at constant generator current one can write $U_{\min} = U_{\max}\sqrt{\left(1+a_f^{\,2}\right)/\left(1+a_0^{\,2}\right)}$

Minimal required power of generator

$P_g(U_{\max}) \Rightarrow I_g \,||\, U_c \Rightarrow$ final normalized detuning $a_f = 2I_0R_s / U_{\max}$;

Starting normalized detuning $a_0 = \left(f_0/2\right)/\Delta f_r$.

For RF3 there is an additional condition - absence of static instability, i.e.

$U_{\min 3} > U_{ind3}(t_{start3}) = I_0 A_{66} R_{s3} / a_{30} = \left(11 \div 21\right) kV, \quad I_0 = 0.4A, \quad A_{66} = 1 \div 2$.

Combining these conditions at different energies, we get $U_{2\min} \approx 1.5kV$, $U_{3\min} \approx 22.5kV$.

Longitudinal tracking

1) The number of macroparticles Np=500-5000
2) Numerical integration of equations of motion in the limits of 1 separatrix (method of Runge-Kutta of 4-th order; variable time step proportional to the period of small synchrotron oscillations; arbitrary kicks $\sqrt{D(\sigma_{p,}\sigma_s)\Delta t}\cdot rnd_i$ to each macroparticle at each time step, for σ_p, σ_s calculated over current distribution of macroparticles.

Equations of motion:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\Delta p}{p}\right) = \dfrac{Zef_0\beta^2}{E_s}\big(U(\Delta\varphi,t) - U(0,t)\big) + \dfrac{1}{p}F_{e\,long\,av}\left(\dfrac{\Delta p}{p}\right) \\ \dfrac{d\Delta\varphi}{dt} = 2\pi q f_0\left(\alpha - \gamma^{-2}\right)\left(\dfrac{\Delta p}{p}\right) \end{cases}$$

$\varphi = q\theta, \quad \varphi \in [0, 2\pi]$, $\varphi_s = -\pi/2$, $\Delta\varphi = \varphi - \varphi_s$, $q = 22$,

$$U(\Delta\varphi,t) = U_2(t)\cos(\varphi_s + \Delta\varphi) + U_{sc}(\varphi) + \begin{cases} U_3(t)\cos(3\varphi_s + 3\Delta\varphi), & U_3(t) > 0, \\ U_{ind\,3}(t), & U_3(t) = 0, \end{cases}$$

$U_{2,3}(t)$ - amplitudes of voltages of RF2, RF3, when switched on.

$U_{sc}(\varphi)$ -voltage induced by the ions' space charge in the vacuum chamber:

$$U_{sc}(\varphi) = \frac{k_0 Ze}{\varepsilon_0} F(\sigma_b) \cdot \frac{Q_1}{\gamma^2} R_{ring} \lambda'(-\Delta\varphi R_{ring} / q_{22}),\ \ \lambda(z) = \frac{1}{\sqrt{2\pi}\sigma_b} e^{-z^2/2\sigma_b^2},\ \ F(\sigma_b) \le 2\ for\ \sigma_b \ge 0.5\ cm.$$

$U_{ind3}(t)$ - amplitude of the voltage induced by the ion beam current on the cavities of RF3 when RF3 is still switched off. It is calculated over current distribution of macroparticles:

$$\Delta U_{ind3}(\Delta\varphi, t) = \mathrm{Re}\left\{ I_3(t) Z_3(a_{30}) \left(\exp\left(i3\Delta\varphi \right) - 1 \right) \right\},$$

$$I_3(t) = -I_0 \frac{1}{N_p} \sum_{n=1}^{N_p} \Delta\varphi_n \exp\left(-i3\Delta\varphi_n \right), \quad \Delta\varphi = \varphi - \varphi_s.$$

3) **Compression of the process time** by the factor 0.1-0.001, together with times of cooling and IBS growth time, while synchrotron frequencies being unchanged. It means decrease of the number of fast oscillations by the factor 0.1-0.001, keeping them still fast in comparison with slow processes of cooling, IBS growth and adiabatic increase of voltage amplitudes.

Approximation.

Rms length and rms impulse spread of the bunch well inside separatrix at adiabatic increase of the voltages are connected as

$$\begin{cases} \sigma_{p1}\sigma_{s1} = \sigma_{p2}\sigma_{s2}, \\ \sigma_{p1} = k\Omega_{s1}\sigma_{s1}, \\ \sigma_{p2} = k\Omega_{s2}\sigma_{s2} \end{cases} \Rightarrow \quad \frac{\sigma_{p1}}{\sigma_{p2}} = \frac{\sigma_{s2}}{\sigma_{s1}} = \sqrt{\frac{\Omega_{s1}}{\Omega_{s2}}}$$

But for capture of the beam initially fully out of separatrix
These conditions change:

$$\begin{cases} 2\sigma_{p0}P_{ring} / q_2 = \pi\sigma_{p1}\sigma_{s1}, \\ \sigma_{s1} = \dfrac{P_{ring} / q_2}{2\sqrt{3}} \\ \sigma_{p1} = k\Omega_{s1}\sigma_{s1} \end{cases} \Rightarrow \quad \sigma_{p1} = \sqrt{k\Omega_{s1}\sigma_{p1}\sigma_{s1}} = \sqrt{k\Omega_{s1}\frac{2}{\pi}\sigma_{p0}P_{ring} / q_2}$$

These relations together with increase of rms impulse spread due to diffusion and estimation of cooling to the stationary value given above give a rough approximation for the check of results of tracking and for scaling some dependencies on the parameters.

Stationary parameters

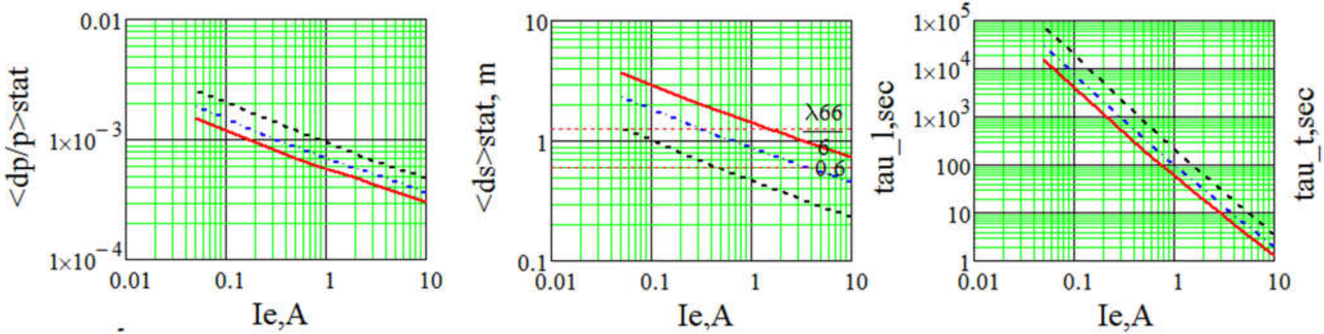


Fig. Stationary rms impulse spread and rms length versus electron beam current; averaged longitudinal and transverse cooling times at stationary parameters.

Red line - at $U_{2\max}, U_3 = 0$. It defines bunch length before increasing U_3: at $I_e = 1A$ $\sigma_{s1st} = 1.425\ m$.

Blue dash-dots - at $U_{2\max}, U_3 = U_{2\max}$ (arising the side separatrices). Comparing this bunch length with $\lambda_{66} / 6 = 1.27\ m$ - 1/6 of wavelength of 66-th harmonic defines number of ions in side separatrices. (at $I_e > 0.3A$ $\sigma_{s2st} < \lambda_{66} / 6$).

Black dashes - at $U_{2\max}, U_{3\max}$. These stationary parameters define final parameters after cooling. $\sigma_{s3st} < 0.6\ m$ - necessary for experiment. It can be achieved at $I_e > 0.5A$.

Results of calculation of beam capture with RF2

Below are the results of calculation of capture and bunching of ion beam with described above 1-dimentional tracking code . Initial parameters: $\sigma_{p0} = \left(\delta_{p\,sep}\right)_{RF1} / 3,\quad \varepsilon_{x0} = 0.1\pi \cdot mm \cdot mrad$ (values after accumulation), $I_e = 1A$.

The calculations are fulfilled for the rate of detuning change $da / dt = 3\sec^{-1}$ time of tuning for RF2 ~60 sec and for RF3 - ~30 sec. After reaching maximal voltages of RF2 and RF3 cooling is switched on to reach necessary parameters.

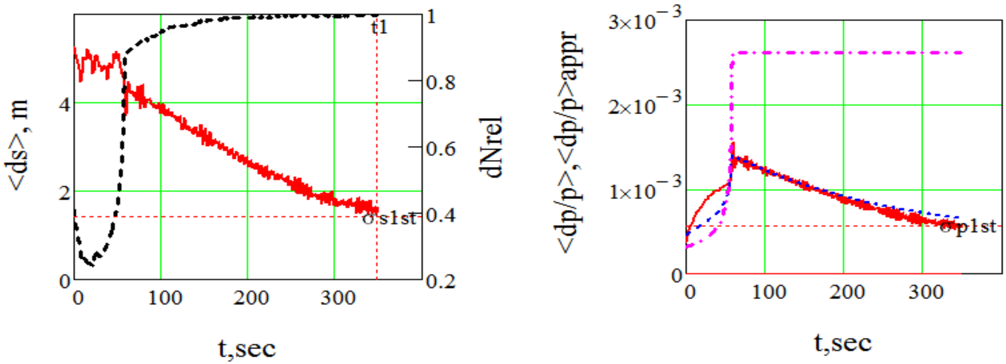


Fig. Left: rms bunch length versus time (red line) and relative number of ions captured into separatrix. (black dashes). Right: rms impulse spread versus time, tracking (red line) and approximation (blue dashes); separatrix amplitude (pink dash-dotes).

A beam can be cooled bis $\sigma_{s\,st} = 1.425\ m$ with the electron beam current $I_e = 1A$.

Induced voltage by the ion beam current voltage on the cavities of RF3 (66-th harmonic).

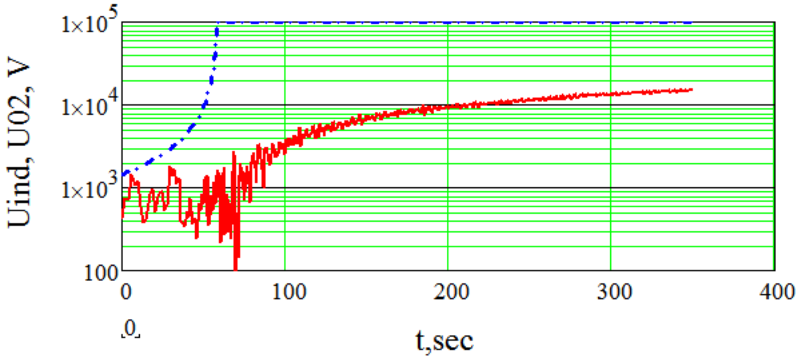
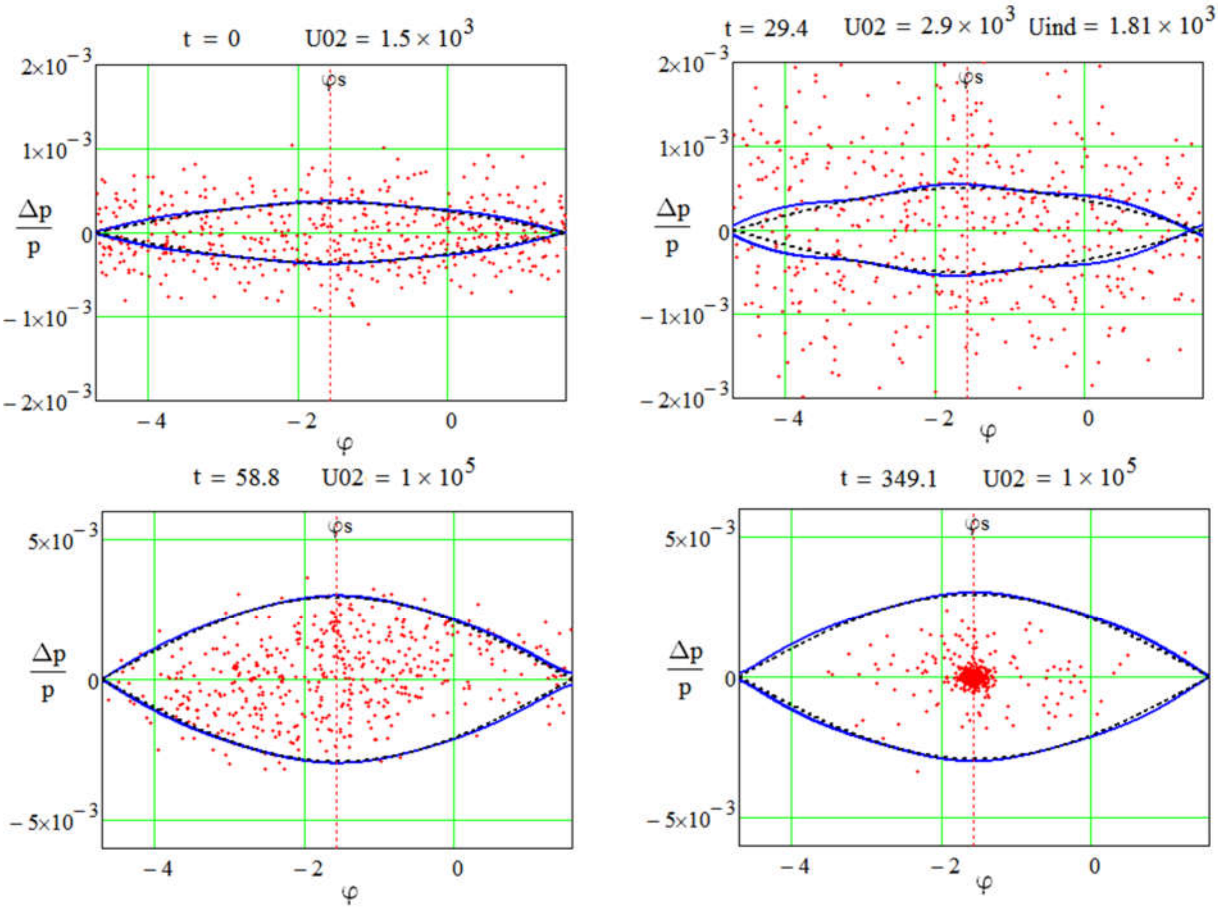


Fig. Amplitude of voltage RF2 (22-nd harmonic) and of induced voltage.
Induced voltage is small in comparison with voltage of RF2, so it practically does not influence the dynamics of the ions.

Fig. (below): Phase portret of the beam during capture. 1) *t=0* - at the start of capture, $U_2 = U_{2\min}$, $U_{ind} = 0$; 2) *t=29.4* $U_{ind} \sim U_2$, maximal distortion of the separatrix; 3) $U_2 = U_{2\max}$, start of cooling; 4) $U_2 = U_{2\max}$, end of cooling, 100% capture of ions into separatrix.



t = 0 U02 = 1.5 × 10^3
t = 29.4 U02 = 2.9 × 10^3 Uind = 1.81 × 10^3
t = 58.8 U02 = 1 × 10^5
t = 349.1 U02 = 1 × 10^5
φs
Δp/p
φ
2×10^-3
1×10^-3
0
− 1×10^-3
− 2×10^-3
5×10^-3
− 5×10^-3
− 4
− 2
0

Further bunching with RF2+RF3.

Increase of voltage of RF3 leads to formation of 3 separatrixes of 66-th harmonic instead of 1 separatrix of 22-nd harmonic, with further compression of the bunch length, ideally bis $\sigma_s < \lambda_{66} / 6$, so that the bunch as a whole is located in the cenral separatrix, and the side separatrices contain a small share of ions. The goal is to minimize this share.

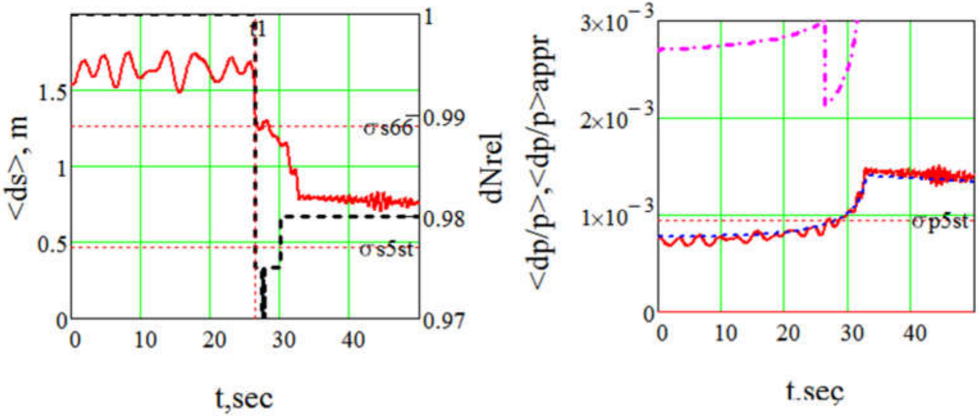
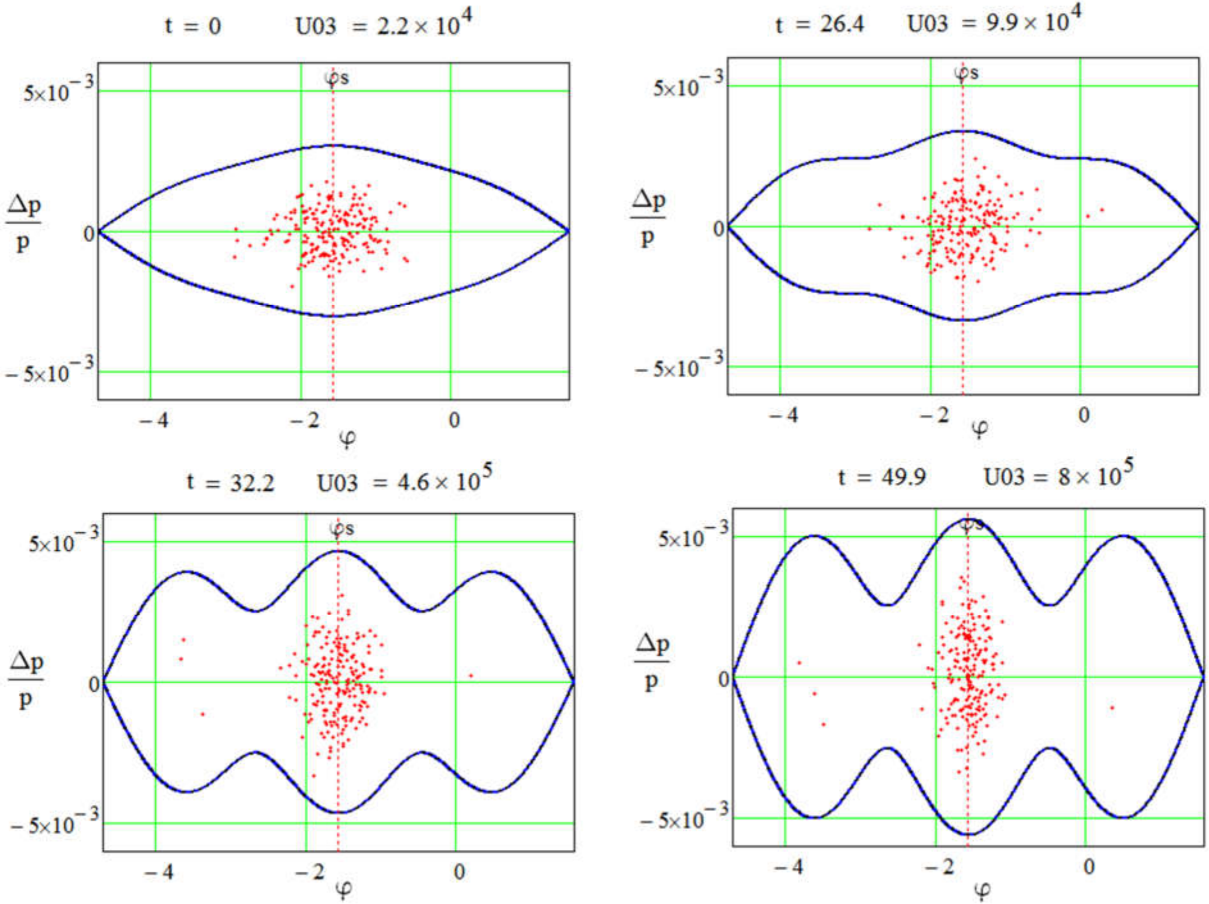


Fig. Left: rms bunch length versus time (red line) and relative number of ions captured into the central separatrix. (black dashes). Right: rms impulse spread versus time, tracking (red line) and approximation (blue dashes); separatrix amplitude (pink dash-dotes).

Fig. (below): Phase portret at increasing voltage of RF3. 1) *t=0* at the start, $U_3 = U_{3\min}$; 2) t_1*=26.4* $U_3 = U_{2\max}$, arising side separatrices; 3) *t=32.2* $U_3 = U_{3\max}$; 4) after cooling.



t = 0 U03 = 2.2 × 10^4
t = 26.4 U03 = 9.9 × 10^4
t = 32.2 U03 = 4.6 × 10^5
t = 49.9 U03 = 8 × 10^5
φs
5×10^−3
−5×10^−3
0
Δp/p
−4
−2
0
φ

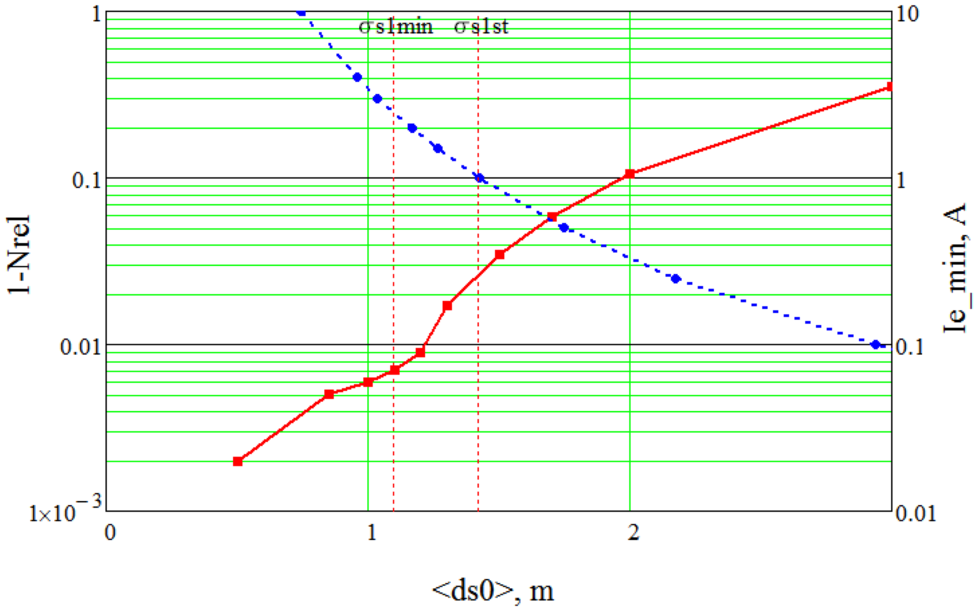


Fig. Relative number of particles outside the central separatrix versus rms bunch length at the start of increasing voltage of RF3 (red line) and minimal electron beam current at which this length could be reached.

At $\sigma_{s0} = 1.5m$ 2.5% of ions are outside the central separatrix. In order to decrease this number, one should increase the electron current. At $I_e = 1.5\ A$ $\sigma_{s1st} = 1.2m$, ~1% of ions are outside the central separatrix.

Comparison with Betacool.

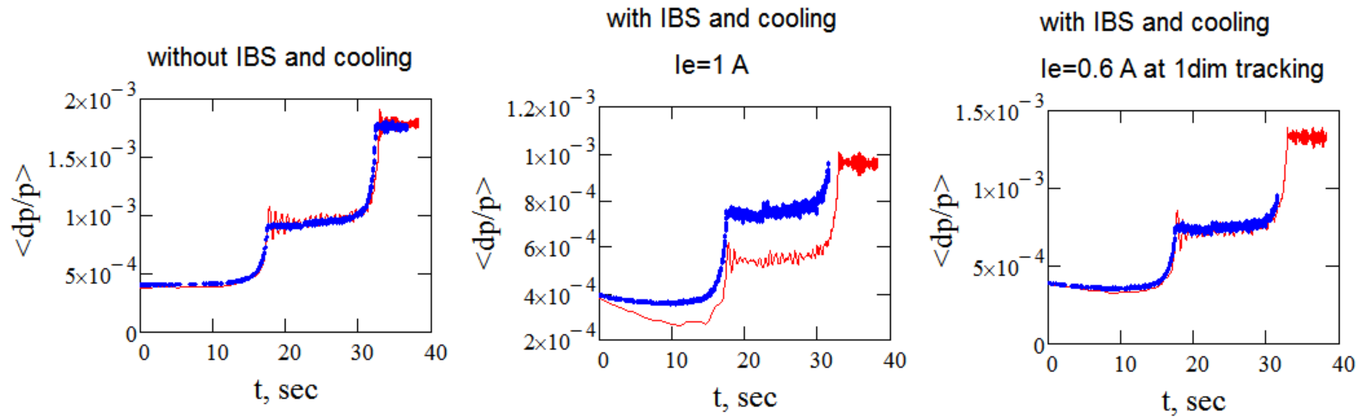


Fig. Impulse spread versus time at capture and bunching of ions. Red thin line - 1-dimentional tracking code, blue thick dashes - Betacool. Left - without cooling and IBS (times of calculation of one order); central - with cooling ($I_e = 1A$) and IBS (Betacool requires ~15 times more time); right - 1-dimentional tracking with IBS and lesser cooling ($I_e = 0.6A$), Betacool result - the same ($I_e = 1A$).

Number of ions in side separatrices:

	without cooling and IBS	with cooling and IBS
1-dimentional tracking ($I_e = 1A$)	10.4%	3.2%
1-dimentional tracking ($I_e = 0.6A$)		4.8%
Betacool	9.6%	5.1%

Conclusion.

1. $5.5\cdot 10^{10}$ ions can be accumulated in 55 injections at $I_e = 1\ A$ (nonlinear model) or in 60 injections at $I_e = 1.25\ A$; maximal number $4\cdot 10^{10}$ in 50 injections (gaussian model). Betacool calculations give $4\cdot 10^{10}$ in 50 injections and predict $5.5\cdot 10^{10}$ ions in ~80 injections (extrapolation).

2.At $I_e = 1\ A$ 2.5% of ions are outside the central separatrix. In order to decrease this share till 1% the electron current should be increased at least up to $I_e = 1.5\ A$.

3. Final parameters ($\sigma_{sf} = 0.6\ m$) can be reached at electron current $I_e > 0.5\ A$.

4. At comparison with Betacool one can see that stationary solutions have sufficient accordance, but the time-dependent solutions have a certain difference. It looks like cooling in 1 dimensional tracking is overestinated ~1.7 times. The side separatrices contain less than 1% at $I_e = 1.5\ A$ (1-dimentional tracking). For Betacool this current should be greater ~1.7 times.

So, the 1-dimentional approach codes can be used for estimation of dependences on variating parameters, but final calculation requires more accurate 3D calculation.

THANKS FOR YOUR ATTENTION.