

# STOCHASTIC COOLING OF BUNCHED IONS SIMULATED IN THE TIME DOMAIN

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## Abstract

To include the influence of synchrotron oscillations, beam feedback and IBS a particle by particle and turn by turn treatment in the time-domain has been tried out. – Complete pickup, amplification and kicker characteristics, defined in the frequency domain, are introduced via inverse Laplace transformation, and so is the Dirac function representing the single ion. – The computation time is kept within reasonable limits thanks to the rule that cooling times scale proportionally with the particle number [1]. Typically 10000 simulation particles are cooled in 1200 turns. A recent proposal [2], based on signal binning and FFTs, offers the potential for at least one order of magnitude more ions and turns.

## INTRODUCTION

The usual arrangement of radial pickup, preamplifier, phase equalizer, power amplifier and longitudinal kicker is shown in Fig. 1 for momentum cooling.

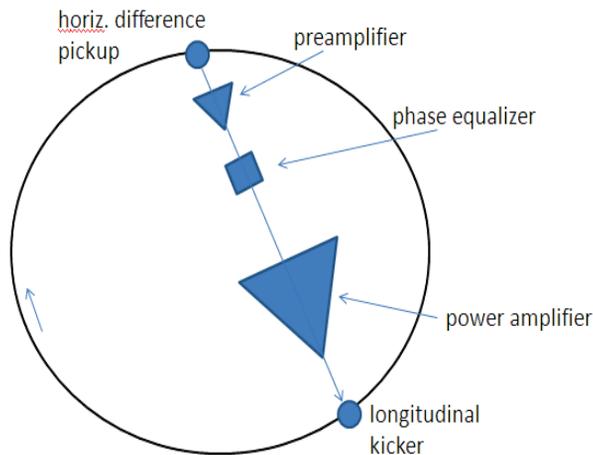


Figure 1: Palmer momentum cooling lay-out.

The method consists in cascading all loop components in the complex frequency plane and to perform the inverse Laplace transformation on the product of the cascaded elements, integrating over the frequency range (in our case 3 to 6 GHz). There are thus 6 elements, the first one being the Dirac function, representing the single particle. Optimum pickup and kicker loop geometries, both with the usual frequency dependency  $\sin(2\pi f/4.5e9)$  (amplitude) are included. - The system gain is defined as the fraction of the ion energy error (E) removed per turn (coherent effect and assuming no unwanted mixing). Considering the high charge/ion, the preamplifier noise is neglected and the amplification chain (benefiting from the phase equaliser) is assumed to have ideal phase. Due to the high momentum compaction factors ( $\eta$ ) of most low-energy ion rings, only a Palmer cooling set-up (based on radial pickups) with simultaneous betatron

cooling has been considered.

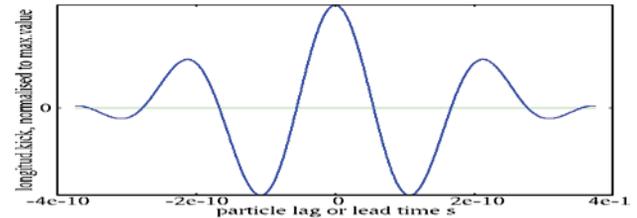


Figure 2: Single-particle correction pulse from an octave-band difference pickup obtained via the inverse Laplace transformation of the Dirac function followed by the 5 cooling components shown in Fig. 1.

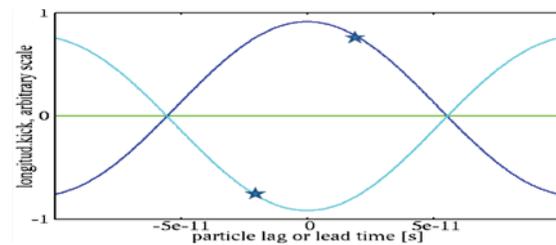


Figure 3: Expanded single particle coherent kick pulses for 3-6 GHz bandwidth. The dark blue curve represents the pulse created by a high energy ion grazing the outer pickup loop; this particle arrives in advance (with respect to the peak) by about 19 ps due to its high energy  $E=3\sigma E$ , whereas an ion with  $E= -3\sigma E$  grazing the inner loop (light blue curve) will arrive late by the same amount. The moments of correction are indicated by stars.

The correction that particle N exerts on itself via the system (coherent cooling effect) will be reduced by the unwanted mixing between pickup and kicker which is introduced as a time-lead or lag with respect to its passage at the kicker, depending on the particle energy, see Fig 3. Kicks from all other particles (incoherent heating effects) are also applied, taking into account their arrival times at the kicker. Furthermore, since we simulate step by step what actually happens in a real cooling process, the treatment inherently also includes:

- a) the beam feedback effect: change of Schottky signals when the feedback loop is closed (a potential source of instabilities).
- b) the synchrotron motion due to the bunching cavity: synchrotron resonances are not neglected.
- c) IBS, introduced via its rms kick value, using a randomly changing sign between particles and turns.
- d) betatron cooling results are obtainable for 2 separate distributions (2 betatron phase spaces), taking into account the evolving longitudinal distribution.

Example: cooling results for NICA

Circumference: 503.04 m  
 Gamma=4.76, particles: 1e9 ions, 197Au97  
 Initial rms dp/p = 0.00069 , eta = 0.02423  
 Initial rms length = 1.11e-9 s  
 H = 96 cavity voltage: 500 V peak

Time of flight pickup to kicker = 0.58 e-6 s  
 Local eta pickup to kicker = 0.014  
 Dispersion at pickup = 3 m  
 Band = 3 – 6 GHz, loop pickup and kicker

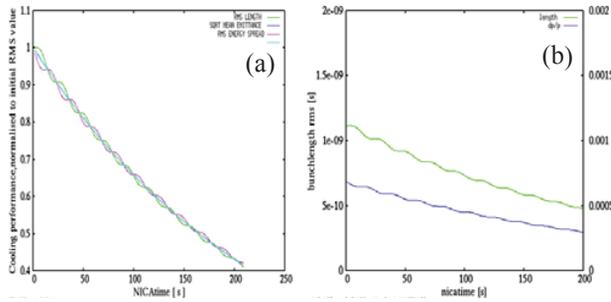


Figure 4: (a) rms bunch length, (b) rms energy spread and sqrt mean emittance reduction versus time, all normalised to initial. Small oscillations are due to synchrotron motion. - Only coherent cooling effects are accounted for. 10 000 sim. ions, gain -0.0016/turn. 1200 sim. turns.

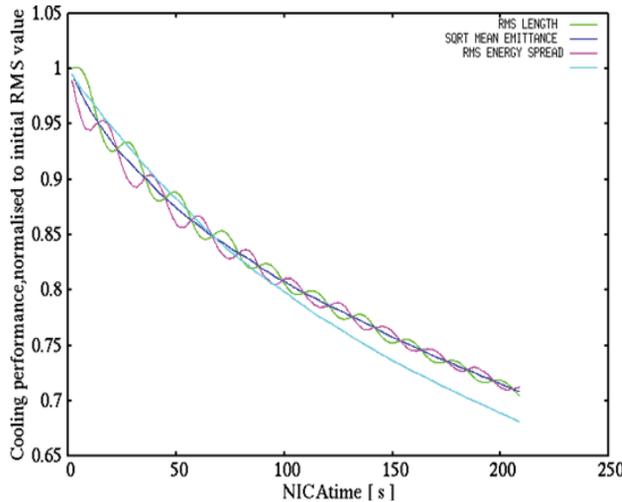


Figure 5: Same as Fig.3b but the incoh. heating effect is also included. Betatron rms amplitude decrease: light blue with gain also -0.0016/turn.

Increased Speed with Binning and FFTs

Evaluating for each particle the effects caused by all particles needs computation times proportional to  $N^2$  per turn ( $N = \text{nb. of sim. ions}$ ). In addition, the simulation (for a given case) requiring a number of turns in the ring proportional to  $N$ , the overall computation time becomes proportional to  $N^3$ . Typically 4 hours are needed with an HP 7900 pc for the case of Fig.5.

Following a suggestion by Daniel Schulte [2], shorter simulation times have been obtained by binning ( $M$  bins) of the pickup signal in time (at each turn), followed by an

FFT yielding  $M/2$  complex harmonic phasors in the freq. domain (also at each turn).

Then the cooling system features (such as pu & kicker responses, amplifier gains, equalizer, all in complex notation) are introduced by multiplication with the complex harmonic phasors (which implies that all phasors outside the feedback band are cancelled).

Subsequently an inverse FFT will deliver the complete kicker time pulse (blue in Fig. 6). This innovation yields cooling times proportional to  $N^2$  (instead of  $N^3$ ), a substantial gain of time, even if  $M > N$ . - The Fig.4 case is typically reduced to 10 mins computation time, seemingly with some small loss of precision. Figure 7 illustrates a simulation, like Fig. 5, with 1200 turns and 10 000 sim. ions.

At each turn, at the pickup, a sorting procedure places in chronological order the  $N$  particles in the  $M$  bins with a weight proportional to their horizontal deviation for  $dp/p$  and horiz. betatron cooling (see green background in Fig. 6) and with a weight proportional to their vertical deviation for vertical cooling.

There are typically  $M = 2 \times 14$  bins stretching over one bunching cavity period (~15 ns) leading via FFT to  $2 \times 13$  harmonic phasors.  $M$  should be chosen such that the bin length is less than 10 deg. for the highest frequency.

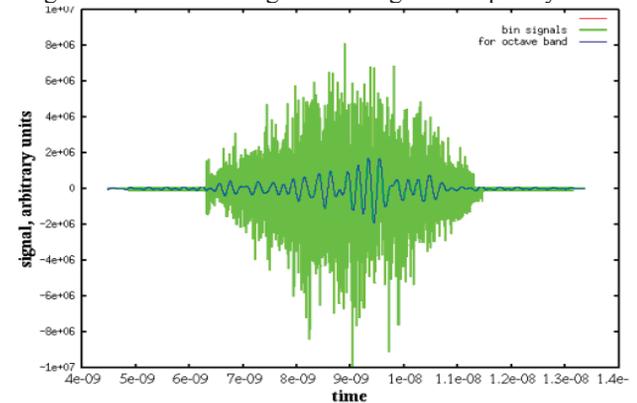


Figure 6: Bunch single passage Green background: signals in the bins, blue: resulting octave (3-6 GHz) stochastic correction signals experienced by passing ions.

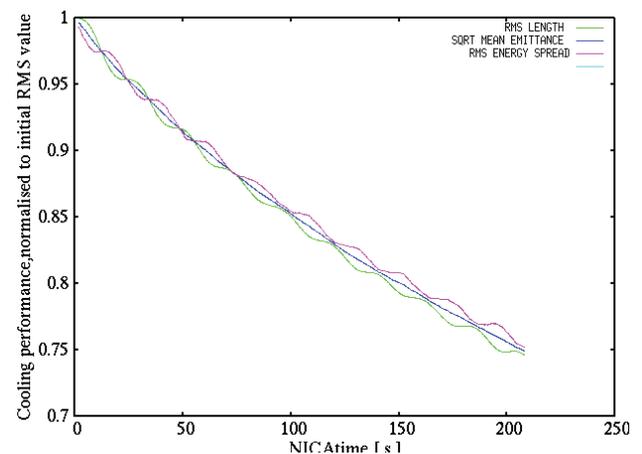


Figure 7: Binning:  $M = 2 \times 14$  bins,  $2 \times 13$  harmonics, 1200 turns, 10 000 sim. ions, gain = -0.0016/turn.

*Longitudinal Instability*

Increasing the sim. gain from -0.0016/turn by a factor 3 produces within 12 sim. turns (or 1.2e6 turns in NICA) a bunching instability, as shown in the longitudinal phase plane near midband (4.5 GHz, see Fig. 8). Note that the bunching pattern of the upper half (high energies) is in antiphase with the lower half (low energies); this is the pattern that produces high pickup signals favourable for the instability. Note also that particles pushed to extreme energies drift out of the phase conditions that drive the resonance.

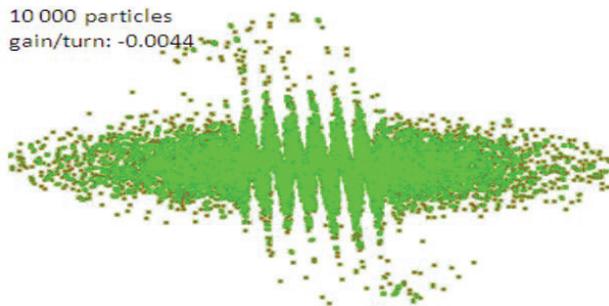


Figure 8: Longitud. instability, longitud. phase plane.

*Nyquist Diagram, Positive Feedback at the Bunch Centre*

The complex open-loop gain  $S$  (stable case of Fig. 5) has been obtained through longitudinal perturbation by the kicker at 100 frequencies (over a span of 1 rev. freq., centred on  $H=7856$ , the system midband). The system response from all particles is the mean value over 1000 turns (~30 synchrotron oscillations, to average away Schottky signals).

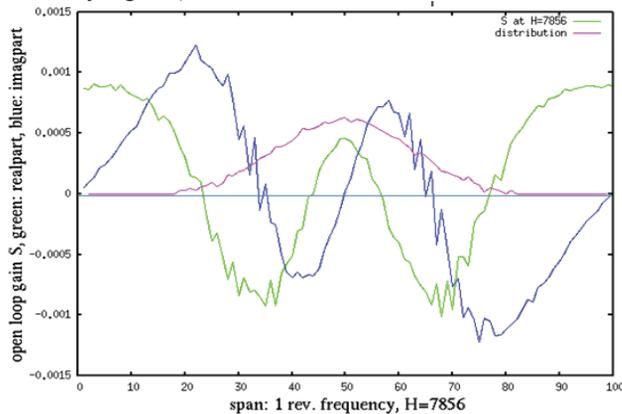


Figure 9: Shows real: green and imag: blue terms of  $S$ , using the pu in the position foreseen for NICA (the unwanted mixing is included). Red curve: bunch time-distribution, stable case, gain -0.0016.

Note that there is positive feedback in the bunch centre and half-way between harmonics, caused by the modulating bunching. But open-loop gains not exceeding 0.003 (extrapolated value to above unstable case, Fig.8), cannot alone explain the observed instability. - If however something like 300 harmonics (out of the 5200 of the band) were involved, somewhat coherently, then an

instability may turn up? Obviously, Landau damping would make signals decohere over a number of turns depending on eta.

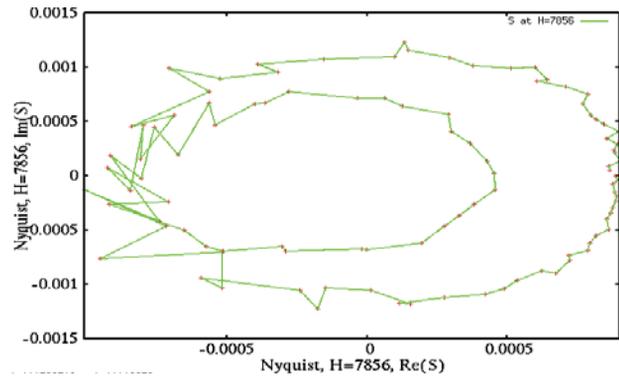


Figure 10: Nyquist diagram, stable case, gain -0.0016.

For very long bunches the coasting-beam conditions are approached; there is no positive feedback at the bunch centre.

*Conclusions*

- a) Stoch. momentum cooling simulations in the time-domain of bunches including synchrotron resonances is possible over a few thousand turns evaluating for each ion, at each turn, the effects from all ions via the feedback system.
- b) Cooling of betatron oscillations, taking into account the shrinking longitudinal emittance, can easily be part of the simulation.
- c) Binning of the cooling signals permits the use of at least one order of magnitude more simulation ions and simulation turns.
- d) Further work remains to understand the instability: could lack of smoothness of the initial distribution be another explanation? Or is this instability not real at all?
- e) A generalised definition of  $S$ , accounting for the intermodulation effects at all rev. harmonic bands, may be useful.

**ACKNOWLEDGEMENTS**

Thanks to Takeshi Katayama for the NICA cooling case [3] used for testing of the method and for his extensive advice and explanations during the numerical experimentations.

Daniel Schultes binning proposal [2], making simulations over more than 10 000 turns feasible, should be particularly useful for future investigations. Dieter Moehl helped with the understanding of the bucket dynamics during stoch. cooling. The study was supported by JINR.

**REFERENCES**

- [1] M. Blaskiewicz and J. M. Brennan, "Bunched beam stochastic cooling in a collider", Physical Review special topics-Accelerators and beams 10, 061001 (2007)
- [2] Daniel Schulte, private communication, March 2013.
- [3] Takeshi Katayama, private communication, April 2011