

Study of Helical Cooling Channels for Intense Muon Sources

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Cool'15 Workshop at JLab

Ionization cooling for muon beam



Highlights of D. Kaplan's talk

- Energy loss collision with atoms/molecules via ionization process
 - Very high collision frequency (therefore a high cooling rate) since a high density cooling media is available
 - Lost-energy is immediately recovered by RF accelerations
 - Often a large angle scattering takes place by collision with nuclei of the cooling media (i.e. multiple scattering)
 - Low Z material is ideal to minimize the large angle scattering

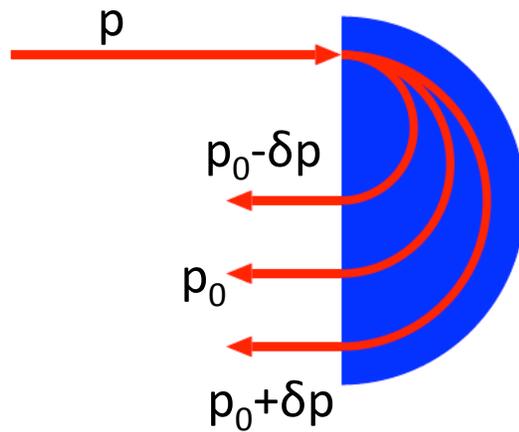
Ionization cooling for muon beam



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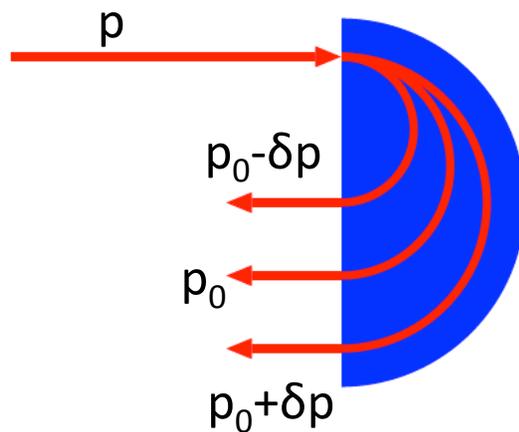
- Energy loss collision with atoms/molecules via ionization process
 - Very high collision frequency (therefore a high cooling rate) since a high density cooling media is available
 - Lost-energy is immediately recovered by RF accelerations
 - Often a large angle scattering takes place by collision with nuclei of the cooling media (i.e. multiple scattering)
 - Low Z material is ideal to minimize the large angle scattering
 - Gaseous hydrogen is the best cooling material
 - High energy-loss rate (dE/dx)
 - Small scattering angle via protons (long radiation length X_0)
 - GH2 can also be used to suppress dark currents
 - Eliminate a RF electric breakdown due to strong magnetic fields (See B. Freemire's talk)

Concept of hydrogen gas-filled Helical Cooling Channel (HCC)



Homogeneous gas
absorber in
a dipole magnet

Concept of hydrogen gas-filled Helical Cooling Channel (HCC)



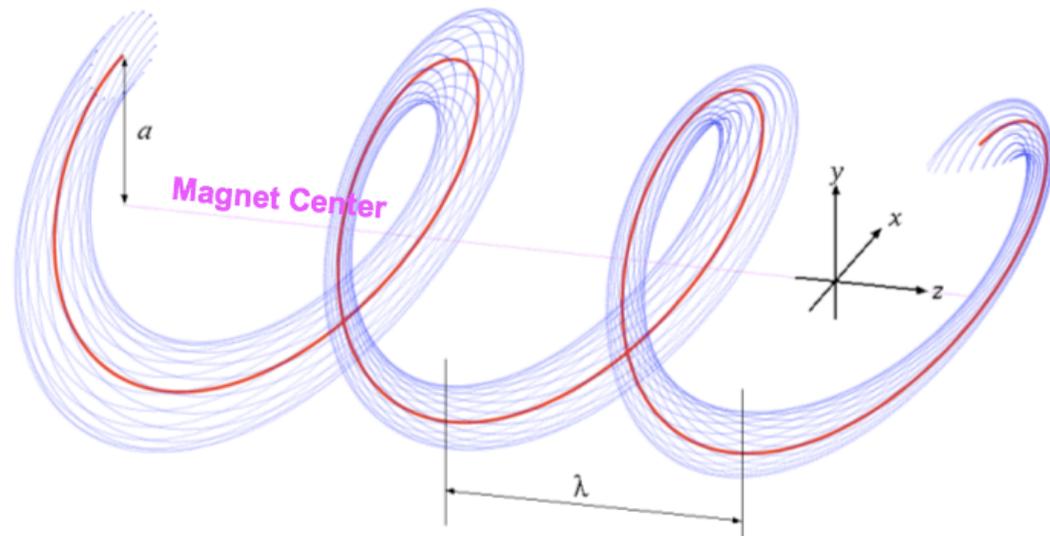
Homogeneous gas absorber in a dipole magnet

New conceptual accelerating system

Key feature:

- Dense hydrogen gas distributed homogeneously in a lattice with constant dispersion

→ Non-periodic lattice structure



Particle tracking in HCC (red: reference)

Particle motion (blue) is periodic due to the solenoid and helical dipole magnetic fields

Complete linear theory: Ya.S. Derbenev & R.P. Johnson, PRSTAB 8 041002 (2005)

Linear beam parameter in HCC



- **Betatron tune:** $Q^2 = Q_{\pm}^2 \equiv R \pm \sqrt{R^2 - G}$ where $R^2 \geq G$ is a stability condition

Equation of motion for a reference particle

$$f_{\text{central}} = \frac{e}{m}(b \cdot p_z - B_z \cdot p_{\perp})$$

$$B_z = b_z + \kappa b$$

$$p = \frac{(b_z + \kappa b)(1 + \kappa^2)^{1/2}}{k(q + 1)}$$

k and q are a geometry parameter

$$\kappa = \frac{p_{\perp}}{p_z} = \frac{2\pi a}{\lambda} = \kappa a, \quad q = \frac{b(1 + \kappa^2)}{\kappa(b_z + \kappa b)}$$

R and G are given by q , k , and \hat{D}

$$R = \frac{1}{2} \left(1 + \frac{q^2}{1 + \kappa^2} \right), \quad G = \left(\frac{2q + \kappa^2}{1 + \kappa^2} - \hat{D}^{-1} \right) \hat{D}^{-1}$$

- **Dispersion factor:**

$$\hat{D}^{-1} = \left(\frac{p}{a} \frac{da}{dp} \right)^{-1} = g + \frac{\kappa^2 + (1 - \kappa^2)q}{1 + \kappa^2}, \quad b' \equiv \frac{db}{da} = \frac{gpk}{(1 + \kappa^2)^{3/2}}$$

- **Beta function:**

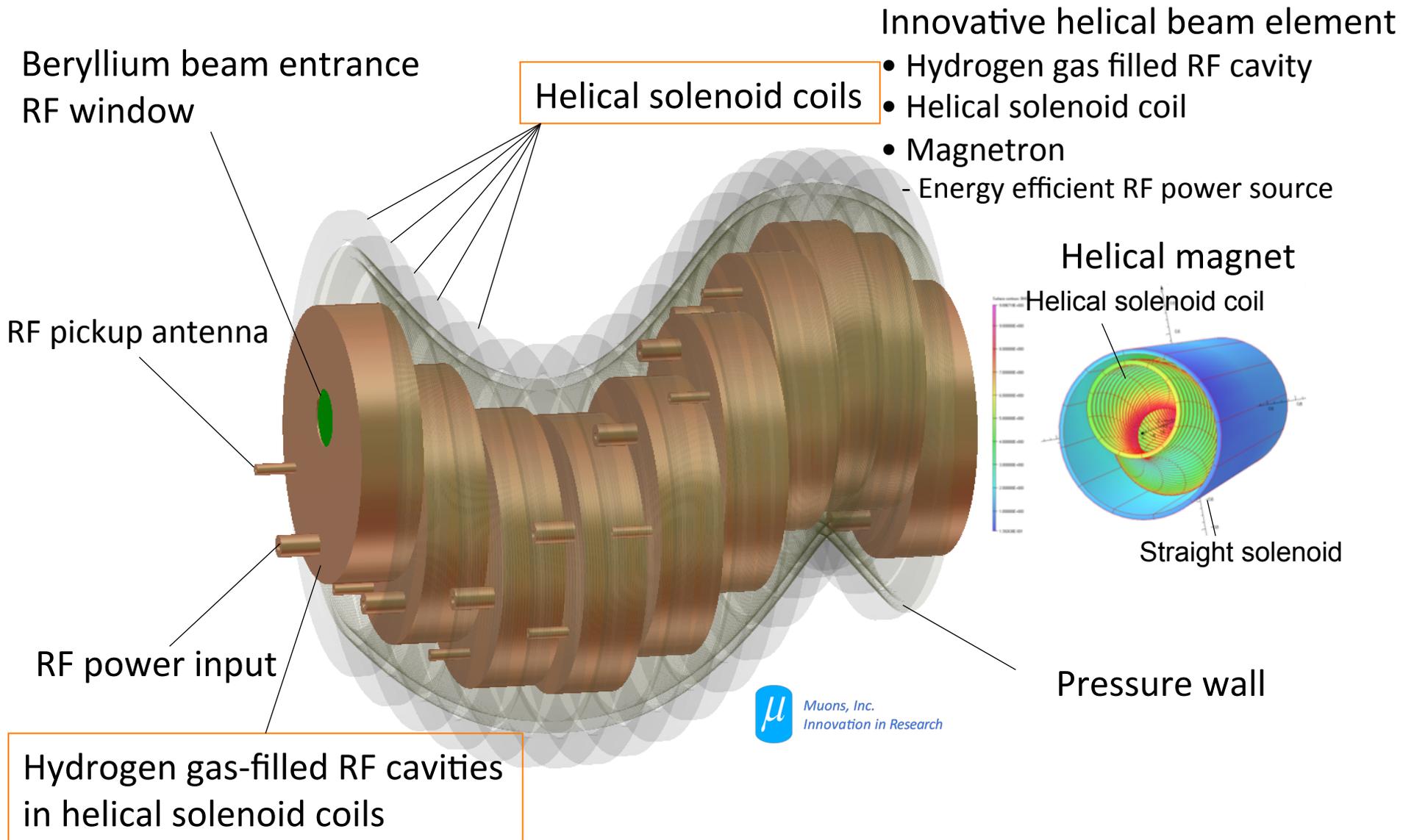
$$\beta_{\pm} = \frac{1}{kQ_{\pm}} = \frac{\lambda}{2\pi Q_{\pm}}, \quad \beta_L = \sqrt{\frac{m_{\mu} c}{\eta \omega e V'} \frac{1 + \sin(\phi_s)}{1 - \sin(\phi_s)}}$$

Phase slip factor

$$\eta = \frac{d}{d\gamma} \frac{\sqrt{1 + \kappa^2}}{\beta} = \frac{\sqrt{1 + \kappa^2}}{\gamma \beta^3} \left(\frac{\kappa^2}{1 + \kappa^2} \hat{D} - \frac{1}{\gamma^2} \right)$$

Note: HCC has a positive slip factor

Design concept of helical beam element



Validate HCC theory with numerical simulation



Emittance evolution

$$\varepsilon_r(s) = (\varepsilon_{r,0} - \varepsilon_{r,eq}) \exp(-\Lambda_r s) + \varepsilon_{r,eq}$$

Equilibrium emittance

$$\varepsilon_{T,eq} \approx \frac{\beta_T (13.6 \text{ MeV})^2}{2m_\mu \beta g_T X_0 \langle dE/ds \rangle}$$

$$\varepsilon_{L,eq} \approx \frac{m_e c^2 \gamma^2 \beta (1 - \beta^2/2) \beta_L}{2m_\mu g_L \left(\ln \left(\frac{2m_e c^2 \beta^2 \gamma^2}{I} \right) - \beta^2 \right)}$$

Cooling rate (decrement)

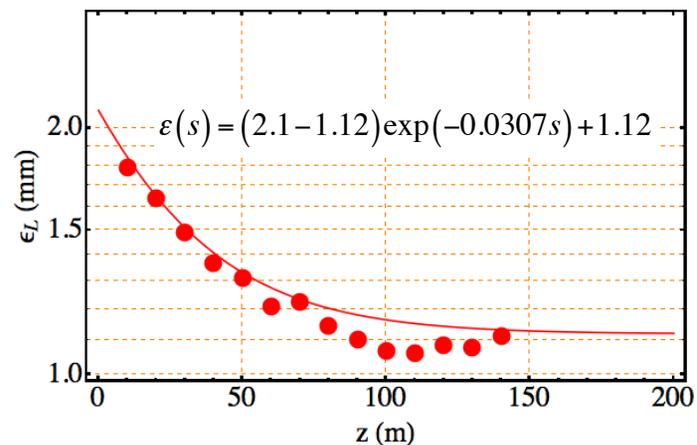
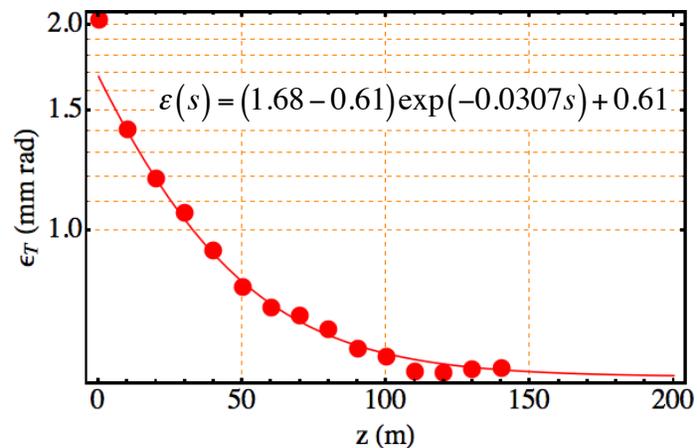
$$\Lambda_T = \frac{g_T}{\beta^2 E} \frac{dE}{ds}$$

$$\Lambda_L = \frac{g_L}{\beta^2 E} \frac{dE}{ds}$$

$$g_L \rightarrow g_{L,0} + \delta g_L, \quad g_{T(=x,y)} \rightarrow 1 - \frac{\delta g_L}{2}$$

$$\delta g_L = \frac{\kappa^2}{1 + \kappa^2} \hat{D}$$

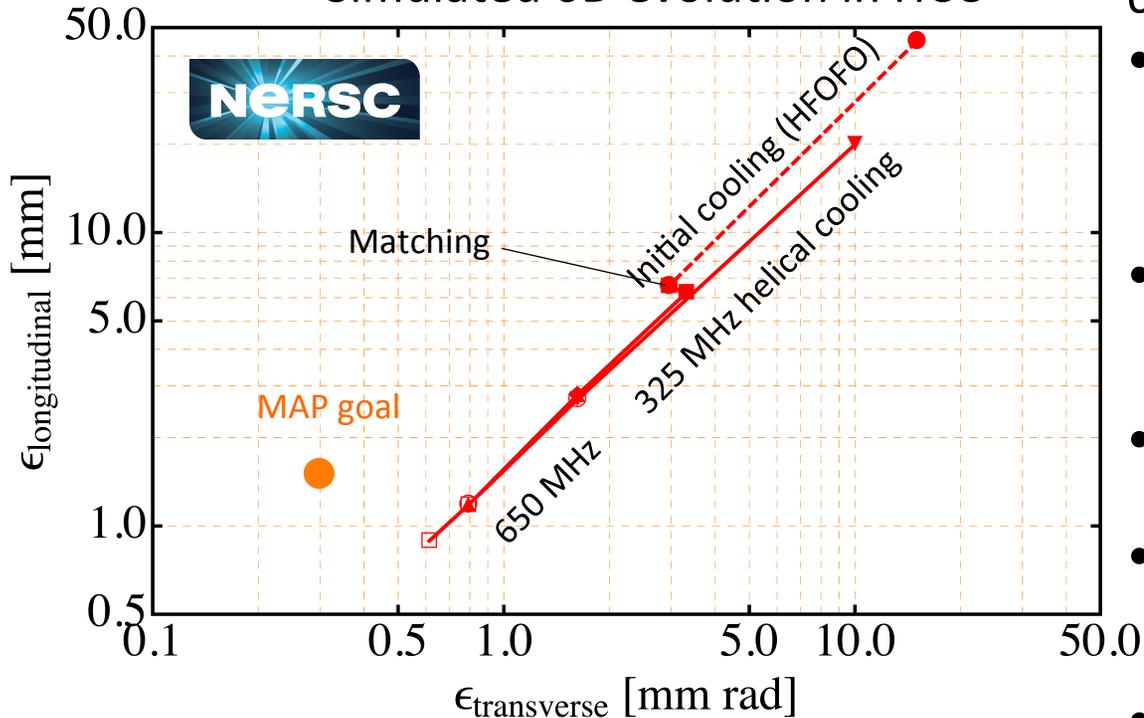
$I = 0.5 \text{ m}$, $n = 650 \text{ MHz}$, Gas Pressure = 160 atm @ 300 K
 $E = 20 \text{ MV/m}$, RF window thickness = 60 mm, 10 RF cells / I



Solid line is the prediction
(Not a fitting curve!)

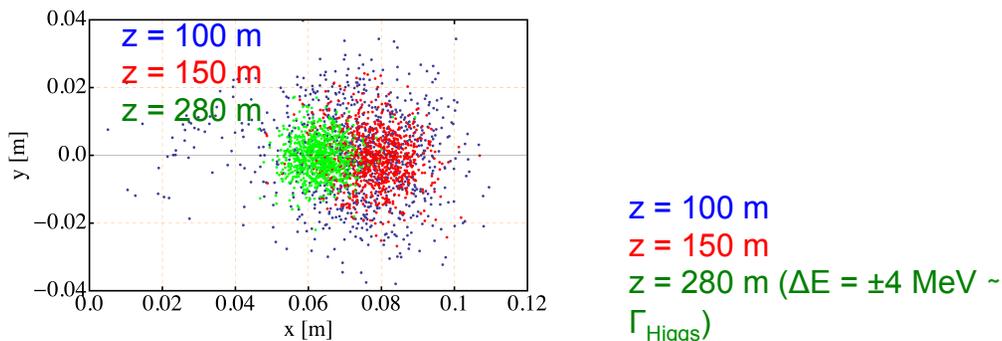
Six-dimensional phase space evolution in helical cooling channel for muon collider scheme

Simulated 6D evolution in HCC



6D HCC

- RF parameter
 - $E_{\text{peak}} = 20 \text{ MV/m}$
 - $n = 325 \text{ \& } 650 \text{ MHz}$
 - 60 mm thick Be window
- Gas pressure
 - 160 atm at 300 K
 - 43 atm at 80 K
- Magnetic fields
 - $B_z = 4 - 12 \text{ Tesla}$
- Equilibrium emittance
 - $e_T = 0.6 \text{ mm}$ (goal: 0.3 mm)
 - $e_L = 0.9 \text{ mm}$ (goal: 1.5 mm)
- Transmission (one cooling path)
 - 60 %
- Channel length (one cooling path)
 - 280 m

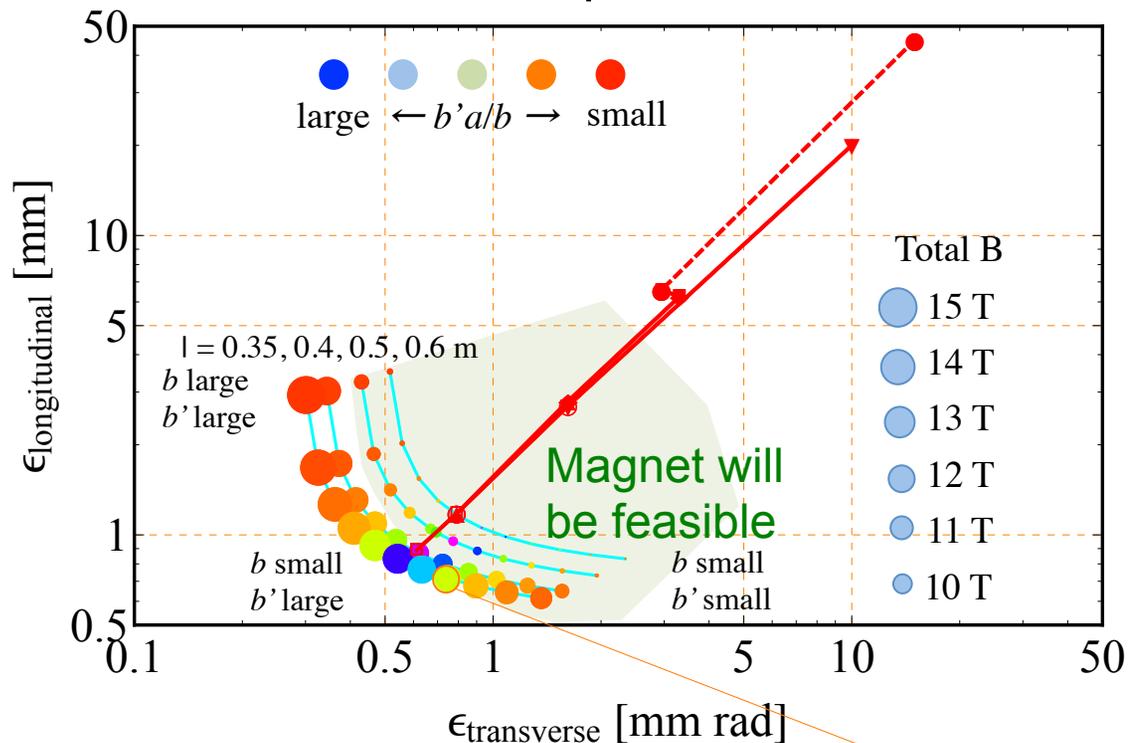


Equilibrium emittance was limited by the magnetic field strength

Variable cooling rate and equilibrium emittance by tuning helical lattice

To overcome equilibrium emittance limit...

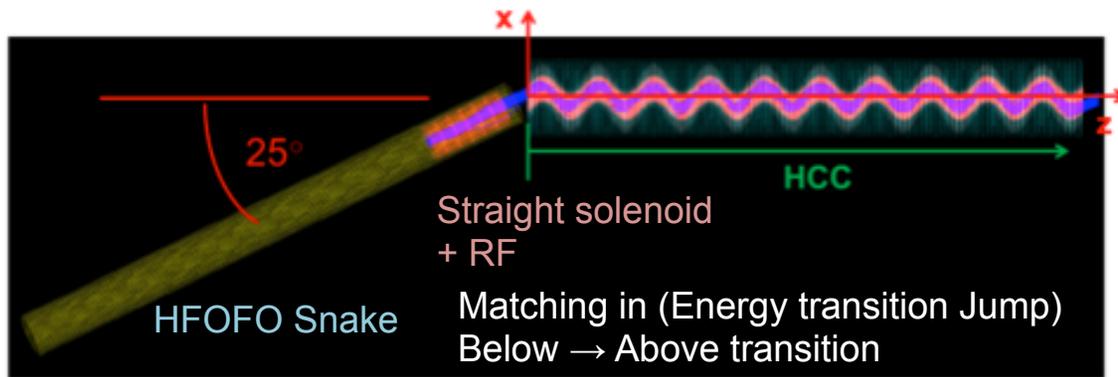
Feasibility of helical magnet vs achievable equilibrium emittance



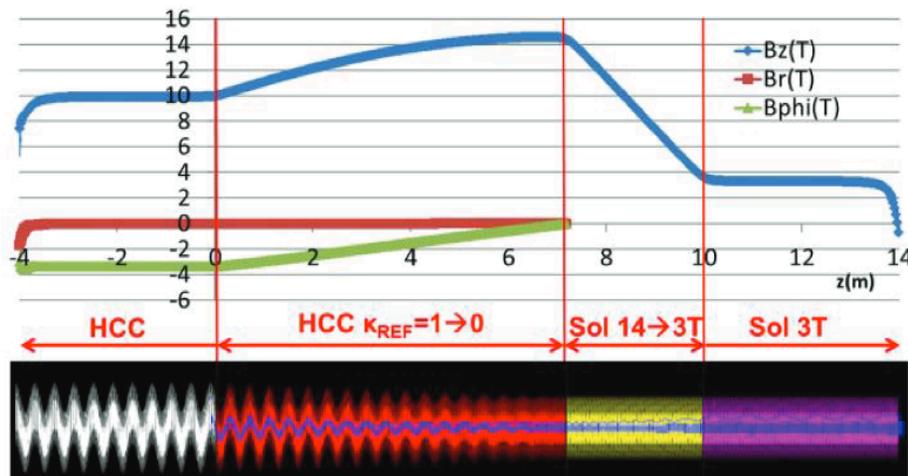
- Shorter l generates lower emittance
- Shorter l requires stronger B
- Equal cooling decrements require large $b'a/b$
- Lower longitudinal emittance requires lower $b'a/b$
- **Space charge is not important**
 - Longitudinal space charge focuses for positive η
 - Transverse space charge is neutralized by gas-plasma (see next slide)
- For example, it will be possible to reach $\epsilon_t = 0.75$ mm and $\epsilon_l = 0.75$ mm at a total $B = 12$ T and $l = 0.35$ m

Longitudinal enhance cooling will be applied for a Higgs factory

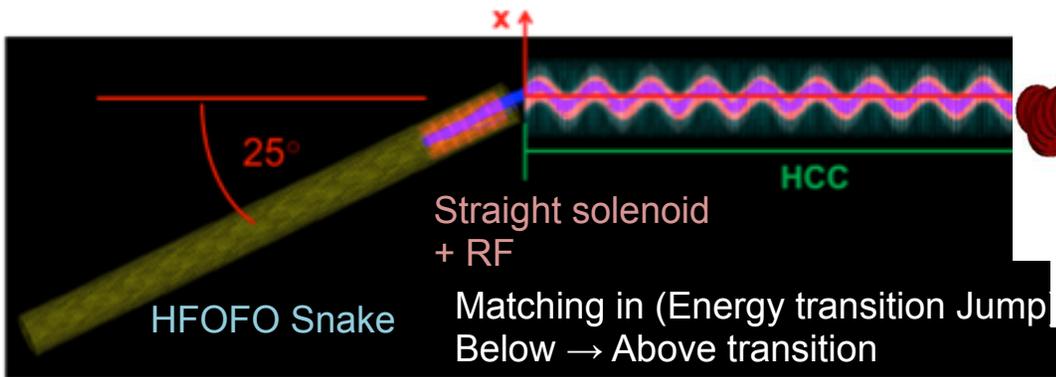
Matching and Low Energy Bunch Merging based on HCC



B Field Components in HCC Matching Out Section
 HCC(-4.1to0m) HCCtaper(0to7.2m) BsolMatch(7.2to10m)



Matching and Low Energy Bunch Merging based on HCC



Helical bunch merge channel
 = Accelerator + Isochronous channel
 Transmission ~90 % in 120 m



Helical accelerator for $b = 1$ channel

Phase space in a helical bunch merge chanr

B Field Components in HCC Matching Out Section
 HCC(-4.1to0m) HCCtaper(0to7.2m) BsolMatch(7.2to10m)

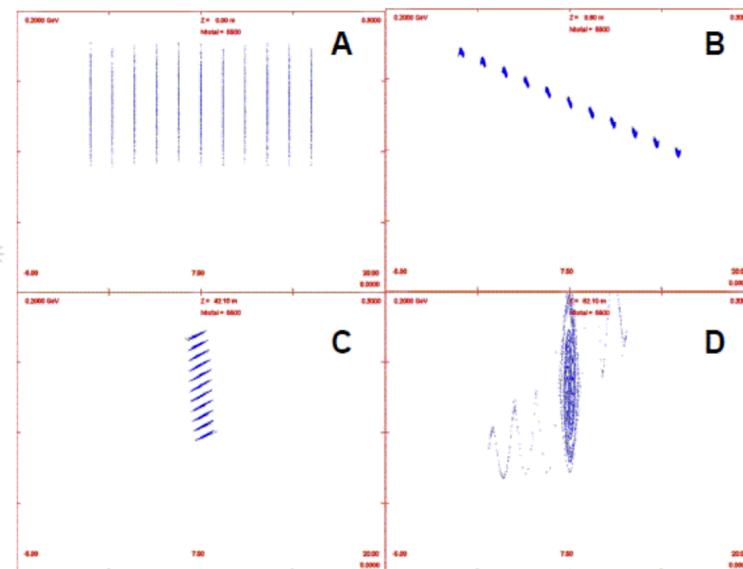
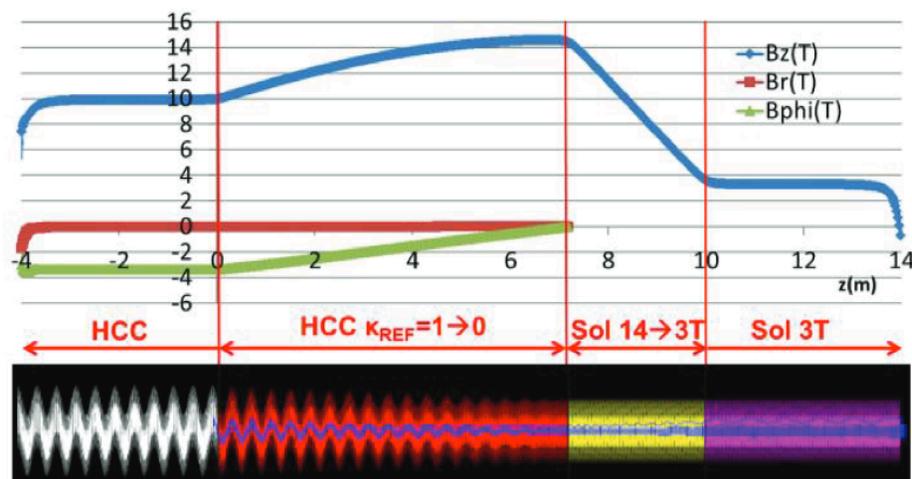


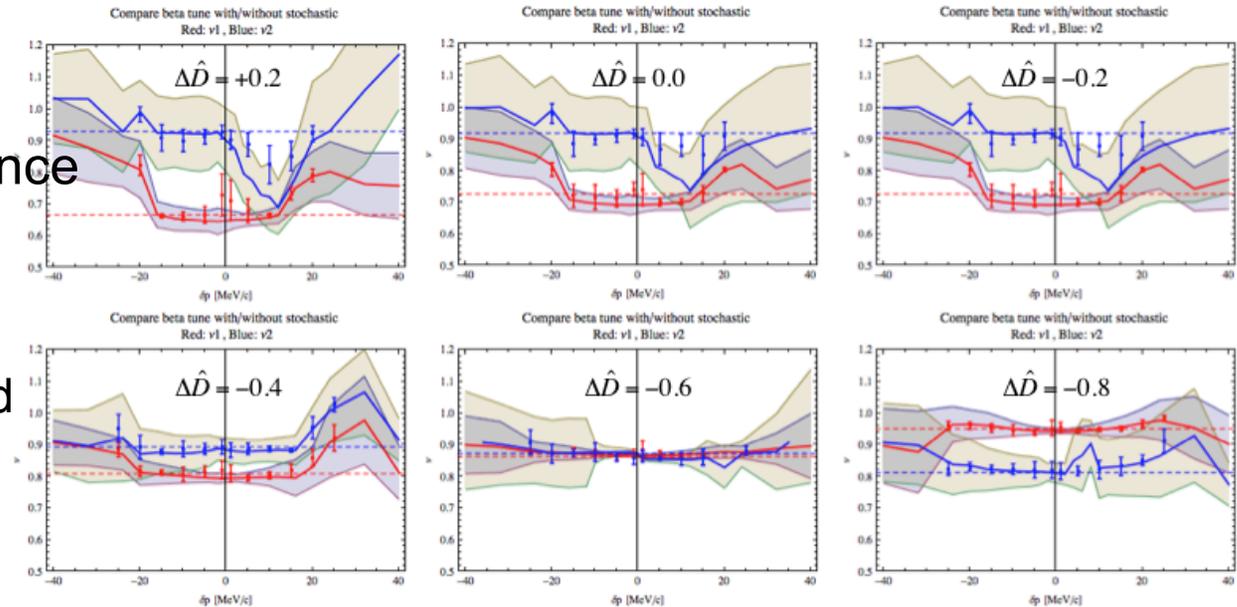
Figure 4: Overview of bunch recombiner with initial quasi-isochronous bunch formation.

Acceptance in non-linear fields

Chromaticity in HCC

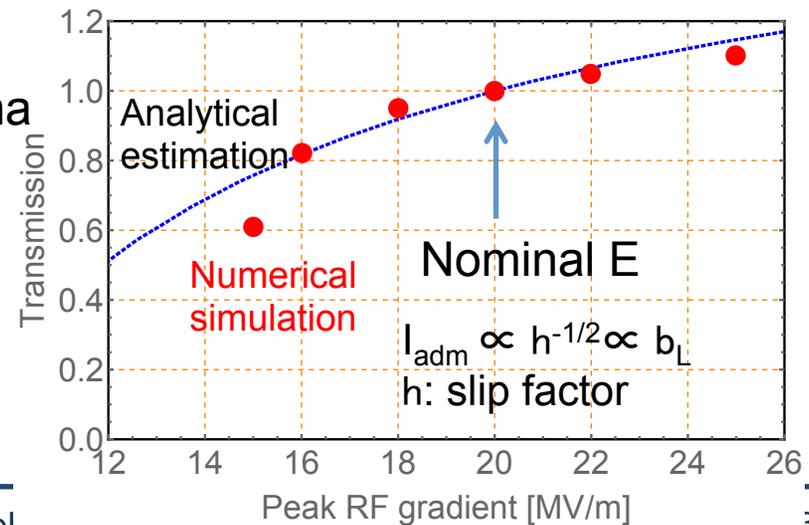
Study transverse acceptance

- Added helical sextupole component to compensate a chromaticity
- Transmission was improved by 10 %



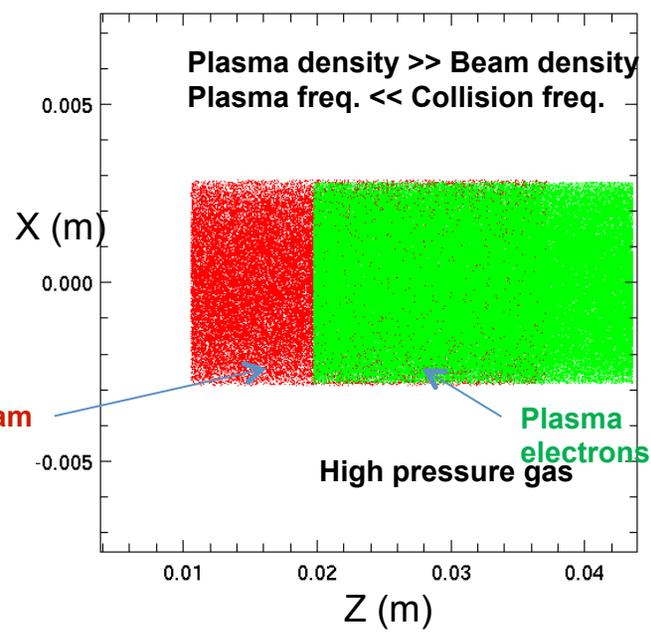
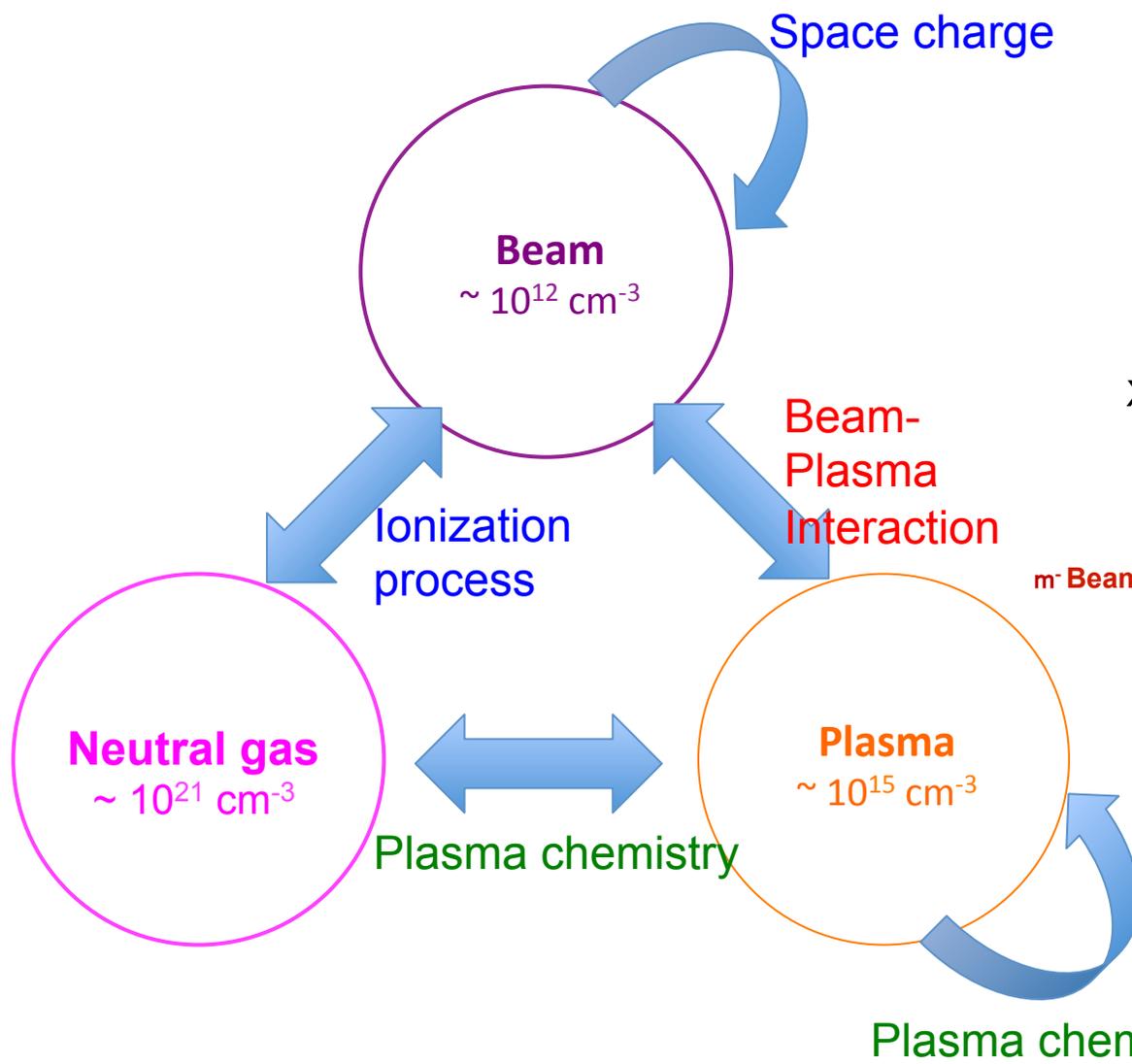
Study longitudinal acceptance

- RF power is consumed by beam-induced plasma
- RF gradient drops by 20 % for the 21st bunch
- Numerical study agrees well with the prediction except for $E = 15$ MV/m



Space Charge Neutralization and Plasma Lens

Beam-plasma interaction in gas-filled RF cavities

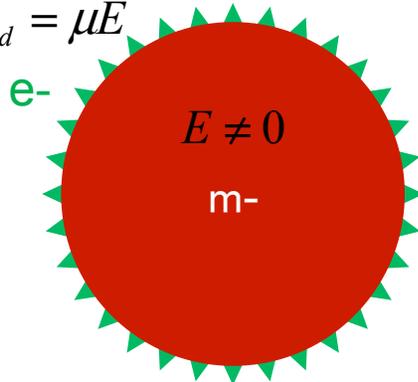


Electrons are quickly thermalized and the spatial distribution is “frozen”

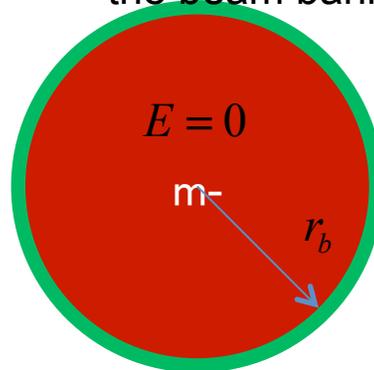
Space Charge Neutralization

1. Space-charge of the muon beam pushes plasma electrons outward

$$v_d = \mu E$$



2. Electron column expands slightly, making E-field inside the beam vanish



3. This space-charge neutralization can change the beam dynamics

$$\frac{d^2 r_b}{dz^2} + \left(\kappa_{sf} + \frac{\alpha K_b}{r_b^2} \right) r_b = \frac{\epsilon_{KV}^2}{r_b^3}$$

$$\alpha = \begin{cases} -1 & \text{for no neutralization (defocusing)} \\ \gamma^2 \beta^2 & \text{for full neutralization (focusing)} \end{cases}$$

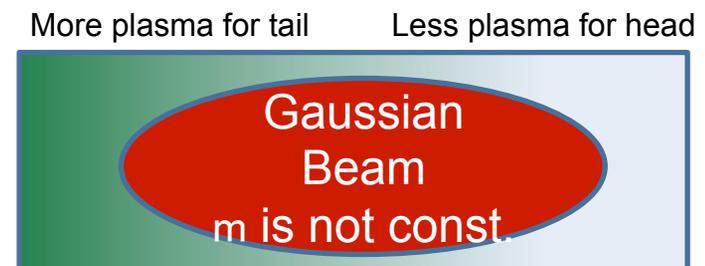
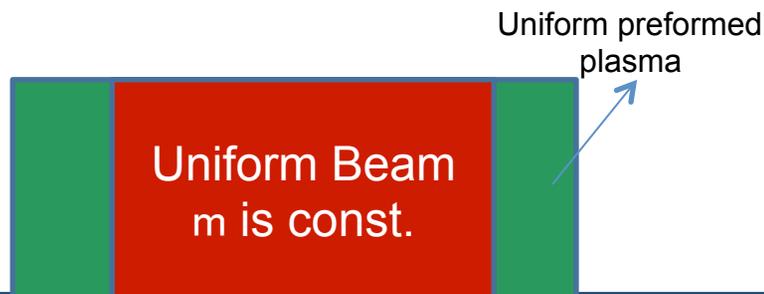
$$K_b = \frac{2r_e}{\gamma^3 \beta^2} \frac{N}{\sqrt{2\pi}\sigma_z}$$

4. Analytical estimation of space-charge neutralization time for a simple configuration

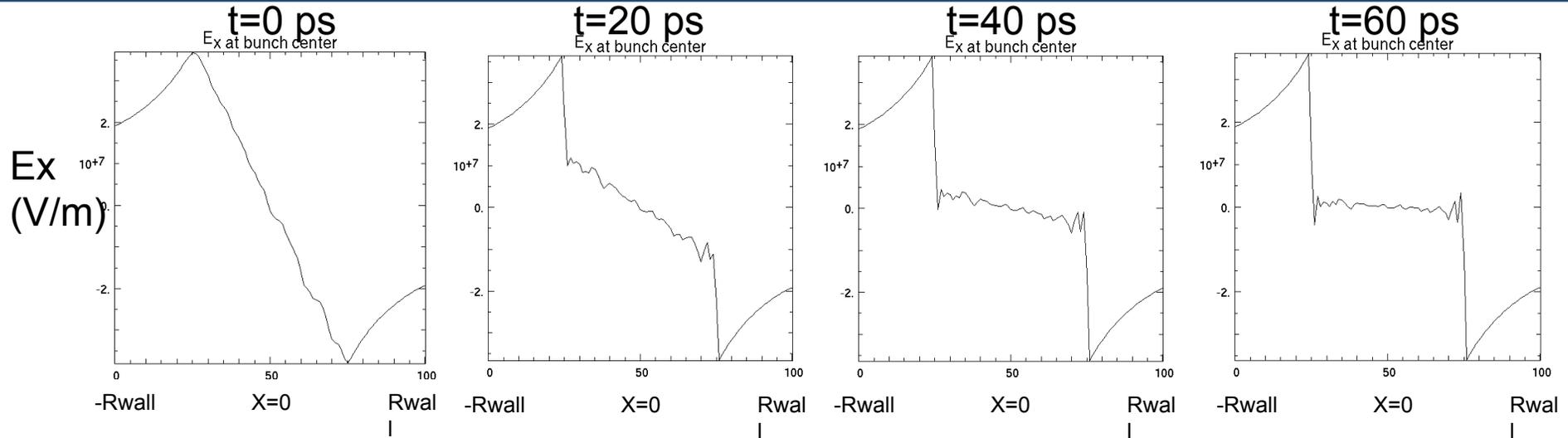
$$\tau = \frac{\epsilon_0}{(n_e/n_b)\mu|e|} < \text{typical bunch length } (\sim 100 \text{ ps})$$

5. Good simulation is required to predict the beam dynamics in the real configuration

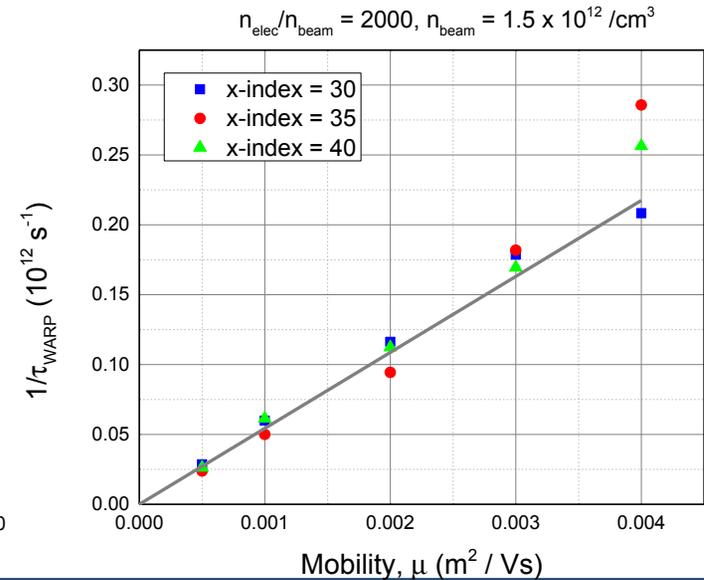
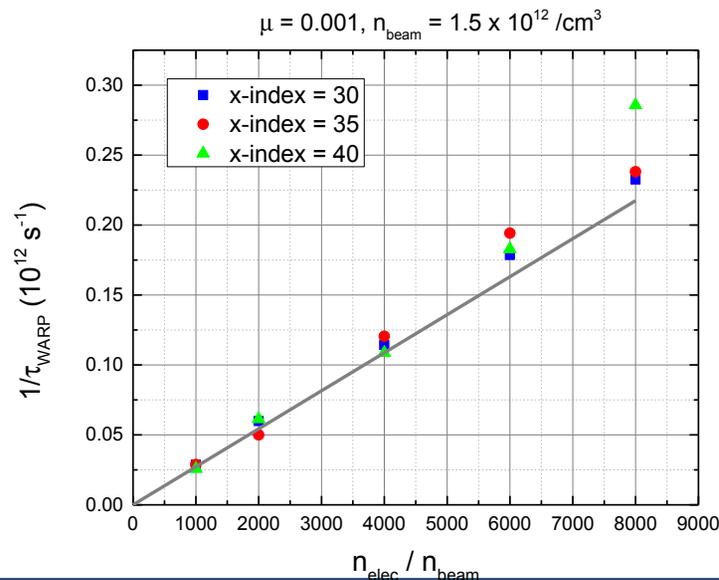
- Fermilab: WARP PIC code
- BNL: SPACE code with molecular processes



Time constant of charge neutralization

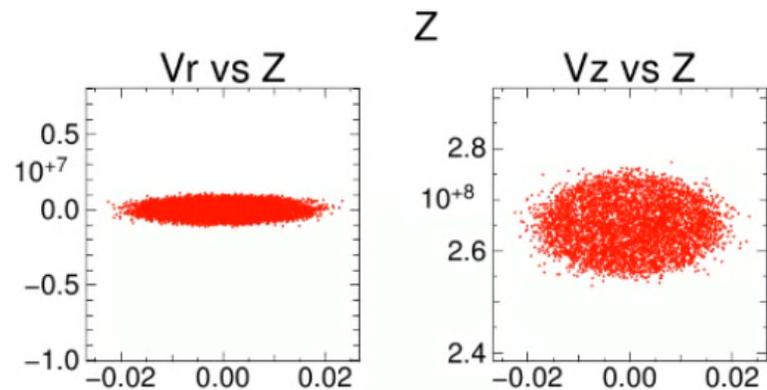
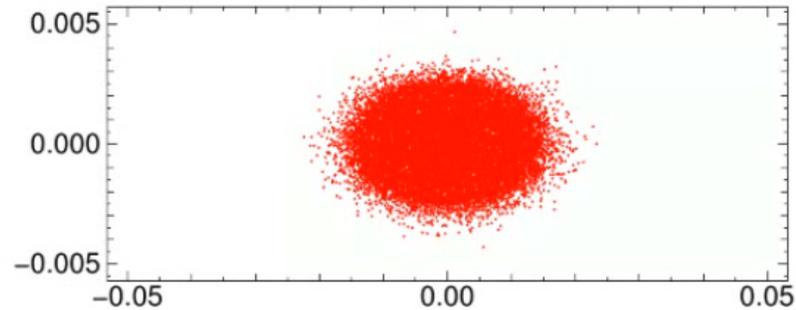


Good agreement between theory and simulation



Beam-Plasma interaction

5



z window $Z_0 = -5.3016e-02, 5.3016e-02$

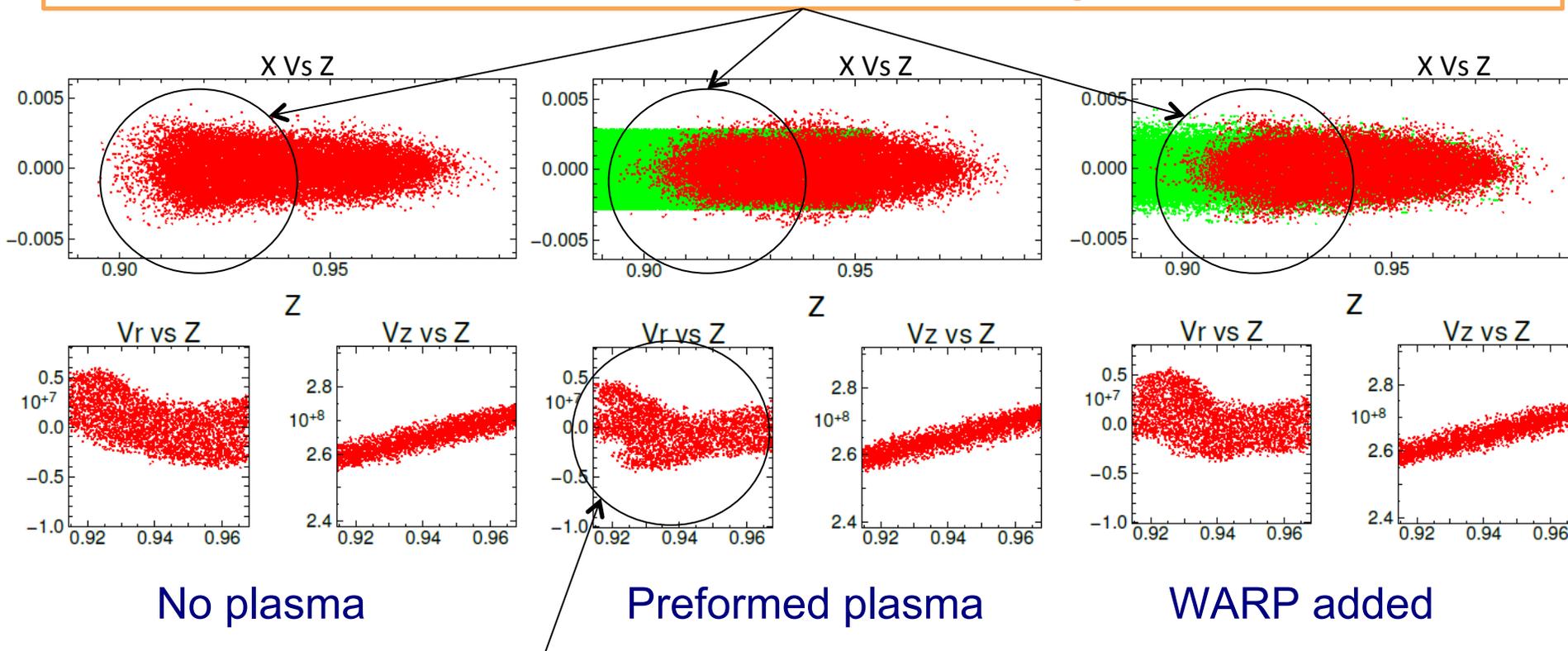
Step 0, T = 0.0000e+0 s, Zbeam = 0.0000e+0 m

HPRF: Beam-Plasma Interaction

- No RF for longitudinal focusing
- No dispersion magnet
- A straight 5-T solenoid

Beam-plasma interaction in gas-filled RF cavities

Less spread in bunch tail due to charge neutralization

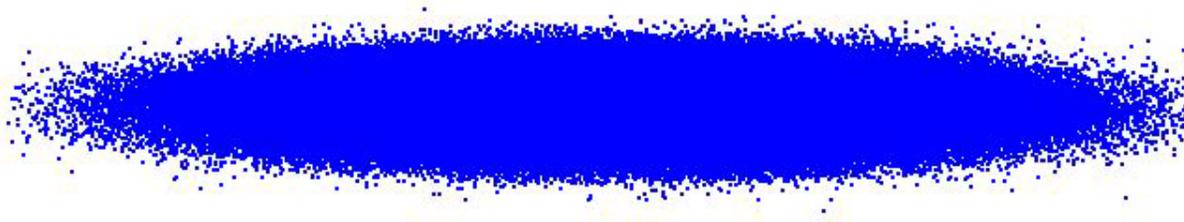


Edge effect from uniform cylinder

Beam-plasma interaction study in non-WARP simulation



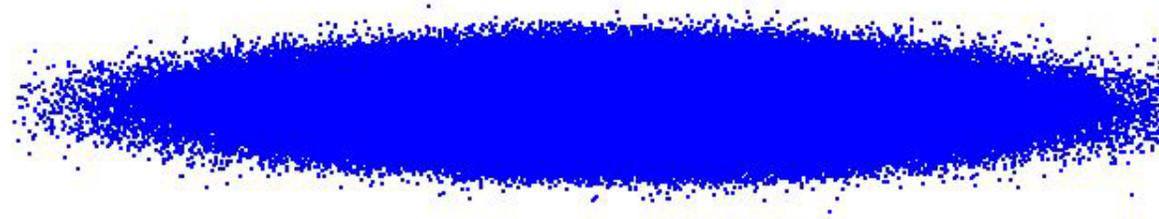
Vacuum



Beam-plasma interaction study in non-WARP simulation



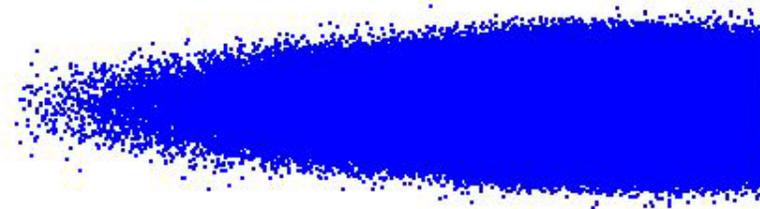
Vacuum



Beam-plasma interaction study in non-WARP simulation



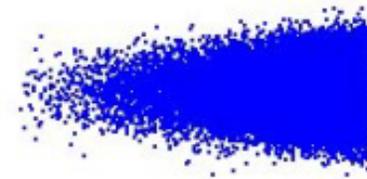
Vacuum



Beam-plasma interaction study in non-WARP simulation



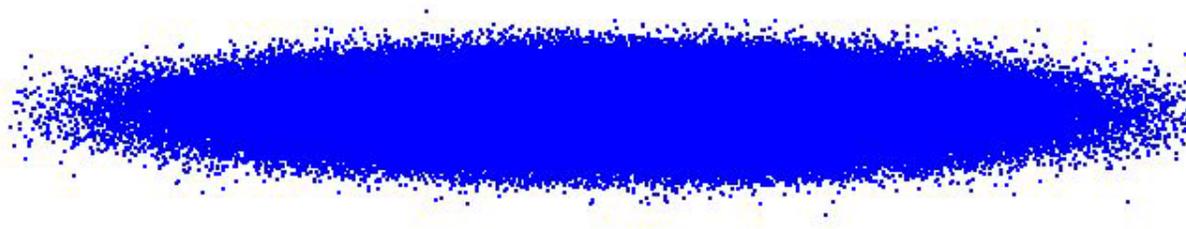
Vacuum



Beam-plasma interaction study in non-WARP simulation



In a dense H₂ gas



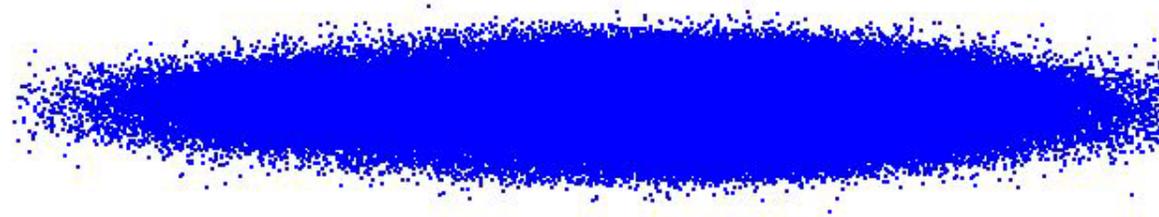
150 ps



Beam-plasma interaction study in non-WARP simulation



In a dense H₂ gas



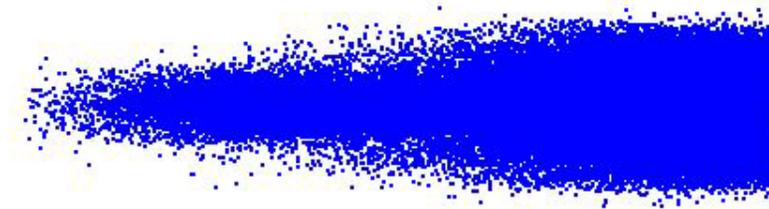
200 ps



Beam-plasma interaction study in non-WARP simulation



In a dense H₂ gas

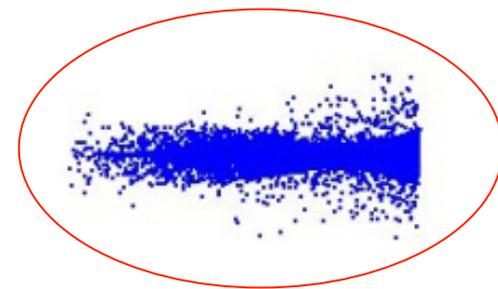


250 ps



Beam-plasma interaction study in non-WARP simulation

In a dense H₂ gas



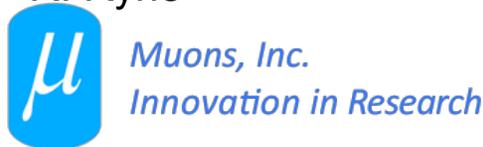
300 ps

Collaborators



6D Cooling Design & Simulation

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K. Yonehara¹
S. Kahn²
T. Roberts²
Y. Derbenev³
R. Johnson²
V. Morozov³
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R. Ryne⁸



Helical magnet

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V. Kashikhin¹
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Helical RF

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R. Samulyak^{5,6}
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J. Ellison⁴
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²Muons, Inc.
³Jlab
⁴IIT
⁵BNL
⁶STONY BROOK
⁷TRIUMF
⁸LBNL



Summary of HCC Design Effort



- **Verified helical cooling theory**
 - Understood linear dynamics
 - Studied non-linear dynamics (in progress)
 - Demonstrated ability to tune the cooling lattice
 - Enhanced longitudinal cooling to overcome the field limit
- **High pressure RF cavities**
 - Oxygen doped high pressure RF cavities tested
 - Plasma was modeled and compared to measurement
 - High pressure RF HCC should work
- **Beam-plasma interaction**
 - Plasma lens effect may increase beam focusing
 - Requires re-evaluating cooling models/simulations (in progress)