BEAM CRYSTALLIZATION — ARE WE THERE YET?*

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Abstract

A brief review is made of Coulomb crystallization of a charged particle beam circulating in a storage ring at high speed. An ideal crystalline state is reached when the beam is cooled to near the absolute zero temperature. The corresponding emittance is also nearly zero, which means that the crystalline beam has the highest quality achievable in principle. Through past theoretical studies, it has been revealed that beam crystallization is feasible only in a storage ring that satisfies several physical conditions. This paper summarizes those necessary conditions and illustrates why they are so important in establishing the ultimate state of a beam.

INTRODUCTION

In any practical applications of particle beams, we certainly care about the beam quality, or in other words, the *emittance* that represents the phase-space volume occupied by the beam. A beam has better quality and is thus more useful as the emittance becomes smaller. Since the emittance cannot be negative, its ultimate limit is zero. An interesting question is whether such an ultimate state is physically allowable. If it is, we might raise more questions including "what conditions are required to stabilize the zero-emittance state?", "how can we reach there in practice?", etc.

To the best of the author's knowledge, the phase transition of a charged particle beam toward an ultralow emittance state was first discussed by Russian researchers when they tried to explain an anomalous behavior of electron-cooled ion beams in the NAP-M storage ring [1]. Later, John Schiffer and his co-workers performed systematic molecular dynamics (MD) simulations demonstrating that a one-component plasma confined by a uniform external restoring force can form a spatially ordered structure at very low temperature [2]. This seminal work was followed by further MD studies by Wei, Li, and Sessler who incorporated effects from realistic alternating-gradient (AG) lattice structures of modern accelerators into their simulations [3]. It is now believed that the periodic and dispersive nature of a cooler storage ring imposes severe restrictions upon the realizability of beam crystallization [3, 4]. In fact, nobody has succeeded in generating a crystalline beam while very low-energy, moving Coulomb crystals were produced in a tabletop circular Paul trap [5].

This paper focuses on the dynamics of crystalline beams, outlining past theoretical progress. After clarifying the definition of beam crystallization, we show several conditions essential to establish such an ultimate low-emittance state. We then consider possible cooling schemes toward beam crystallization.

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COULOMB CRYSTALS

In Paul traps [6], it is straightforward to make a variety of Coulomb crystals by employing the laser cooling technique. Figure 1 shows the fluorescence images of actual Coulomb crystals produced in a compact linear Paul trap at Hiroshima University. Each bright spot corresponds to a single $^{40}Ca^{+}$ ion Doppler cooled by a semi-conductor laser system to a mK range. The upper panel is a picture of the so-called *string* crystal where cooled ions are aligned along the trap axis at almost equal intervals. By adding more ions, this simple string formation converts into a *zigzag* structure. Above a certain density threshold, the zigzag formation is eventually transformed to a *shell* structure as shown in the lower panel. The number of the ion shells increases as the line density becomes higher. The structural transitions of infinitely long Coulomb crystals can be well explained by the Hasse-Schiffer theory [7].

The phase transition of a one-component plasma is often characterized by the Coulomb coupling constant defined as the ratio of the average Coulomb potential energy to the thermal energy:

$$\Gamma = \frac{q^2}{4\pi\varepsilon_0 d} \cdot \frac{1}{k_B T}, \tag{1}$$

where q and $2d$ are the charge state and the average distance of particles, k_B is the Boltzmann constant, and T denotes the plasma temperature [8]. Regular *gaseous*

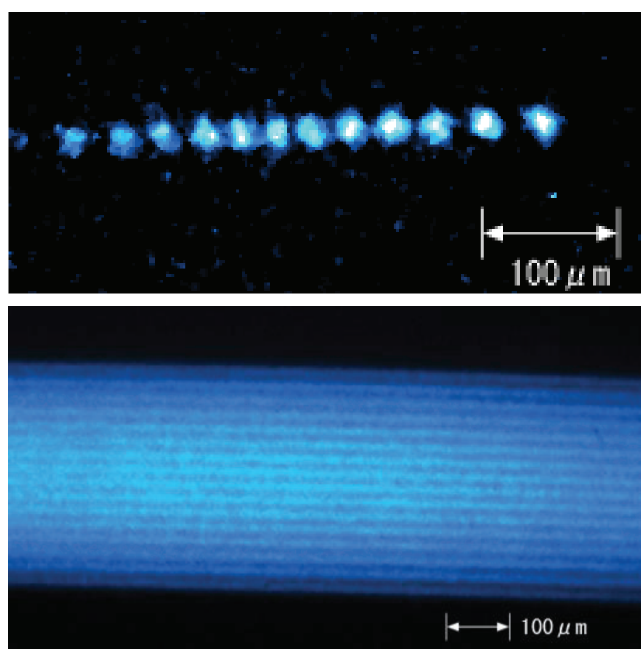

Figure 1: Fluorescence images of a string (upper) and multi-shell (lower) Coulomb crystals produced in a linear Paul trap by Doppler laser cooling.

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beams in regular accelerators have Γ values much smaller than unity. A phase transition to the *liquid* state occurs when Γ approaches unity by cooling. In a *solid* (crystalline) state, Γ exceeds 170.

Note that the temperature is not the simple average of particles' kinetic energies. When the particle ensemble is exposed to time-dependent external potentials just like a beam in an AG machine, we should be careful in defining T. At high temperature, there is no problem to equate T with the average of squared kinetic momenta over all particles. At ultralow temperature, such a definition becomes irrelevant due to strong coherence in the kinetic motions of the particles. Since the temperature is a measure of *random* motions, the contribution from the *coherent* beam oscillation to the kinetic energy must be subtracted before evaluating T. Otherwise, Γ cannot be large even in a perfect crystalline state because the periodic breathing oscillation driven by an AG lattice possesses a significant energy. For a reasonable and quick estimate of beam temperature, it is convenient to use the concept of root-mean-squared (rms) emittance [9]. For instance, the transverse beam temperature $T_\perp$ can be evaluated from $k_B T_\perp = (p_0^2/8m)(\varepsilon_\perp/a)^2$, where m is the particle mass, p_0 is the kinetic momentum of the reference particle, a is the rms beam size, and $\varepsilon_\perp$ is the transverse rms emittance.

CRYSTALLINE BEAMS

Crystal Orbit

Needless to say, the dynamic nature of a crystalline beam is very different from that of an ordinary high-temperature beam. Each particle no longer executes the regular betatron and synchrotron oscillations in a crystalline state. Instead, all particles coherently oscillate at the same frequency and at the same phase. The oscillation period perfectly coincides with a unit lattice period. In order to maintain a spatially ordered structure, the transverse trajectories of individual particles along the design closed orbit must be proportional to each other; the horizontal and vertical coordinates (x, y) of any particles forming a coasting crystalline beam can be expressed as

$$(x, y) = (C_x D_x, C_y D_y), \tag{2}$$

where the coefficients $C_{x(y)}$ are particle-dependent constants while the periodic functions $D_{x(y)}$ are universal among all particles and satisfy [10]

$$\begin{cases} \dfrac{d^2 D_x}{ds^2} + K_x(s) D_x - \dfrac{\zeta}{D_x + D_y} = \dfrac{1}{\rho}, \\ \dfrac{d^2 D_y}{ds^2} + K_y(s) D_y - \dfrac{\zeta}{D_x + D_y} = 0. \end{cases} \tag{3}$$

Here, $K_{x(y)}$ are the beam focusing functions determined by the lattice design, ρ is the local orbit curvature, s is the path length along the reference orbit, and ζ is a constant parameter related to the beam perveance and average momentum spread. According to the author's past experience, these equations are applicable even for bunched crystalline beams.

Equation (2) indicates that the ratio of the coordinate $x(y)$ and the angle dx/ds (dy/ds) is independent of which particle we choose. All particles are located on a straight line in both horizontal and vertical phase planes. The transverse emittance $\varepsilon_\perp$ is thus exactly equal to zero in an ideal crystalline state except for quantum fluctuation. Since $\varepsilon_\perp = 0$, the temperature $T_\perp$ is also zero by definition.

Stability

If the external beam focusing force is spatially uniform and time-independent, we have no trouble stabilizing Coulomb crystals. In reality, however, a beam in a storage ring is exposed to periodic forces depending on the magnet arrangement. Furthermore, the beam orbit is not linear but closed. Two lattice conditions must then be met to ensure the stability of crystalline beams:

- The betatron phase advance per lattice period should be below 90 degrees [3, 11].
- The ring has to operate below its transition energy [3].

The first condition is required to avoid crossing a dangerous, linear coherent resonance during a cooling process toward a crystalline state. If the phase advance over a unit lattice exceeds 90 degrees, the operating point of the ring inevitably encounters the linear instability band at which the beam is strongly heated. This condition is essential also in a linear Paul trap employing a periodic field for particle confinement. The second condition is peculiar to circular machines. Once a crystalline state is reached, all particles clearly have the same revolution frequency and their orbits never intersect. A particle traveling along a radially outer orbit must run slightly faster than inner particles whenever the crystal has a finite horizontal extent. Above the transition energy, this requirement cannot be met because a higher-energy particle has a longer revolution period.

The second condition is usually fulfilled in most cooler rings operating at relatively low energy. On the other hand, the betatron phase advance of a storage ring is often much greater than 90 degrees per lattice period. For example, the ASTRID ring where Doppler cooling experiments aiming at beam crystallization used to be done in 1990's has the superperiodic number of 4 [12]. Since its typical operating tune was over 2, the tune per single period exceeds 0.5 corresponding to the phase advance of above 180 degrees. The experimental observation reported by Madsen et al. has actually pointed out the importance of minimizing low-order resonance-induced heating [13]. The situation is even worse in the TSR ring [14], that is, another storage ring equipped with a laser cooling system. This is, however, just a matter of lattice design; it is always possible to construct a storage ring satisfying both conditions above. A real issue is how to create a proper cooling force that guarantees an ultralow temperature and the stability of crystalline states.

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COOLING CONDITIONS

Coulomb crystallization has been realized in ion traps all over the world. By contrast, crystallizing a fast circulating beam in a storage ring is far more difficult to achieve in practice due to several technical reasons. In the past, two European groups tried to make a Coulomb crystal in their storage rings (TSR and ASTRID) applying the state-of-the-art Doppler laser cooling technique, but unfortunately, one of the two lattice conditions were completely broken which made it impossible to avoid the strong transverse instability of the linear collective mode [15]. It is interesting to ask what would happen if these rings satisfied both conditions. The most likely answer to this question is that it is still quite difficult to reach a crystalline state. Although the string or zigzag structure could be formed with minor modification to the lattice [16], we have no chance to establish a stable shell crystal. There are a couple of reasons for that, both of which relate to the cooling conditions necessary for crystal formation.

Shear Force

The most essential difference between a storage-ring system and a plasma-trap system is whether or not momentum dispersion exists. We always need dipole fields in any ring to make the beam orbit closed, which inevitably causes the dispersive effect; the local curvature of each individual particle depends on its kinetic momentum. Figure 2 illustrates the orbits of two particles (A and B) picked from a crystalline beam with a finite horizontal extent. As already explained in the last section, the average longitudinal velocity of Particle A must be slightly higher than that of Particle B. If the two particles are moving at the same speed, the outer one A gets gradually behind the inner one B every turn and the resultant shear force destroys the crystalline order. This suggests that for the stabilization of a crystalline beam, we need such a special cooling force as to give larger longitudinal velocities to radially outer particles in the final equilibrium state. This is often referred to as *tapered cooling* [4, 17]. It is impossible, according to past MD simulations [18], to generate stable shell crystalline beams without a properly tapered cooling force.

A simple way to *taper* the Doppler cooling force has been proposed by the TSR group of the Max Planck Institute in Heidelberg [19]. Provided that the distribution of laser photons is not uniform but has a spatial gradient, a weak tapered force becomes available by slightly displacing the laser axis from the beam orbit. The tapered force can be enhanced to some degree by the use of two counter-propagating, displaced lasers [20]. It is, however, very difficult in practice to adjust the so-called *tapering factor* to the optimum value [17]. An alternative remedy against the shear heating is to employ the *dispersion-free bending element* where an electric dipole field is superimposed on a magnetic dipole [21]. This unique element is usable unless the beam energy is too high.

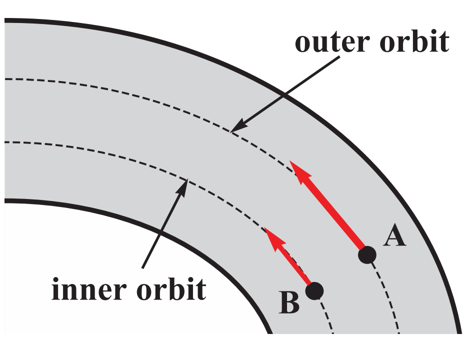

Figure 2: Particle orbits in a bending region.

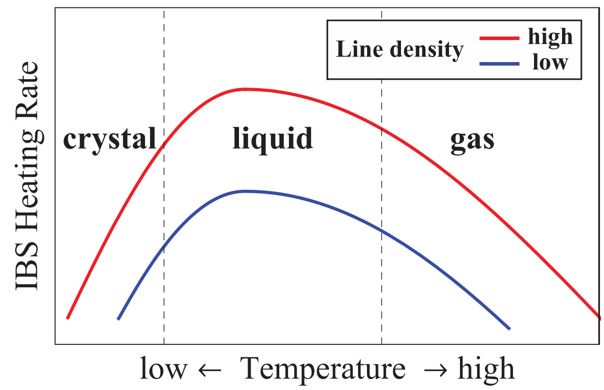

Figure 3: IBS heating rate vs. beam temperature.

Cooling vs. Coulomb Collisions

As the phase-space density of a beam rises due to cooling, the rate of Coulomb collisions among individual particles naturally becomes higher. Conventional theories of intra-beam scattering (IBS) predict that the emittance-growth rate monotonically increases at higher beam density [22]. This is true only for regular gaseous beams whose temperature is high. Systematic MD simulations have revealed an interesting behavior of the IBS heating rate as depicted in Fig. 3 [4]. The collisional heating is most severe in the liquid phase where the Coulomb energy is comparable to the thermal energy; namely, $\Gamma \approx 1$. In the ultralow-temperature regime, IBS starts to be suppressed and eventually disappears when a crystalline state is reached. Therefore, if the cooling rate is higher than the peak IBS heating rate, the beam will encounter no serious obstacle toward crystallization except for the shear heating. In case we fail to compensate for the dispersive effect, the final equilibrium temperature is determined by the balance between the shear heating and external cooling forces. Similarly, if the cooling power is not high enough to overcome the peak in Fig. 3, the temperature reduction stops when the operating point hits the high-temperature side of the heating-rate mountain.

It is worthy to emphasize that the mountain-like curve in Fig. 3 is obtained only in a storage ring that satisfies the stability conditions discussed above. While we have no problem designing such a machine, any actual rings contain finite mechanical errors. The lattice periodicity is, therefore, more or less broken depending on how carefully we constructed the machine. When the lattice

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symmetry breakdown is strong, the heating rate stays at a high level even near the left end of Fig. 3, which means that crystallization is not achievable any more [23]. It is very important to maintain the original lattice symmetry as precisely as possible, so that the betatron phase advance per lattice period can approximately be kept low.

POSSIBLE COOLING SCHEMES

If there is an extremely efficient cooling method that can almost instantly reduce the beam temperature to near the absolute zero, we no longer need a storage ring but a linear cooling channel might suffice for our purpose. Then, the major beam instability sources described in the last section just disappear. The system has no transition energy and probably no severe resonance (because the cooling channel is too short to excite resonances). Most importantly, we do not have to worry about the shear heating. No such ideal cooling method has, however, been invented so far.

A beam must be cooled very close to the absolute zero in order to achieve crystallization. Among modern cooling techniques, laser cooling is presently the only choice that meets this essential requirement. The Doppler cooling limit is typically in a mK range or even below. One practical problem of laser cooling, when it is applied to fast circulating beams, is that the radiation pressure operates only in the longitudinal direction of beam motion. It is, of course, possible to apply the laser light from the transverse directions, but the cooling efficiency cannot be high because of a very poor overlap between the beam line and the laser. Although we can expect some sympathetic transverse cooling via Coulomb collisions [24], such an indirect effect is not only weak but also beam-intensity dependent.

Several simple methods have been proposed to extend the powerful longitudinal Doppler cooling force to the transverse degrees of freedom [17, 25-27]. The so-called *resonant coupling method* (RCM) is particularly easy to implement in practice [25, 26]. All we need is to introduce linear synchro-betatron and betatron-betatron coupling sources into the ring, and then, drive the operating point onto linear difference resonances:

$$\nu_H - \nu_V \approx \text{integer}, \quad \nu_H - \nu_L \approx \text{integer}, \tag{4}$$

where ν_H, ν_V, and ν_L are the horizontal, vertical and longitudinal tunes of the ring. Linear synchro-betatron coupling can readily be developed either by a regular radio-frequency (RF) cavity sitting at a dispersive position [26] or by a special coupling RF cavity excited in a deflective mode [25]. For linear horizontal-vertical coupling, we can use a skew quadrupole magnet or solenoid. Recently, the RCM was employed in the cooler ring "S-LSR" [28] and successfully generated ultracold ion beams whose temperature is below a few K in all three dimensions [29]. The use of a Wien filter in a laser cooling section is another simple solution to artificially enhance the transverse cooling efficiency [27]. This scheme is applicable, unlike the RCM, to coasting beams because it is free from the synchro-betatron resonance condition in Eq. (4). The tapered cooling considered above for dispersion compensation is also a promising means toward beam crystallization [17].

CONCLUDING REMARKS

In spite of past extensive efforts on beam cooling, nobody has yet reached a crystalline state in a storage ring. Although several very interesting reports have been made regarding an anomalous behavior of Schottky signals from electron-cooled hadron beams [30-32], the observed "one-dimensional (1D) ordering" should be distinguished from the formation of a crystalline string. In fact, this unique phenomenon is known to take place only at extremely low line density. The average spacing between neighboring ions are on the order of centimeters or even greater while the final beam temperature exceeds ~1K. Each particle is moving back and forth longitudinally in-between the potential barriers created by its neighbors [33, 34]. The coupling constant in Eq. (1) is then on the order of 1 at most, which suggests that 1D ordering should not be categorized as Coulomb crystallization by definition [35].

Figure 4 is a conceptual sketch of a cooler storage ring intended for ultracold beam production. The ring has very high lattice symmetry (superperiodicity = 10), but even higher symmetry is preferred for crystal stability. The bending magnets can be switched to the dispersion-free mode by inserting dipole electrodes [21]. Since the betatron phase advance must be low to suppress the linear coherent resonance, we do not need too many quadrupole magnets. In the present example, the bare tunes around the ring should be smaller than $(1/4)\times 10 = 2.5$ in both transverse directions. Reducing the number of lattice elements is advantageous also from a practical point of view because inevitable field imperfections and

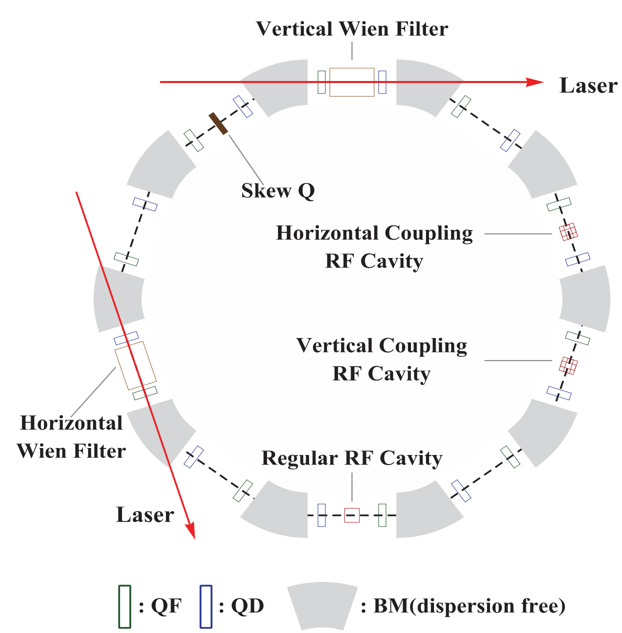

Figure 4: A possible storage ring for ultracold ion-beam generation.

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misalignments of those elements always become possible beam-heating sources. Furthermore, fewer magnets leave us with wider spaces for the installation of other electromagnetic devices such as the coupling RF cavities, Wien filters, skew quadrupoles or solenoids, etc.

We have no other choice than the Doppler laser cooling in order to challenge beam crystallization. $^{24}Mg^{+}$ ion is a good candidate for laser cooling. Ideally, cooling should be carried out in all straight sections. At least two frequency-tunable lasers, one co-propagating and the other counter-propagating with a beam, are desirable to improve the cooling efficiency (and to allow several experimental options). It may be useful to have an electron cooler as well for precooling initial hot beams. As long as we rely on the Doppler cooling technique, the RCM and/or other schemes should be employed to ensure sufficiently high transverse cooling efficiency that is indispensable for the beam to go over the IBS heating-rate mountain. It is thus beneficial to introduce as many independent coupling sources as possible, so that we can try a variety of indirect transverse cooling schemes. Note that regular RF cavities are not usable as a synchro-betatron coupling source when the ring is operated in the dispersion-free mode. In that case, the special coupling RF cavities must be turned on. In addition to these cavities, Wien filters are installed in laser-cooling sections.

Past MD simulations have demonstrated the feasibility of generating string or zigzag crystalline beams in a properly designed storage ring with linear coupling sources [16, 18, 36]. It is, however, practically difficult to form a stable multi-shell crystalline structure even in such an elaborate ring as illustrated in Fig. 4. For perfect stabilization of a shell crystal, it is very important to hold high lattice symmetry including coupling sources and even cooling forces [18, 36].

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