Recent results from MICE on multiple Coulomb scattering and energy loss

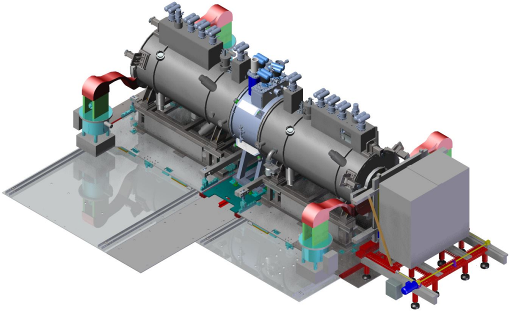
John Nugent
on behalf of the MICE Collaboration

University of Glasgow

john.nugent@glasgow.ac.uk

18/9/2017

The MICE Experiment: Step IV



Ionization Cooling

The rate of change of normalised emittance due to ionization cooling is:

$$\frac{d\varepsilon_n}{dz} \approx -\frac{\varepsilon_n}{{\beta_{\mathrm{rel}}}^2 E}\left\langle\frac{dE}{dz}\right\rangle + \frac{\beta(13.6\ \mathrm{MeV})^2}{2{\beta_{\mathrm{rel}}}^3 E m X_0} \quad (1)$$

Overview of multiple Coulomb scattering

- The PDG recommends this formula, based on work by Lynch and Dahl [1, 2] incorporating path length effects, (accurate to $\sim$11%)

$$\theta_0 \approx \frac{13.6\,\mathrm{MeV}}{p_\mu \beta_{\mathrm{rel}}} \sqrt{\frac{\Delta z}{X_0}} \left[1 + 0.038 \ln\left(\frac{\Delta z}{X_0}\right)\right] \tag{2}$$

- Goal of MICE is to measure $d\varepsilon_n/dz$ to precision of 0.1%
- MUSCAT [3] showed poor agreement between theory and low Z material scattering data
- MICE has taken scattering data for muons on a LiH target.
  - LiH composition: 81% ^{6}Li, 4% ^{7}Li, 14% ^{1}H (trace of C, O, and Ca)

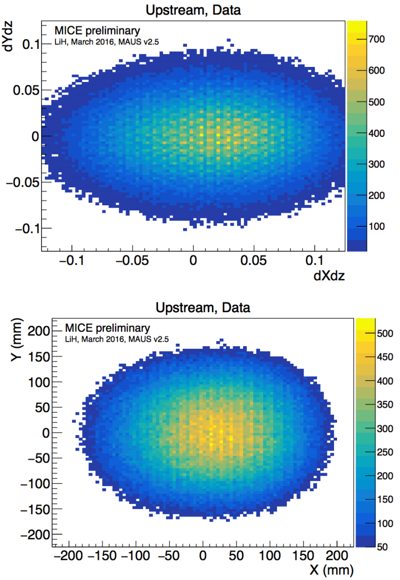
Overview of multiple Coulomb scattering

- GEANT4, full Legendre polynomial expansion & evaluates the Urban cross-section [4] for most particles and the Wentzel single-scattering cross-section for muons.
- Moliere [5] calculation solves the scattering transport equation describing scattering with a single variable χ_a
- ELMS covering both energy loss and multiple scattering (ELMS) based on electromagnetic first principles, was developed by Allison and Holmes [6, 7] and shows good agreement with hydrogen data.
- Cobb-Carlisle model [8, 9], samples directly from the Wentzel single-scattering cross-section and simulates all collisions with nuclei and electrons. Cut-off for the nuclear cross-section and seperate contributions from the nuclear and atomic electron scattering

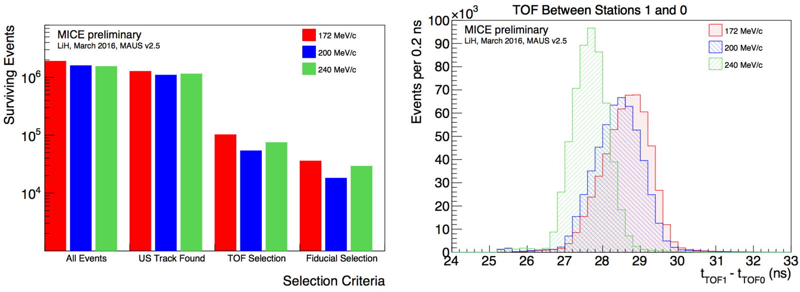
$$\frac{\theta_0^2}{z} = 8\pi N_A \frac{Z^2}{A} r_e^2 \left(\frac{mc}{p_\mu \beta} \left[\ln\left(\frac{\theta_2}{\theta_1}^2 + 1 \right) - 1 \right] + \frac{1}{Z} \left[\ln\left(\frac{\theta_2^e}{\theta_1^e}^2 + 1 \right) - 1 \right] \right) \quad (3)$$

Scattering Data

- Field off data sets were collected in ISIS run periods 2015/03 and 2015/04
- A momentum dependent multiple scattering measurement is made
  - Measure empty channel scattering
  - Convolved with physics model of scattering in absorber - prediction.
  - Measure absorber scattering
  - A Bayesian deconvolution algorithm unfolds absorber scattering distribution
  - χ^2 comparison between data and prediction
  - Width of scattering distribution: Θ as a function of P



Selection

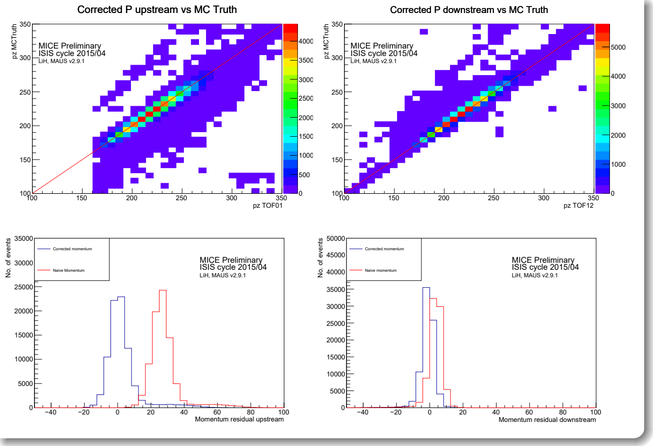


Procedure

- Require a US track. If a DS track not extant, statistics are set to overflow values.
- Analysis done in 200 ps bins, as shown in TOF plot
- Require projection of US tracks to appear, when 12 mrad radial angle is added, within central 140 mm radius of DS plane 1 projected

Momentum Correction

A correction must be applied to the P as reconstructed by the TOF to account for the additional path length and energy loss in the channel



- The exact P at the centre of the absorber can be described by an analytic expression which is the second order expansion of the Taylor series in p/mc
- Caveat is constant energy loss is assumed in derivation

Scattering Data

- Define projection angles

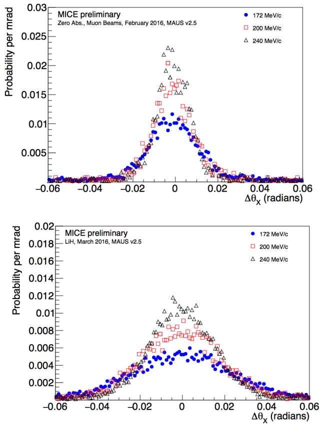
$$\theta_y = \text{atan}\left(\frac{p_{DS} \cdot (\hat{y} \times p_{US})}{|\hat{y} \times p_{US}||p_{DS}|}\right) \quad (4)$$

and

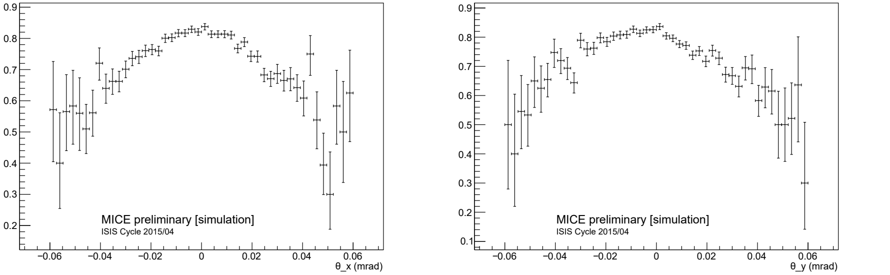
$$\theta_x = \text{atan}\left(\frac{p_{DS} \cdot (p_{US} \times (\hat{y} \times p_{US}))}{|p_{US} \times (\hat{y} \times p_{US})||p_{DS}|}\right) \quad (5)$$

- Where $\theta_x^2 + \theta_y^2 \approx \theta_{scatt}^2$ with

$$\cos\theta_{scatt} = \frac{p_{US} \cdot p_{DS}}{|p_{US}||p_{DS}|} \quad (6)$$



Tracker Acceptance

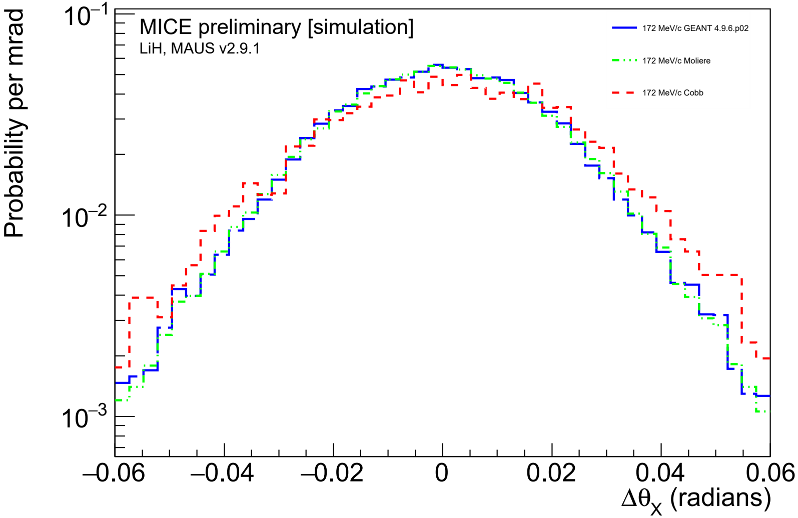


- Match track upstream and downstream
- TOF selection
- Calculate angle θ as per analysis
- Downstream acceptance is defined

$$\frac{\text{No. of tracks in } \theta \text{ bin MC Truth that are reconstructed}}{\text{No. of tracks in } \theta \text{ bin MC Truth}} \tag{7}$$

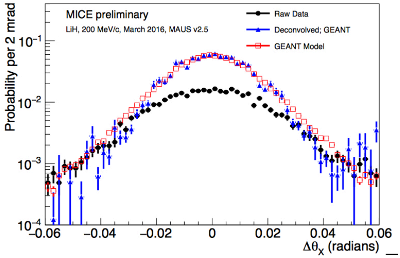
Physics Model

Three different physics models are used to make the scattering prediction, GEANT4, Carlisle-Cobb & Moliere



Deconvolution of Raw Scattering Data

- Use an iterative algorithm that uses the conditional probability to characterize the response of the reconstructed scattering angle to the true scattering angle
- Right: example output from this algorithm



Bayes Theorem

$$P(C_i|E_j) = \frac{P(E_j|C_i)P_0(C_i)}{\sum_{l=1}^{n_c} P(E_j|C_l)P_0(C_l)}$$

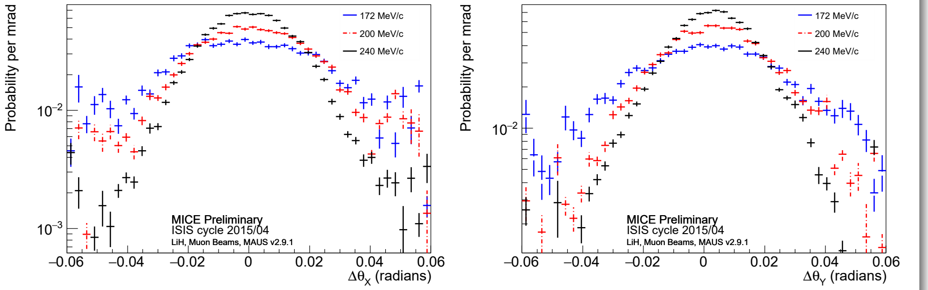
- We want $C_i = \Delta\theta_Y^{abs}$ the deflection angle in the absorber material.
- We measure $E_j = \Delta\theta_Y^{tracker}$ the deflection angle measured at the first tracker plane.

Systematics

- A study of the systematics is in progress
- The results remain preliminary
- Several sources have been considered
  - Material thickness uncertainties
  - Alignment uncertainties
  - TOF uncertainties
  - Fiducial volume uncertainties
- Further work is required to clarify the various contributions

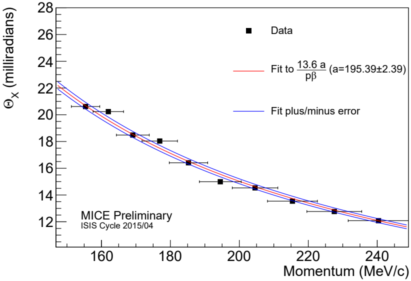
Results slide - deconvolution

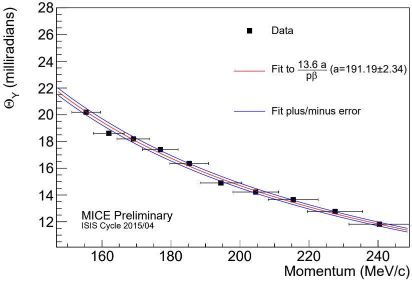
Preliminary MICE result



- Measurement of scattering at each nominal momentum point following the deconvolution proceedure - final value is a gaussian fit to the central -40 to +40 mrad

Θ as a Function of Momentum





- Scan across the entire momentum range and measure scattering in both projections in each bin
- Comparison with PDG formula is made and the fit is made for $a = \sqrt{\frac{z}{X_0}}(1 + 0.038 \ln \frac{z}{X_0})$

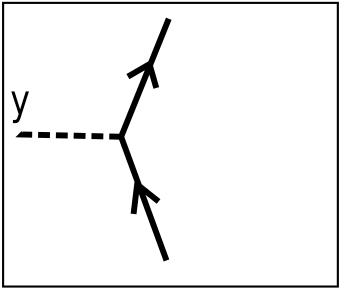
Conclusions

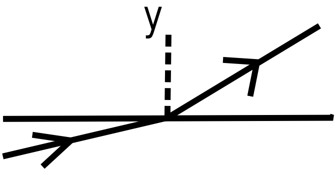
- MICE has measured multiple Coulomb scattering of μ with $140 < P < 240$ MeV/c off lithium hydride
- Data has been compared to popular simulation packages such as GEANT4 and other relevant models such as Moliere and Carlisle-Cobb
- A study of the systematics is in progress, a MICE publication is currently being prepared
- Future work will including a measurement of multiple Coulomb scattering off liquid hydrogen, measurement with magnetic field in the cooling channel and energy loss measurement

Scattering Data

Scattering Angle Definitions

- In the top diagram both the solid vectors are in the plane of the square i.e. the plain of the board. The y-axis is coming out of the board
- If both the up- and downstream vector were in the same plane then the subtraction of the simple projected angle would be sufficient
- The bottom figure is a side on view of the top figure. If the up- and downstream vectors are in two different planes then a more consider apporach is required as detailed in `http://www.ppe.gla.ac.uk/~jnugent/Projected-angles.pdf` by John Cobb





[1] K. A. Olive et al. Review of Particle Physics. *Chin. Phys.*, C38:090001, 2014.

[2] Gerald R. Lynch and Orin I. Dahl. Approximations to multiple Coulomb scattering. *Nucl. Instrum. Meth.*, B58:6–10, 1991.

[3] D. Attwood et al. The scattering of muons in low Z materials. *Nucl. Instrum. Meth.*, B251:41–55, 2006.

[4] S. Agostinelli et al. GEANT4: A Simulation toolkit. *Nucl. Instrum. Meth.*, A506:250–303, 2003.

[5] G. Moliere. Theory of the scattering of fast charged particles. 2. Repeated and multiple scattering. *Z. Naturforsch.*, A3:78–97, 1948.

[6] W. W. M. Allison. Calculations of energy loss and multiple scattering (ELMS) in molecular hydrogen. *J. Phys.*, G29:1701–1703, 2003.

[7] Simon Holmes. The Physics of Muon Cooling for a Neutrino Factory. *DPhil thesis, University of Oxford*, 2006.

[8] Timothy Carlisle. Step IV of the Muon Ionization Cooling Experiment (MICE) and the multiple scattering of muons. *DPhil thesis, University of Oxford*, 2013.

[9] T. Carlisle, J. Cobb, and D. Neuffer. Multiple Scattering Measurements in the MICE Experiment. *Conf. Proc.*, C1205201:1419–1421, 2012.