PROGRESS ON PARAMETRIC-RESONANCE IONIZATION COOLING*

V.S. Morozov#, Ya.S. Derbenev, A. Sy, Jefferson Lab, Newport News, VA 23606, USA
A. Afanasev, George Washington University, Washington, DC 20052, USA
R.P. Johnson, Muons, Inc., Batavia, IL 60510, USA
J.A. Maloney†, Northern Illinois University, DeKalb, IL 60115, USA

Abstract

Proposed next-generation muon collider will require major technical advances to achieve the rapid muon beam cooling requirements. Parametric-resonance Ionization Cooling (PIC) is proposed as the final 6D cooling stage of a high-luminosity muon collider. In PIC, a half-integer parametric resonance causes strong focusing of a muon beam at appropriately placed energy absorbers while ionization cooling limits the beam's angular spread. Combining muon ionization cooling with parametric resonant dynamics in this way should then allow much smaller final transverse muon beam sizes than conventional ionization cooling alone. One of the PIC challenges is compensation of beam aberrations over a sufficiently wide parameter range while maintaining the dynamical stability with correlated behavior of the horizontal and vertical betatron motion and dispersion. We explore use of transverse coupling to reduce the dimensionality of the problem and to shift the dynamics away from non-linear resonances. PIC simulations are presented.

MOTIVATION

Experiments at energy-frontier colliders require high luminosities of order 10^{34} cm^{-2} sec^{-1} or more in order to obtain reasonable rates for events having point-like cross sections. High luminosity colliders require intense beams with small transverse emittances and a small beta function at the collision point. For muon colliders, high beam intensities and small emittances are difficult and expensive to achieve because muons are produced diffusely and must be cooled drastically within their short lifetimes. The muon does not interact by the strong interaction, and its high mass relative to the electron means that it can pass through matter without hadronic or electromagnetic showers. Thus, it is the perfect candidate for ionization cooling. Muons lose energy by passing through a low-Z material and only the longitudinal component is replaced by an RF cavity. This technique allows the angular spread of a beam of muons to be reduced in a very short time close to the limit determined by multiple scattering.

Ionization cooling as it is presently envisioned will not cool the beam sizes sufficiently well to provide adequate luminosity without large muon intensities. For example, a muon-collider s-channel Higgs factory, a logical prerequisite to an energy-frontier muon collider, would be compelling if the luminosity were high enough. The 4 MeV energy resolution needed to directly study the Higgs width is only possible with such a machine. Also, the mass-dependent muon-Higgs coupling gives a factor of over 40,000 cross-section advantage relative to an electron collider. Numerical simulations of muon cooling channels based on technical innovations made and experimentally tested in this millennium have shown 6D emittance reductions of almost 6 orders of magnitude. Parametric-resonance Ionization Cooling (PIC) can achieve an additional two orders of emittance reduction for an additional factor of 10 in luminosity.

In addition, to the extent that the transverse emittances can be reduced further than with conventional ionization cooling, several problems can be alleviated. Lower transverse emittance allows a reduced muon current for a given luminosity, which implies:

- a proton driver with reduced demands to produce enough proton power to create the muons,
- reduced site boundary radiation limits from muons decaying into neutrinos that interact with the earth,
- reduced detector background due to electrons from muon decay,
- reduced proton target heat deposition and radiation levels,
- reduced heating of the ionization cooling energy absorber,
- less beam loading and wake field effects in the accelerating RF cavities,
- reduced space charge effect.

Smaller transverse emittance has virtues beyond reducing the required beam currents, namely:

- smaller higher-frequency RF cavities with higher gradient can be used for acceleration,
- beam transport is easier with smaller-aperture magnetic and vacuum systems,
- stronger collider interaction point (IP) focusing can be used, since that is limited by the maximum achievable beam extension prior to the IP.

PARAMETRIC-RESONANCE IONIZATION COOLING (PIC)

Concept and Analytic Theory

The limit on the minimum achievable emittances in muon ionization cooling comes from the equilibrium between the cooling process and multiple Coulomb scattering in the absorber material. The concept of

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# morozov@jlab.org
† now at TRIUMF, Vancouver, BC, Canada

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Parametric-resonance Ionization Cooling (PIC) [1, 2] is to push this limit by an order of magnitude in each transverse dimension by focusing the muon beam very strongly in both planes at thin absorber plates. This creates a large angular spread of the beam at the absorber locations, which is then cooled to its equilibrium value resulting in greatly reduced transverse emittances. Achieving adequately strong focusing using conventional magnetic optics would require unrealistically strong magnetic fields. Instead, PIC relies on a resonant process to provide the necessary focusing. A half-integer parametric resonance is induced in a cooling channel, causing focusing of the beam with the period of the channel's free oscillations.

The resonant perturbation changes the particles' phase-space trajectories at periodic locations along the channel from their normal elliptical shapes to hyperbolic ones as shown in Figure 1. Thus, at certain periodic focal positions, the beam becomes progressively narrower in x and wider in x' as it passes down the channel. Without damping, the beam dynamics are not stable because the beam envelope grows with every period. Placing energy absorbers at the focal points stabilizes the beam motion by limiting the beam's angular divergence at those points through the usual ionization cooling mechanism. These dynamics then result in a strong reduction of the beam spot size at the absorber locations leading to transverse beam emittances that are an order of magnitude smaller than without the resonance. The longitudinal emittance is maintained constant against energy struggling by emittance exchange occurring due to dispersion or its slope at the locations of wedge or flat absorbers.

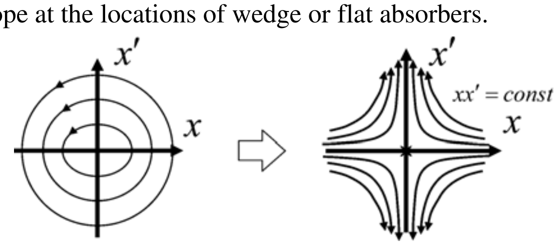

Figure 1: Transformation of a particle's phase-space motion by a half-integer parametric resonance: elliptical phase-space trajectories become hyperbolic. The trajectories are illustrated at the focal points.

The normalized equilibrium transverse emittance achievable in PIC is given by [1, 2]

$$\varepsilon_{n\perp} = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w, \qquad (1)$$

where $\beta = \upsilon / c$ is the relativistic factor, Z is the absorber material's atomic number, m_e and m_μ are the electron and muon masses, respectively, and w is the average absorber thickness in the beam direction.

The expected PIC parameters are summarized in Table 1. Note that the absorbers are thicker at the beginning of the channel in order to produce a higher cooling rate of an initially larger-emittance beam. As the beam being cooled is propagated down the channel, the absorber thickness is gradually reduced in order to reach the minimum practical transverse emittance. Since the cooling rate gets lower for thinner absorbers, the minimum practical absorber thickness is determined by the practically acceptable beam loss due to muon decay.

Table 1: Expected PIC Performance

Parameter	Unit	Initial	Final
Muon beam momentum, p	MeV/c	250	250
Number of particles per bunch, N_b	10^{10}	1	1
Be ($Z = 4$) absorber thickness, w	mm	20	2
Normalized transverse emittance (rms), $\varepsilon_x = \varepsilon_y$	µm	230	23
Beam size at absorbers (rms), $\sigma_a = \sigma_x = \sigma_y$	mm	0.7	0.1
Angular spread at absorbers (rms), $\theta_a = \theta_x = \theta_y$	mrad	130	130
Momentum spread (rms), $\Delta p/p$	%	2	2
Bunch length (rms), σ_z	mm	10	10

Correlated-Optics Channel

To attain simultaneous focusing in both planes at regular locations, the horizontal and vertical betatron oscillation periods must be commensurate with each other and with the channel's period, e.g. as illustrated in Figure 2. A magnetic channel possessing such optical properties, called a twin-helix channel, has been successfully developed and simulated [3]. In a twin-helix channel, two equal-strength helical dipole harmonics with equal periods but opposite helicities are superimposed leading to its name. Analogous to how combining two circularly-polarized waves produces a linearly-polarized one, the magnetic field in the midplane of this configuration is transverse to the plane. This means that the periodic orbit is flat and lies in the midplane. The horizontal and vertical motions are stable and uncoupled. A continuous straight quadrupole is added to the system in order to redistribute focusing between the horizontal and vertical dimensions.

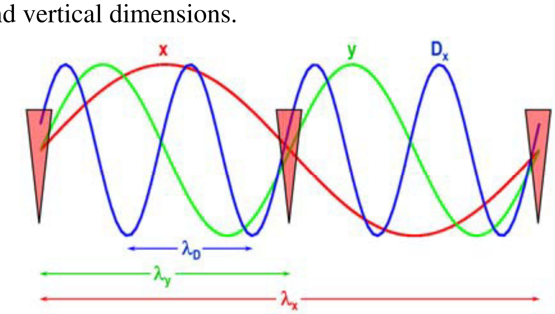

Figure 2: Particle's horizontal x and vertical y betatron trajectories and horizontal dispersion D_x for the case of correlated optics.

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Simulation

A twin-helix channel with wedge absorbers and energy-restoring RF cavities has been implemented [3] in G4beamline [4]. The simulation setup along with muon tracks is illustrated in Figure 3. The RF cavities are placed symmetrically between the absorbers. This allows for ease of compensation of a parasitic parametric resonance caused by the periodic energy modulation in correlated optics [5].

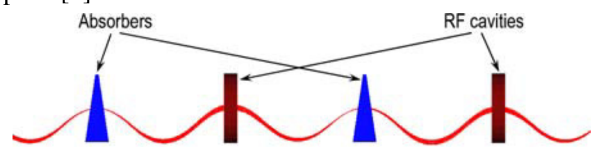

Figure 3: PIC simulation setup in G4beamline.

A parametric resonance is induced in each transverse plane by an additional pair of opposite-helicity but otherwise identical helical quadrupoles. The frequency of each pair is twice the frequency of betatron oscillations in the respective plane. The amplitude and phase of each pair are adjusted to provide appropriate parametric-resonance strength and focal point location, respectively. Figure 4 shows the resulting single-particle horizontal and vertical phase-space trajectories at the focal point without stochastic effects.

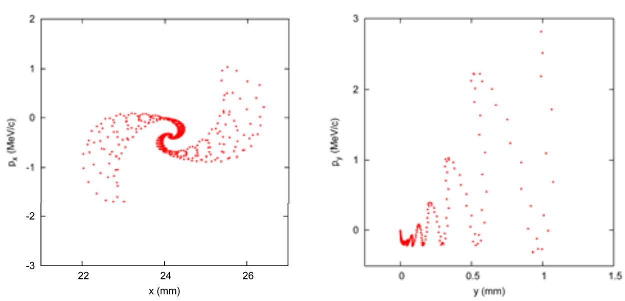

Figure 4: Horizontal (left) and vertical (right) phase-space trajectories of PIC.

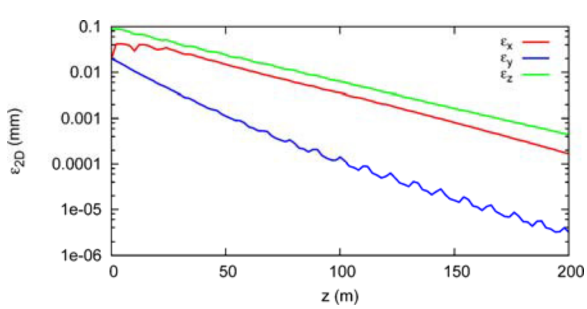

Figure 5: Horizontal ε_x, vertical ε_y, and longitudinal ε_z 2D emittances along the channel without stochastic effects.

Figure 5 shows evolution of the three 2D emittances along the cooling channel obtained in a PIC simulation using this setup. The initial emittance values are intentionally chosen relatively small to stay close to the linear regime to allow comparisons with the first-order simulations in [6]. Since the stochastic effects are off and the dynamics are stable, the emittances cool virtually to zero.

Aberration Impact

To proceed to cooling simulations including stochastic effects, compensation of beam aberrations is required. This was verified by running a first-order simulation with stochastics included [6]. Aberrations from one absorber to another must be compensated to a degree where they are small compared to the beam size at the absorber. Since the equilibrium angular spread in Table 1 is on the order of a hundred milliradians, the angle-dependent aberrations must be precisely compensated over the angular range of a few hundred milliradians. This is a challenging task.

Figure 6 demonstrates one example of aberration compensation. A set of particles with systematically-arranged initial angles is started from a focal point on the reference trajectory and tracked to the next focal point to determine the aberration-induced beam smear at that location. A general optimization procedure is used to minimize the beam smear by introducing various-order continuous multipole fields [3]. Using field harmonics up to octupole allows for compensation of an angular spread of up to ±260 mrad. A more systematic approach to aberration compensation using COSY Infinity has been investigated [6]. However, multipole fields in combination with correlated optics introduce another serious problem, namely, non-linear resonances causing loss of dynamical stability.

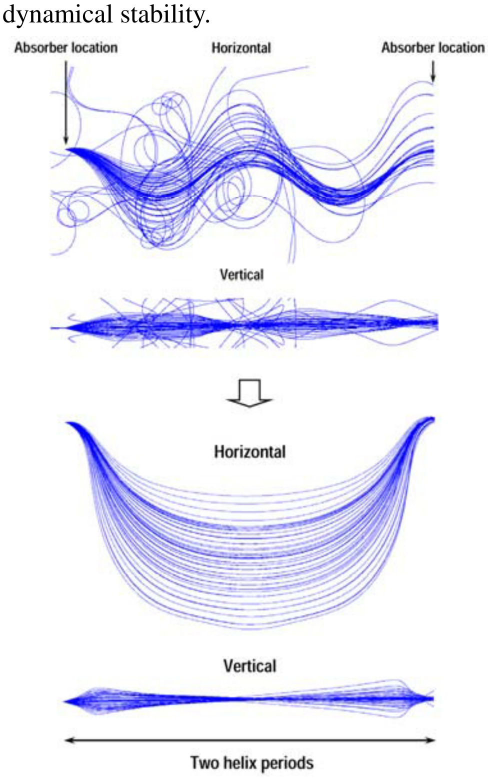

Figure 6: Demonstration of aberration compensation in a twin helix for horizontal and vertical angles of ±260 mrad: 250 MeV/c muon tracks are shown from one focal point to the next before (a) and after (b) aberration compensation using field harmonics up to octupole.

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To illustrate this problem, consider the Hamiltonian term of a continuous harmonically-varying octupole field $H_{oct} = n_{oct}(6x^2y^2 - x^4 - y^4)/4$ where $n_{oct} \sim \cos(2\pi mz/L)$ is the normalized octupole strength, m is an integer, z is the longitudinal coordinate, L is the channel period length, $x \sim \cos(2\pi\nu_x z/L)$ and $y \sim \cos(2\pi\nu_y z/L)$ are the horizontal and vertical transverse betatron coordinates, respectively, and ν_x and ν_y are the horizontal and vertical betatron tunes, respectively. Multiple octupole harmonics are needed in a cooling channel to compensate spherical aberrations. However, as can be clearly seen from the Hamiltonian, with our choice of betatron tunes of $\nu_x = 0.25$ and $\nu_y = 0.5$, any octupole harmonic m causes resonances in both planes. Dispersion further complicates the resonance structure. Selecting different betatron tunes does not help; as long as the betatron periods are integer multiples of the channel period as required by PIC, multipole fields will tend to cause non-linear resonance. This makes it difficult to find a set of multipoles sufficient for aberration compensation that does not cause beam instabilities.

SKEW PIC

Concept

To overcome the non-linear resonance problem, we developed the concept of Skew PIC [7, 8]. We introduce coupling in a cooling channel in such a way that the point-to-point focusing needed for PIC is preserved but the canonical betatron tunes are shifted from their resonant values, i.e. the canonical phase advances in the two planes are shifted from $m\pi$ values. A simple way to think of it is that the beam is azimuthally rotated between consecutive focal points. This moves the dispersion and betatron motion away from non-linear resonances. It also offers a number of other benefits:

- it allows for control of the dispersion size for chromatic compensation;
- it reduces the dimensionality of the aberration compensation problem to just the radial dimension and therefore reduces the number of required compensating multipoles;
- it equates the parametric resonance rates in the two planes, and therefore only one resonance harmonic is needed;
- it equates the two cooling decrements in the two transverse dimensions

Analytic Solution

Skew PIC is based on inducing a linear betatron coupling resonance in a PIC transport line between the horizontal (x) and vertical (y) planes. Previously developed twin-helix magnetic system is supplemented with skew quads that generate coupling between the horizontal and vertical betatron motions. Let us first consider the effect of such coupling using a simplified model. The equations for coupled betatron oscillations are

$$\begin{aligned} &x'' + [K^2(s) - n]x + g(s)y = K(s)\delta, \\ &y'' + ny + g(s)x = 0, \end{aligned} \tag{2}$$

where K is the curvature, n is the normalized straight quadrupole strength, g is the normalized skew quadrupole strength, and δ is the relative momentum offset. We find a solution with one-period linear transfer matrix of the form:

$$\begin{aligned} &\begin{pmatrix} x_f \\ y_f \\ x'_f \\ y'_f \end{pmatrix} = T \begin{pmatrix} x_i \\ y_i \\ x'_i \\ y'_i \end{pmatrix}, \quad T = \begin{pmatrix} M & 0 \\ L & N \end{pmatrix}, \\ &\det(T) = \det(M)\cdot\det(N) = 1, \end{aligned} \tag{3}$$

i.e., where the spatial coordinates are "decoupled" from the angular ones. Such a transformation preserves the point-to-point focusing feature of the correlated optics while not requiring resonant values of the betatron tunes as will be shown later. This solution does not violate the symplecticity requirement. It imposes three independent constraints. The criterion of linear motion stability is $\det(M) = \det(N) = 1$.

For a simplified model of step-like K and g functions illustrated in Figure 7, there exists an analytic solution of Eqs. (2) satisfying Eq. (3) [7]:

$$\begin{aligned} &T = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}, \quad M = N = \begin{pmatrix} \cos(4\theta) & -\sin(4\theta) \\ \sin(4\theta) & \cos(4\theta) \end{pmatrix}, \\ &\tan\theta = \frac{K^2 - 2n - \sqrt{(K^2 - 2n)^2 + 4g^2}}{2g}, \end{aligned} \tag{4}$$

The solution in Eq. (4) represents independent rotations in the spatial and angular phase spaces by an angle 4θ. Thus, we developed a system where the canonical variable pairs are (x, y) and (x', y') rather than the usual (x, x') and (y, y'). The eigenvalues of the transfer matrix in Eq. (4) are $\exp(\pm i4\theta)$. Therefore, the canonical betatron tunes are $\pm 2\theta/\pi$. Note that $\theta \to 0$ when $g \to 0$ (uncoupled case).

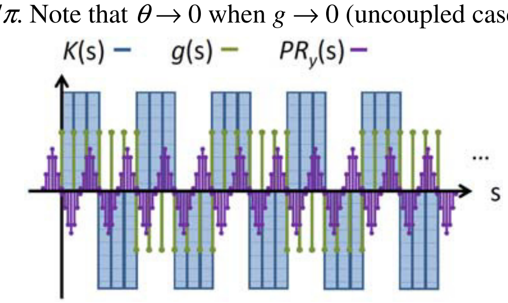

Figure 7: Curvature $K(s)$, skew quadrupole strength $g(s)$, and parametric-resonance harmonic $PR_y(s)$ along the cooling channel's length s.

Simulation

We verified the solution in Eq. (4) by numerically solving Eq. (2) [7] and by implementing the model shown in Figure 7 in MAD-X [8]. Tracking in MAD-X demonstrated the expected rotational behavior of the (x, y) and (x', y') coordinates as shown in Figure 8. The canonical tune values calculated by PTC in MAD-X also agreed with the analytic solution.

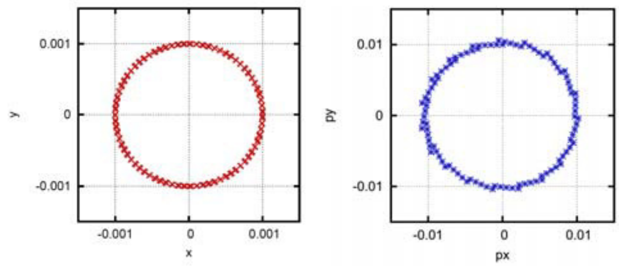

Figure 8: (x, y) and (x', y') phase-space trajectories obtained using PTC tracking in MAD-X.

We then tested the effect of a sextupole in PIC channels with *uncoupled* and *coupled correlated* optics. The radial displacement of a particle along each of the channels is shown in Figure 9. Clearly, there is a sextupole-induced non-linear resonance in the uncoupled case. In the skew channel, optics is still radially correlated; however, the betatron tunes are shifted from rational values and the resonance does not occur.

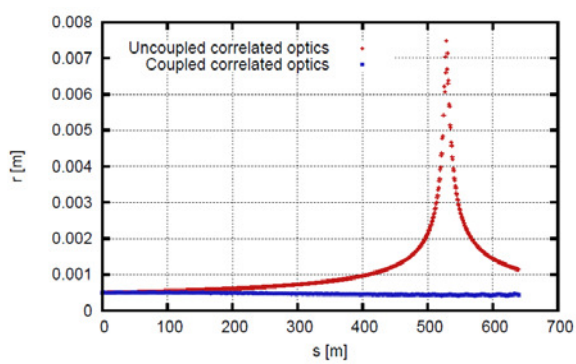

Figure 9: Radial displacement of a particle along PIC channel with uncoupled and coupled correlated optics in the presence of a sextupole harmonic.

We next induced a half-integer parametric resonance using a resonant quadrupole harmonic in the vertical plane as shown in Figure 7. We confirmed that a parametric resonance excited in one plane is equally distributed by the coupling resonance between the two transverse dimensions. Figure 10 shows the x and y coordinates of a particle at consecutive focal points along the channel. The two transverse coordinates are damping at the same rates. It is also quantitatively confirmed by the fact that the two transverse canonical betatron tunes have imaginary components of equal magnitudes.

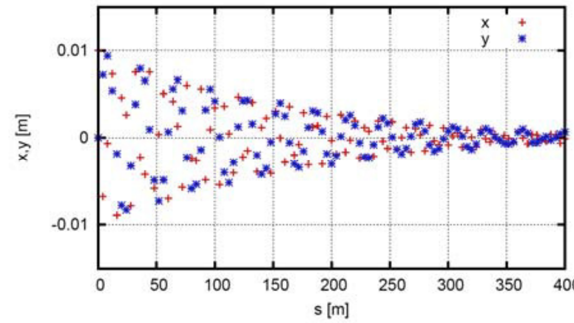

Figure 10: x and y coordinates of a particle at focal points along a skew PIC channel with a half-integer parametric resonance induced in one of the planes.

CONCLUSION

PIC combines muon ionization cooling with parametric resonant dynamics to allow final equilibrium transverse beam emittances that are an order of magnitude smaller than those achievable with conventional ionization cooling alone. Linearized PIC simulations with full stochastic effects are in good agreement with the analytic theory [6]. PIC has also been demonstrated in non-linear G4beamline simulations, however, with stochastic effects ignored [3]. A self-consistent simulation combining both stochastic and non-linear effects requires sufficient linearization of the beam dynamics through aberration compensation. Correction of aberrations in a transversely uncoupled channel has been challenging due to many non-linear resonances associated with correlated optics. We solve the problem by introducing a special type of coupling into the system that preserves the point-to-point focusing feature of correlated optics while shifting the canonical betatron tunes away from resonant values. Some of the expected features of Skew PIC were confirmed by numerical simulations [7, 8].

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