Progress on Parametric-resonance Ionization Cooling (PIC)

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Outline

- Motivation
- PIC concept
- Cooling channel design
- PIC simulations
- Aberration compensation
- Skew PIC
- Future plans

Motivation

- A factor of up to 100 luminosity increase due to lower transverse emittances expected in PIC and REMEX would make a **Muon Collider s-channel Higgs Factory** a very compelling new project

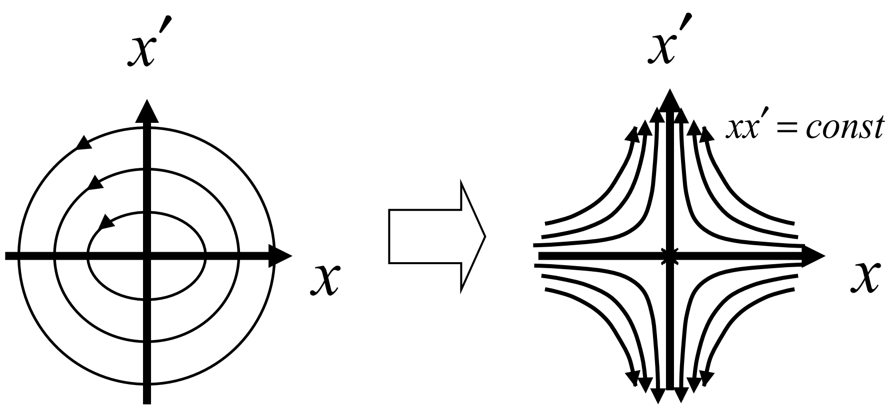
- Lower required muon beam current
  - Lower power proton driver
  - Reduced site boundary radiation
  - Reduced detector background
  - Reduced proton target requirements
  - Reduced heating of the cooling absorbers
  - Lower beam loading and wake field effects in RF cavities

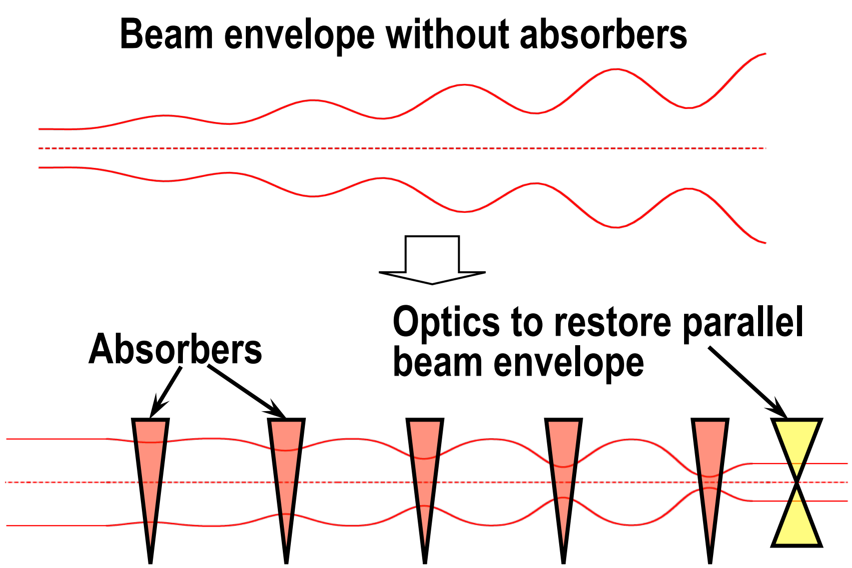
- Beyond reducing the required muon beam currents
  - Smaller higher-frequency RF cavities with higher gradient
  - Smaller magnet and vacuum system apertures
  - Stronger focusing at the IP

PIC Concept

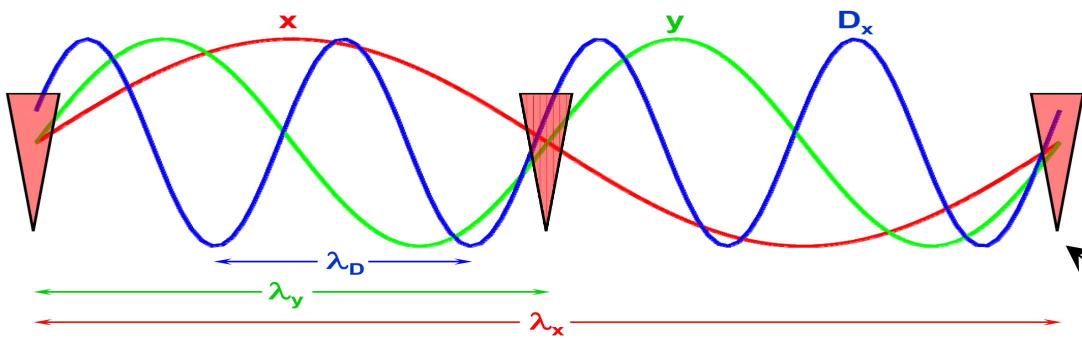
Muons, Inc.

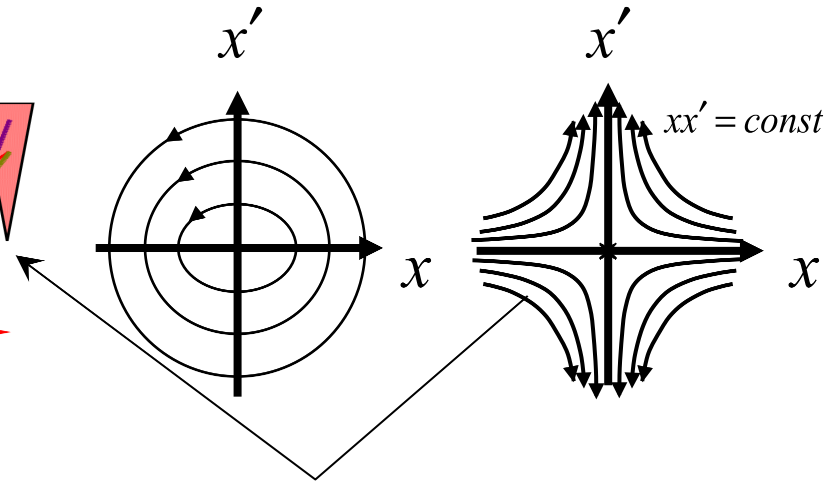
- Half-integer parametric resonance induced in muon cooling channel
- Muon beam naturally focused with period of free betatron oscillations
- Absorber plates placed at focal points and followed by energy-restoring RF cavities
  - Parametric resonance causes strong beam size reduction
  - Ionization cooling maintains constant angular spread
  - Emittance exchange at absorbers (wedges + *D* or tilted flat plates + *D'*) produces longitudinal cooling
- Equilibrium transverse emittances an order of magnitude smaller than in conventional ionization cooling





PIC Schematic





Correlated optics: $\lambda_x = 2\lambda_y = 4\lambda_D$

- **Equilibrium angular spread and beam size at absorber**

$$\theta_a^2 = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}, \quad \sigma_a = \frac{1}{2\sqrt{3}}\theta_a w$$

- **Equilibrium emittance**

$$\varepsilon_n = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w$$

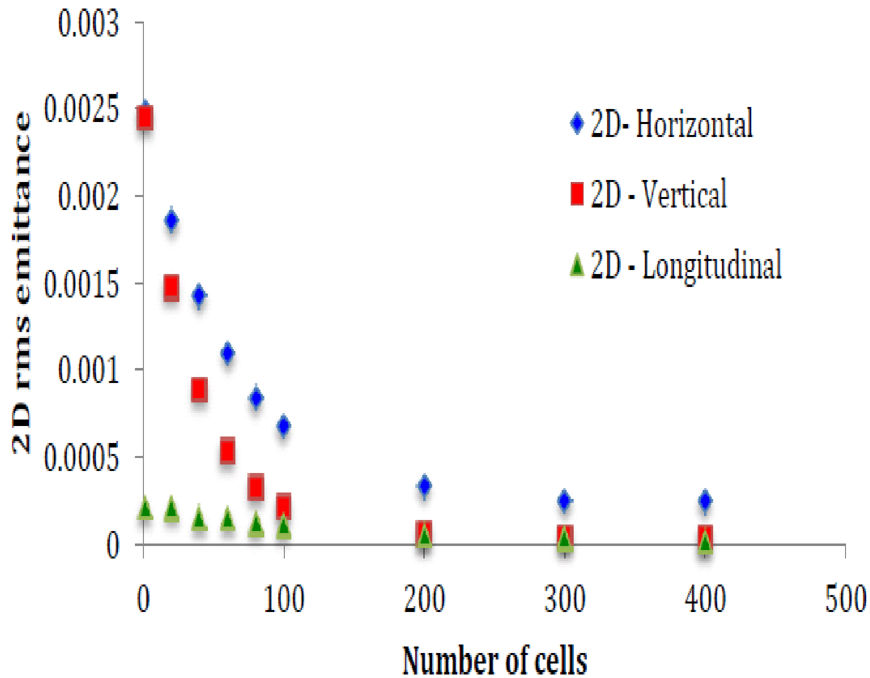
improvement by a factor of

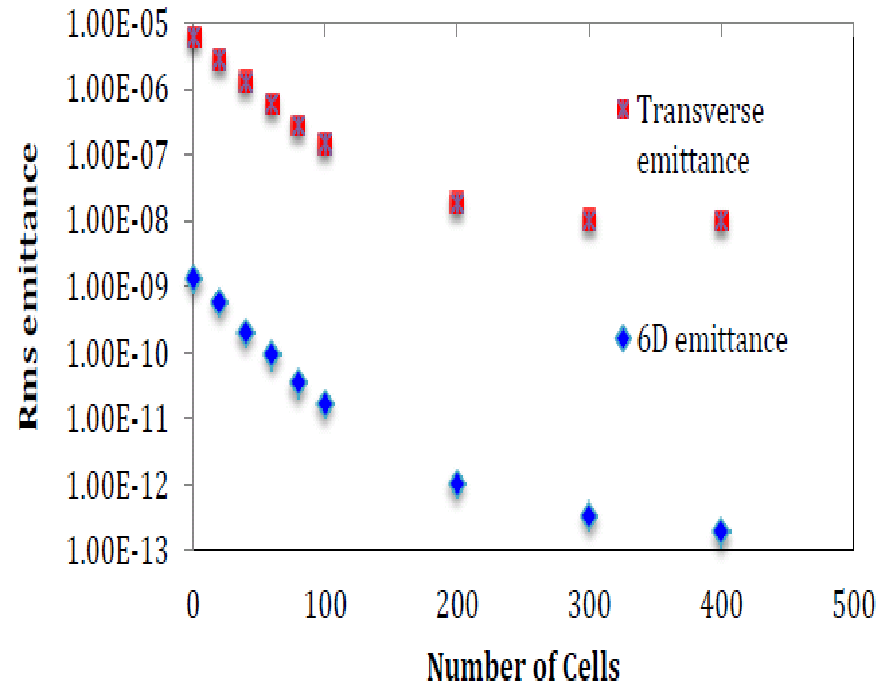
$$\frac{\pi}{\sqrt{3}}\frac{w}{\lambda} = \frac{\pi}{2\sqrt{3}}\frac{\gamma'_{acc}}{\gamma'_{abs}}$$

Parameter	Unit	Initial	Final
Muon beam momentum, *p*	MeV/c	250	250
Number of particles per bunch, N_b	10^{10}	1	1
Be (*Z* = 4) absorber thickness, *w*	mm	20	2
Normalized transverse emittance (rms), $\varepsilon_x = \varepsilon_y$	μm	230	23
Beam size at absorbers (rms), $\sigma_a = \sigma_x = \sigma_y$	mm	0.7	0.1
Angular spread at absorbers (rms), $\theta_a = \theta_x = \theta_y$	mrad	130	130
Momentum spread (rms), $\Delta p/p$	%	2	2
Bunch length (rms), σ_z	mm	10	10

PIC Simulation

- Linearized orbital motion
- Full stochastics
- 6D emittance reduced by 2 orders of magnitude after ~100 periods

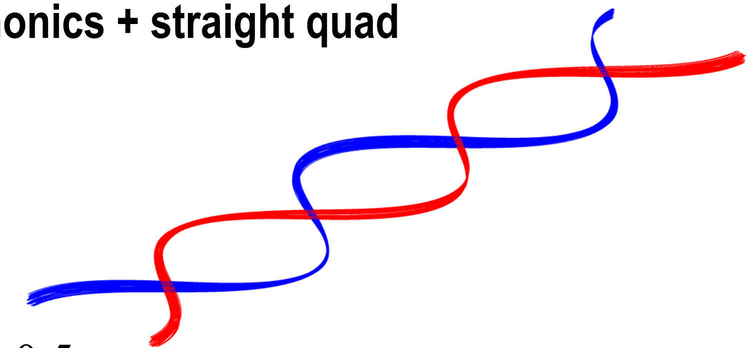


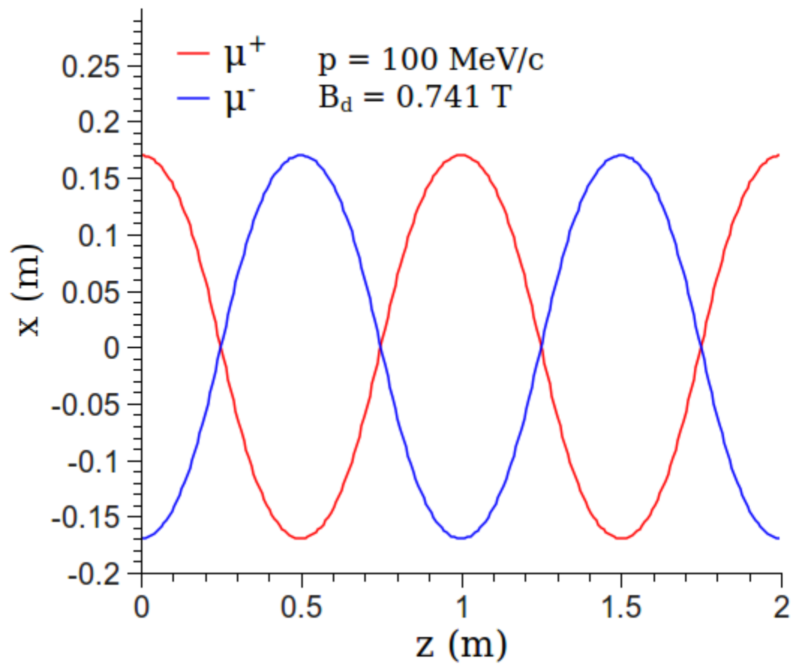


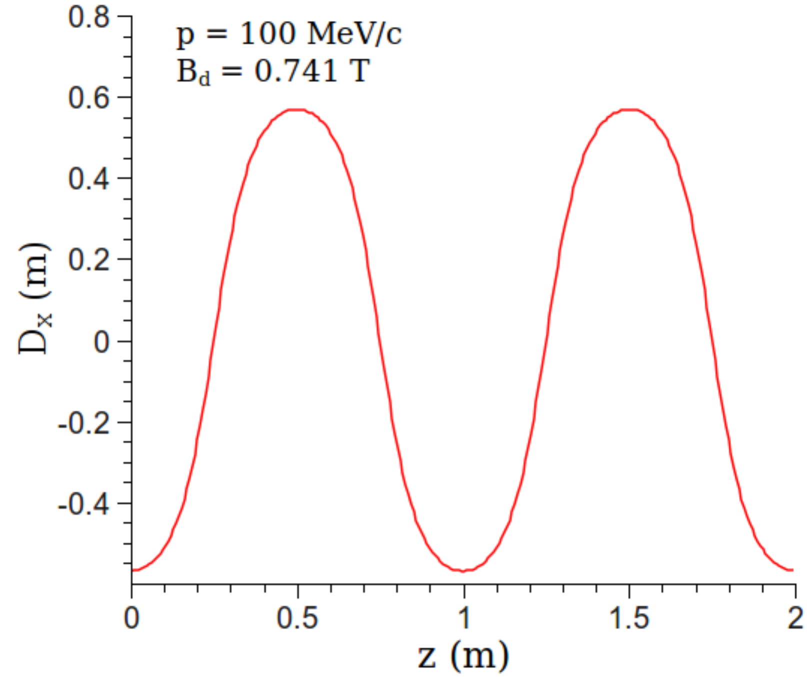
PIC Channel: Twin Helix

Muons, Inc.

- Two equal-strength opposite-helicity helical dipole harmonics + straight quad
- No fringe fields
- Orbit in the horizontal plane
  Horizontal and vertical motion uncoupled
  Transverse motion stable in both planes
  [illegible]$_D$ = [illegible] [illegible] [illegible]$_x$ = 2[illegible]$_y$ = 4[illegible] [illegible] [illegible]$_x$ = 0.25, [illegible]$_y$ = 0.5
- One may even think about both muon signs in the same cooling channel

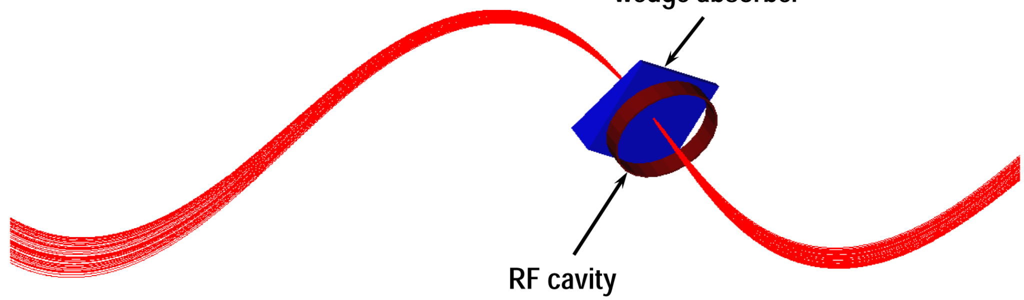




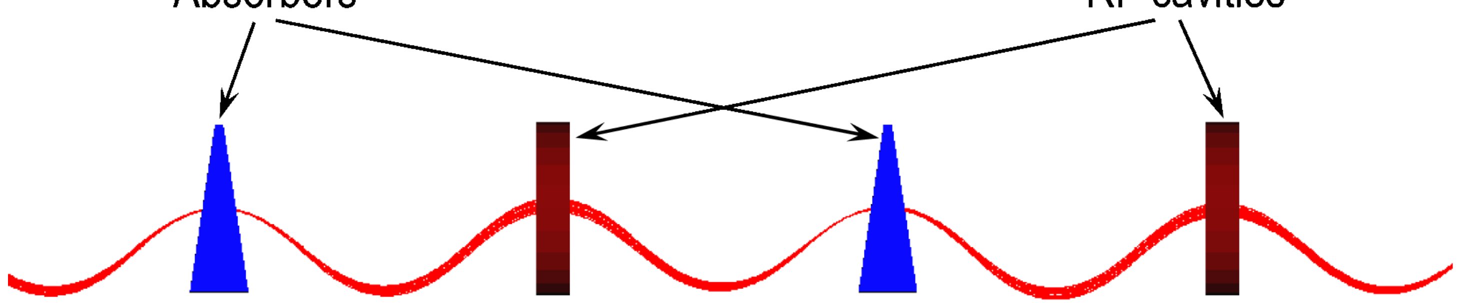


G4beamline Simulation Setup

- Parasitic parametric resonance due to energy kicks correlated with betatron motion
- With an asymmetric setup, compensation complicated



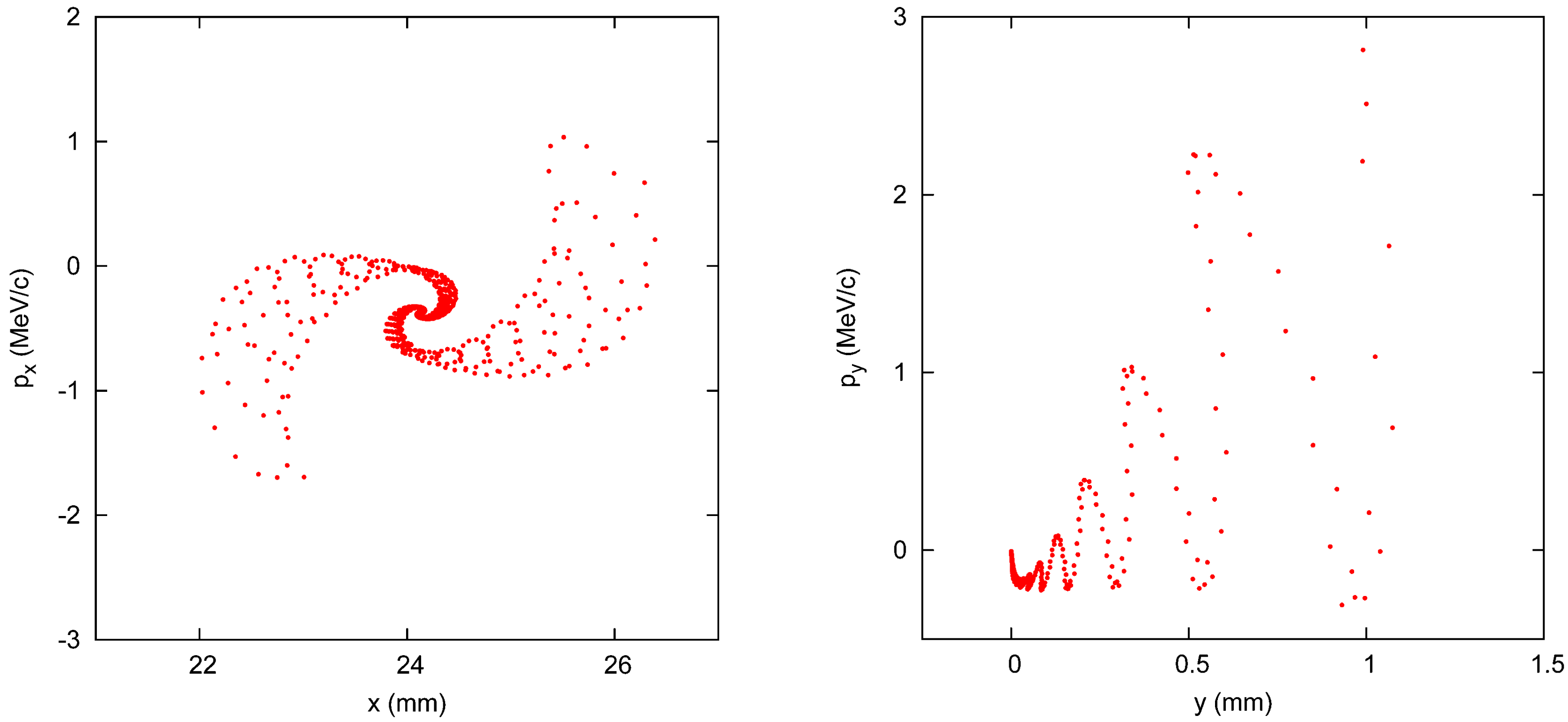
- With symmetric setup, parasitic resonance exactly out of phase with induced one
  - 0.2 m helix period
  - 2 cm Be wedges with 0.3 thickness gradient at $D_{x\,max}$ ☒ 4 cm
  - Short RF cavities placed symmetrically between the absorbers



Phase Space Dynamics

Muons, Inc.

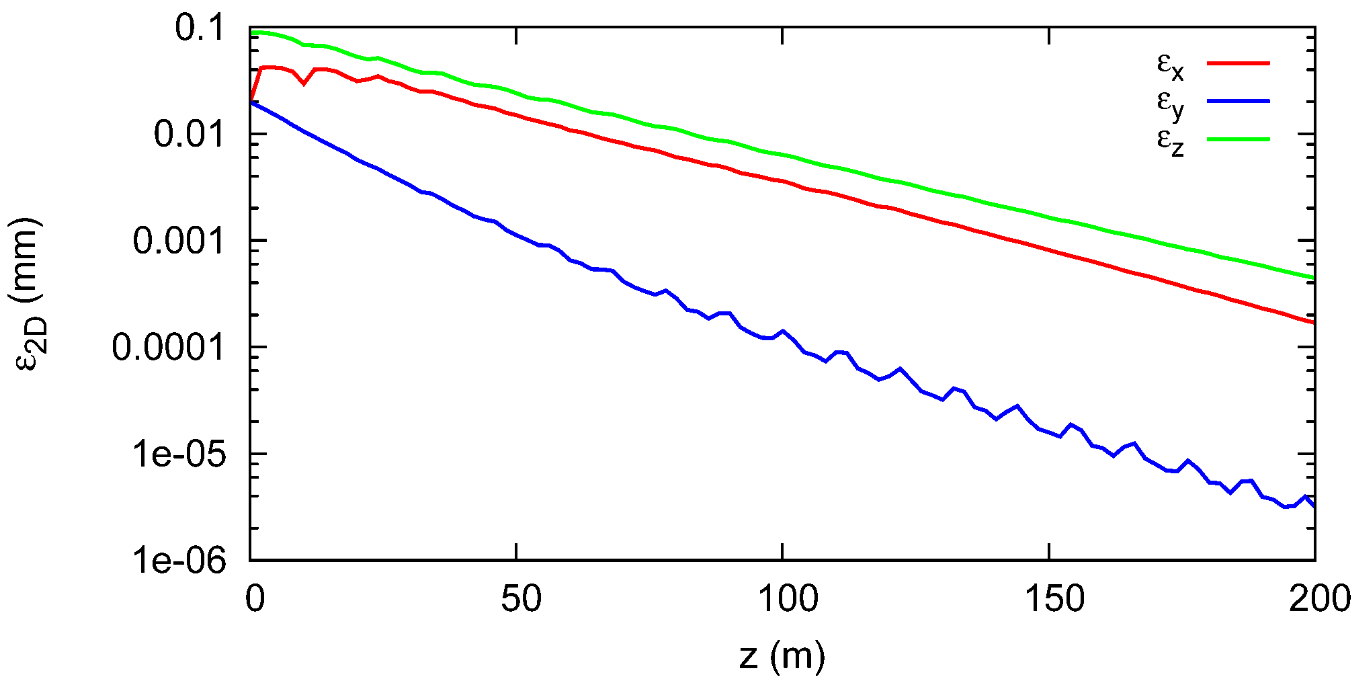
- G4beamline used to track 250 MeV/c muons
- Stochastics: off
- Parametric resonances induced in both planes by two pairs of 1 T/m helical quadrupoles



Emittance Evolution

Muons, Inc.

- G4beamline used to track one thousand 250 MeV/c muons
- 1000 helix periods (500 absorbers/RF cavities)
- Stochastics: off
- Parametric resonances induced in both planes by two pairs of 1 T/m helical quadrupoles
- $\varepsilon_x \equiv \sqrt{\langle \Delta x^2 \rangle \langle \Delta p_x^2 \rangle - \langle \Delta x \Delta p_x \rangle^2} / p, \quad \varepsilon_y \equiv \sqrt{\langle \Delta y^2 \rangle \langle \Delta p_y^2 \rangle - \langle \Delta x \Delta p_y \rangle^2} / p, \quad \varepsilon_z \equiv c\sqrt{\langle \Delta t^2 \rangle \langle \Delta p_z^2 \rangle - \langle \Delta t \Delta p_z \rangle^2} / p$
- Particle transmission 100%
- Cannot turn full stochastics on yet due to beam aberrations



Aberrations

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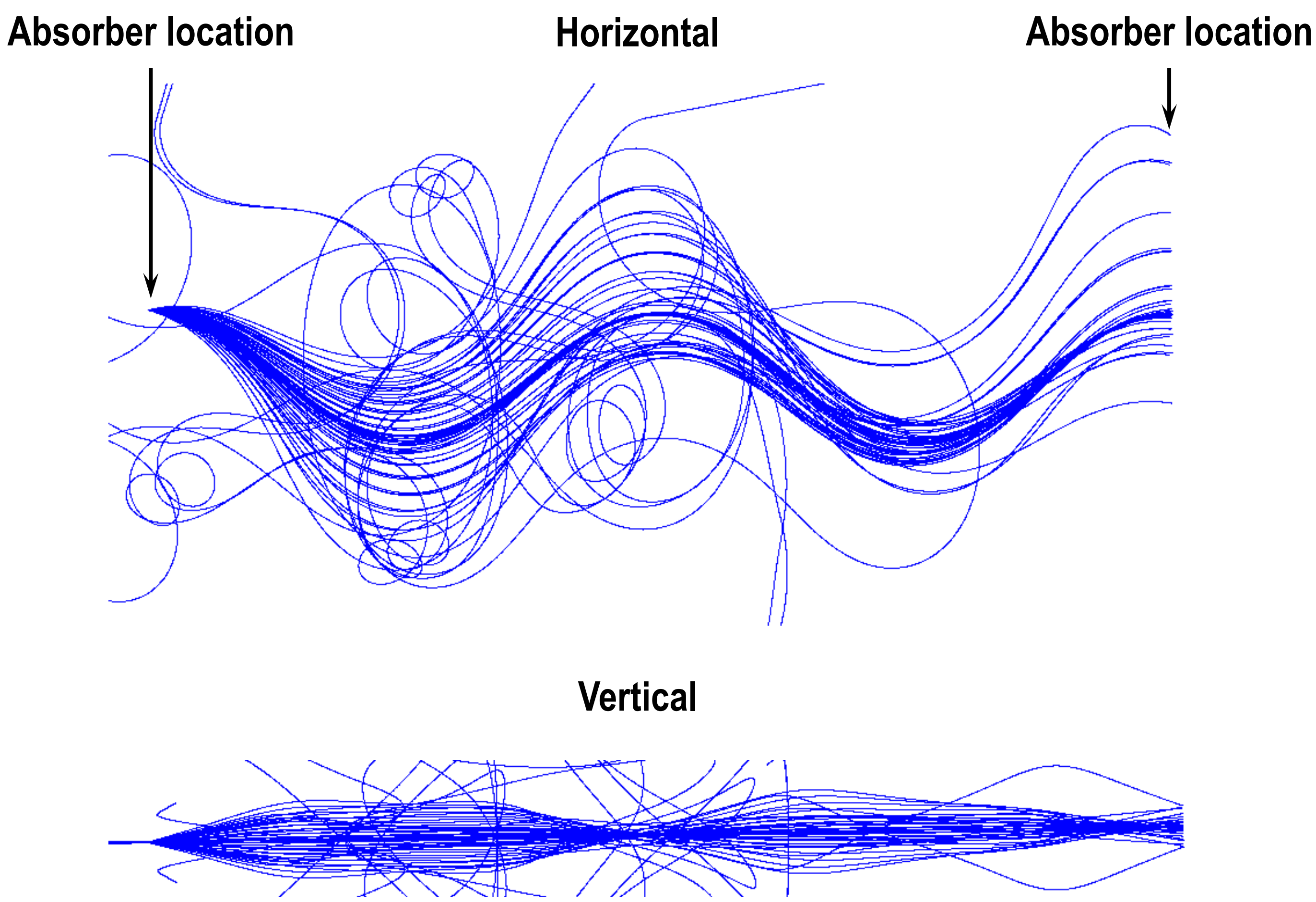
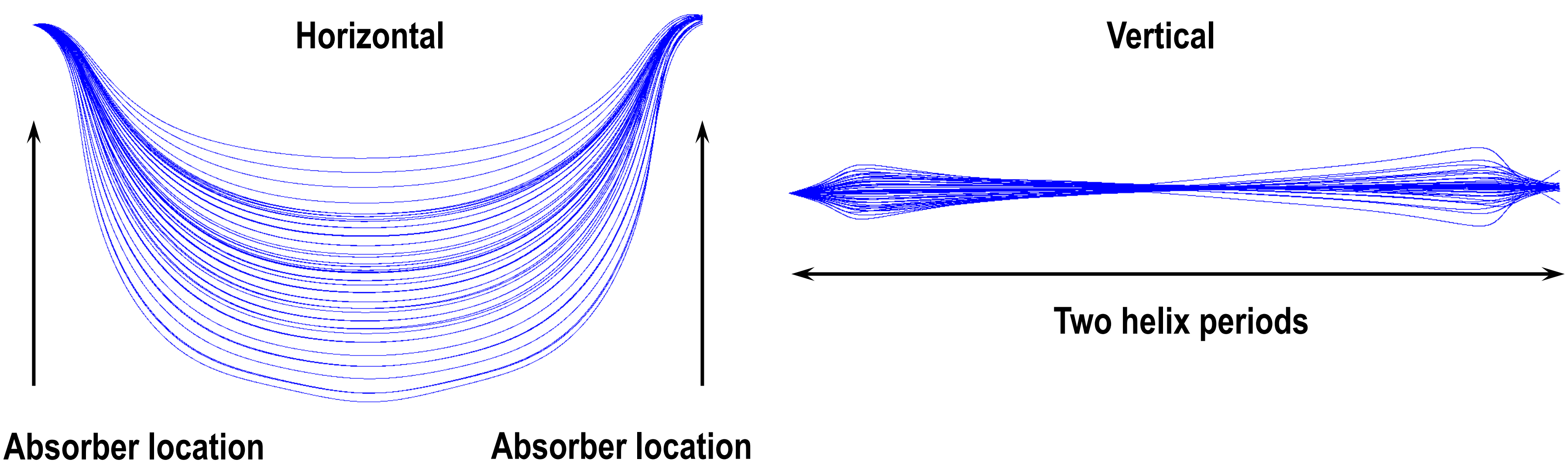
- Basic system: helical dipole pair [of period □] + straight quad
- Tracks in G4beamline for □260 mrad angular spread



Aberration Compensation

- Aberrations
  - chromatic (momentum dependent, compensated using sextupoles at $D \neq 0$)
  - spherical (geometric, compensated using octupoles)
- Grid of particles launched in G4beamline from a point source with $\pm$260 mrad angular spread, beam smear at the next focal point minimized
  - Require 1:2 betatron tune ratio
  - Minimize the number of different helical wavelengths
- Compensated system: basic system + harmonics:
  - Helical: dipole [$2\pi/3$], quadrupole [$2\pi/3$], sextupole [$2\pi/3$] , octupole [$2\pi/3$]
  - Straight: dipole, sextupole, octupole



Nonlinear Resonances

- Many multipole harmonics cause nonlinear resonances in case of correlated optics
- Multiple octupole harmonics needed to compensate spherical aberrations
- Consider, for example, Hamiltonian term of continuous harmonically-varying octupole field

$$H_{oct} = \frac{1}{4} n_{oct} (6x^2 y^2 - x^4 - y^4)$$

where

$$n_{oct} \sim \exp(i2\pi mz / L)$$

$$x = x_\beta + D_x \delta, \quad x_\beta \sim \exp(i2\pi \nu_x z / L)$$

$$y = y_\beta \sim \exp(i2\pi \nu_y z / L)$$

- With $\nu_x = 0.25,\ \nu_y = 0.5$ any octupole harmonic can cause a resonance
- Dispersion further complicates the resonance situation
- Hard to correct aberrations with a limited choice of compensating harmonics

Skew PIC Solution

Muons, Inc.

- Design correlated optics but for radial motion only
- Realized by adding skew quads for strong x-y coupling
- Azimuthal motion not correlated (freedom in tune choice)
- 2d dispersion focused periodically
- Weak parametric resonance quads provide and control beam radial focusing at zero dispersion points
- Beam envelope not axially-symmetric allowing for use of multipoles for compensation of *radial* aberrations
- Advantages:
  - Betatron tunes shifted away from nonlinear resonances
  - Control of dispersion size for chromatic compensation
  - Reduces dimensionality of aberration compensation problem (to just the radial dimension) and number of required compensating multipoles
  - Equates parametric resonance rates in two planes (only one resonance harmonic needed)
  - Equates cooling decrements in the two transverse dimensions

Skew PIC Theory

- **Hill's equations for coupled motion**

$$x'' + [K^2(s) - n]x + g(s)y = K(s)\delta$$
$$y'' + ny + g(s)x = 0$$

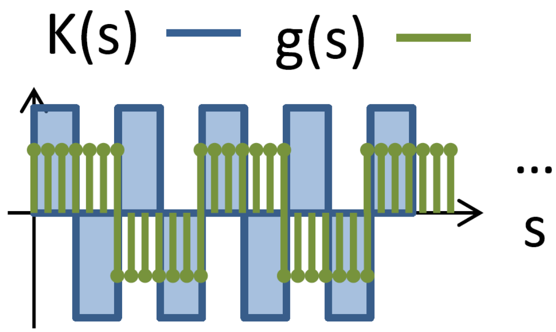
- **Transverse phase space transformation between absorbers**

$$\begin{pmatrix} x_f \\ y_f \\ x'_f \\ y'_f \end{pmatrix} = \mathrm{M} \begin{pmatrix} x_i \\ y_i \\ x'_i \\ y'_i \end{pmatrix}, \quad \mathrm{M} = \begin{pmatrix} M & 0 \\ L & N \end{pmatrix}, \quad \det(\mathrm{M}) = \det(M) \cdot \det(N) = 1$$

- **No symplecticity violation**
- **Three independent constraints**
- **Linear motion stability criterion:** $\det(M) = \det(N) = 1$

Particular Solution

- **Step like curvature and coupling functions**



- **Analytic solution**

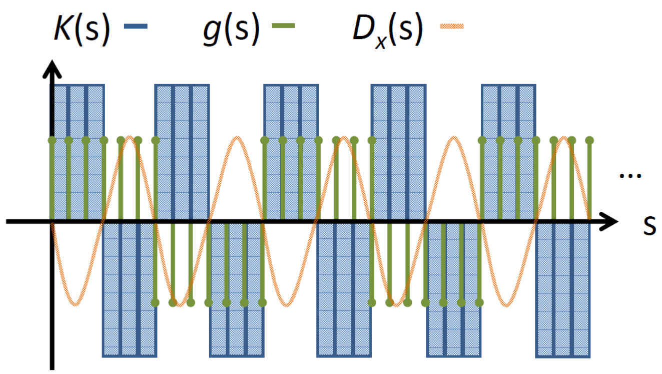
$$\mathrm{M} = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}, \quad M = N = \begin{pmatrix} \cos(4\theta) & -\sin(4\theta) \\ \sin(4\theta) & \cos(4\theta) \end{pmatrix}$$

$$\tan\theta = \frac{K^2 - 2n - \sqrt{(K^2-2n)^2 + 4g^2}}{2g}$$

- **Eigenvalues:** $\exp(\pm i4\theta)$

MAD-X Implementation

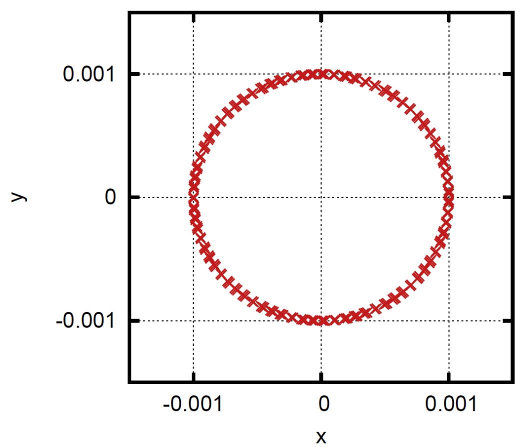
- **Approximately step-like curvature and coupling functions in MAD-X**

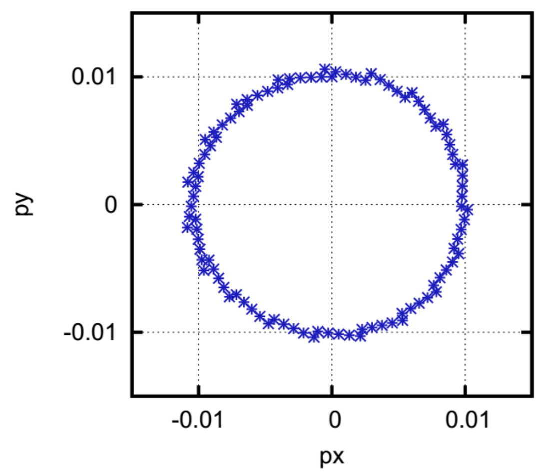


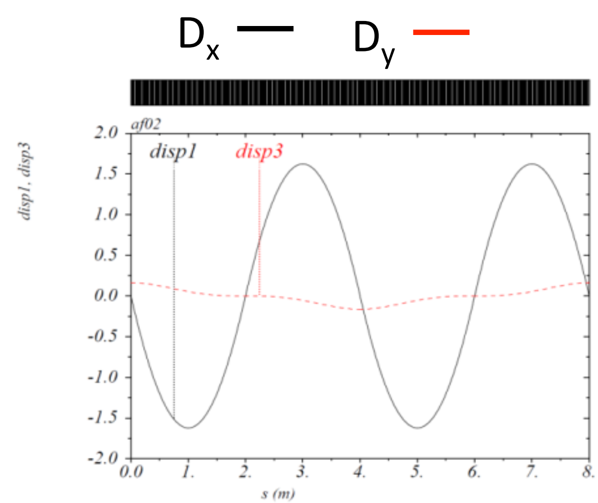
Transfer matrix

```
M =
    0.9444   -0.3287   -0.0000    0.0000
    0.3287    0.9444   -0.0000    0.0000
    0.0000         0    0.9444   -0.3287
    0.0060   -0.0000    0.3287    0.9444

ν1 = 0.0533    detM = 1.0000

ν2 = 0.0533    detN = 1.0000
```

- **x-y, px-py phase-space trajectories**

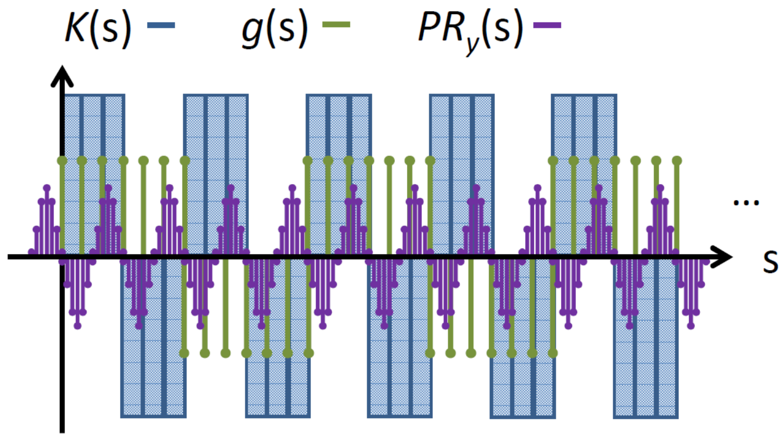






Parametric Resonance

- **Single harmonically-varying quadrupole**

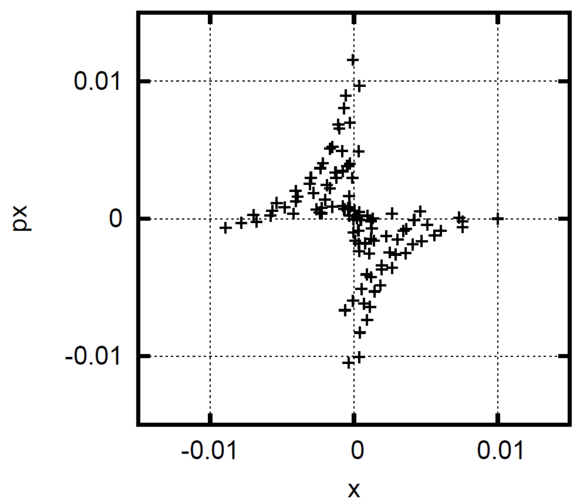


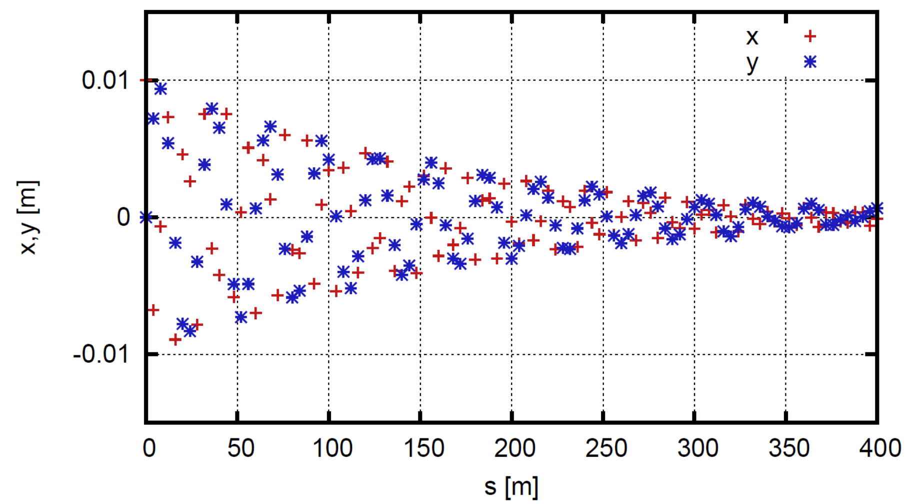

```
M =

  -0.0878   -0.9472   -0.0004   -0.0401
   0.9503   -0.1479   -0.0305   -0.0030
   0.0074   -0.0995   -0.1625   -1.0449
  -0.1153    0.0014    1.0410   -0.0965
```

λ_1 = 0.2697 - 0.0075i   detM = 0.9131

λ_2 = 0.2697 + 0.0075i   detN = 1.1034

- **Hyperbolic behavior in x-px phase space and simultaneous x and y focusing**





Sextupole Resonance

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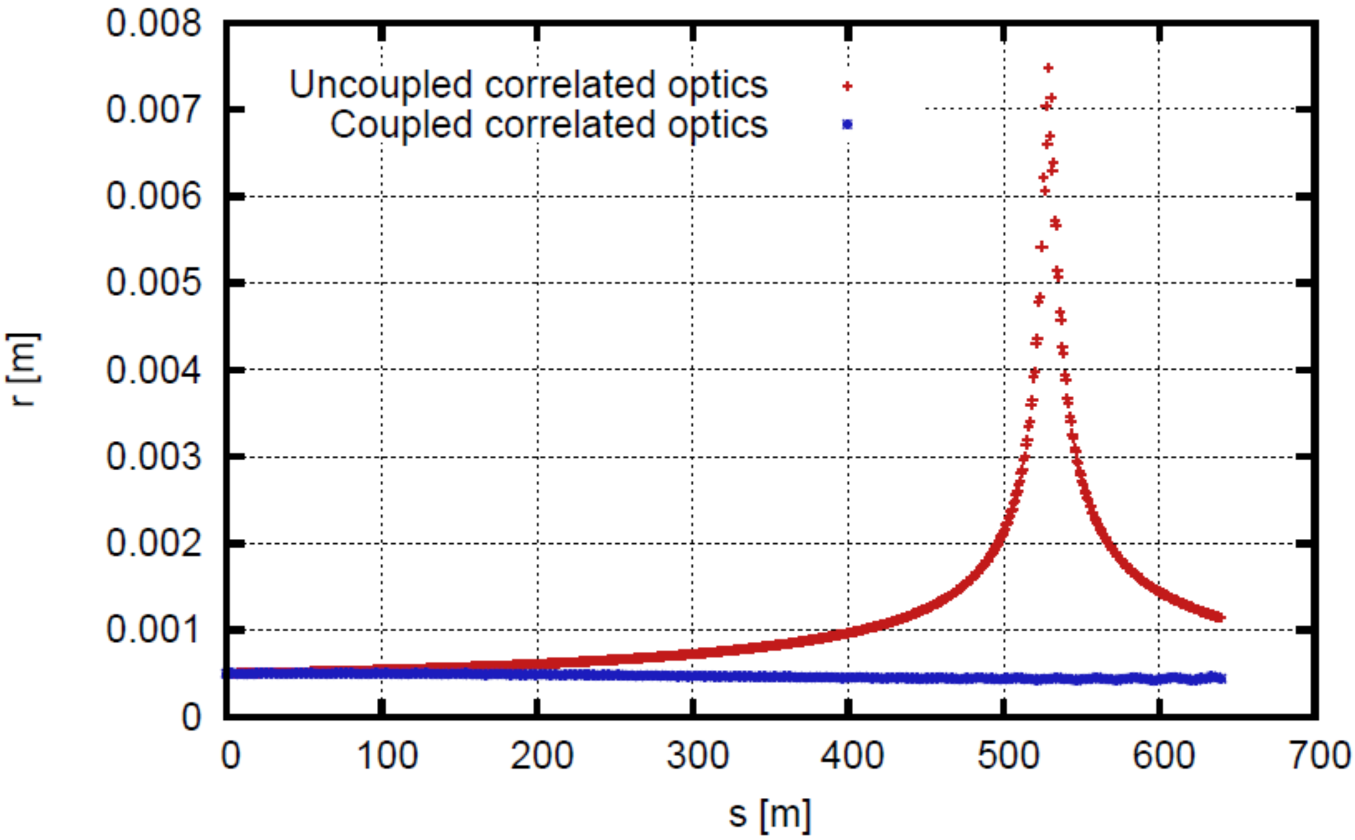
- **Sextupole effect (e.g. due to fringe fields or chromatic compensation) in uncoupled and coupled correlated optics**



Future Plans

- Complete linear Skew PIC design and dynamical studies
- Complete nonlinear analysis
- Compensate aberrations using necessary multipoles
- Compensate dangerous nonlinear resonances if needed
- Implement parametric resonance
- Demonstrate expected dynamical features in Skew PIC channel
- Implement RF and absorbers
- Study ionization cooling in simulations
- Find a feasible technical concept for magnetic lattice of Skew PIC that incorporates RF and absorbers
- Develop beam and optics control
- Extend Skew PIC to REMEX